**1** Obtain the maximum likekihood estimate of the value of a constant $x$ based on $N$ noisy measurements $z_i = x + v_i$, where $v_i$ is a sequence of independent zero mean Gaussian random variables with variance $\sigma^2$. Also obtain the variance of the estimation error.

**2** Consider a sensor that provides measuments at a rate $1/T$. Measurement errors follow a normal distribution with zero mean and variance $\sigma^2$. Also assume that measurement errors are independent.

**(a)** This sensor is used to estimate a constant value. Considering the maximum likelihood estimator, determine the variance of the estimation error after one unit of time.

**(b)** Consider now thar you have another sensor that is 10 times slower. Determine the variance of the measurement errors associated to this sensor that ensures the same variance of the estimation error after one unit of time.

**3** Consider a body moving on a vertical plane and subject to a downward acceleration $g$. The horizontal and vertical coordinates of the body center of mass are denoted by $x$ and $y$ respectively.

**(a)** Consider that the motion in sampled with a period $T$ and that in each sampling interval there are zero mean random accelerations acting in each coordinate (with variances $\sigma_a$), obtain a discrete time representation of the process in a vector form.

**(b)** Also consider that the 2D body coordinates are measured with zero mean observation errors with variances $\sigma_o$. Relate the observations with state variables and observation errors.

**(c)** A Kalman filter is used to estimate the evolution of the body, based on the defined observations. Determine the equations that that provide the evolution of the state estimate and the covariance of the estimation error.

**(d)** Implement the Kalman filter in MATLAB/OCTAVE/Python. Consider the following data: $g = 1$, $T = 0.1$, $\sigma_a = 0.04$, $\sigma_o = 0.2$.

**(e)** Use the data in file `data.txt` to test the filter (the columns are $t, x, v_x, y, v_y, x_m, y_m$, where $t$ is time, $x, v_x, y, v_y$ are the state of the body (not accessible to the filter), and $x_m, y_m$ are the measured coordinates. Try different values for the initial estimate of the body position and estimation error covariance.

**(f)** Consider now that for $10 < t < 18$ measurements are not available. Adapt your filter to take that into account and test whether the filter is still able to track the body. Try different intervals of measurements unavailability.