

- 1 Consider the process $x_{k+1} = 0.5x_k + w_k$, where $\{w_k\}$ is a white gaussian sequence with zero mean and variance σ^2 , and x_0 is a null constant.
 - (a) Obtain the equation for the evolution of the variance of x_k .
 - (b) Determine the steady state variance.
 - (c) Use MATLAB/Python to obtain realizations of the process x_k $\sigma^2 = 1$. Compare in each case the corresponding realizations of x_k and w_k .

- 2 Consider the process $x_{k+2} = \frac{a}{2}x_k + \frac{a}{2}x_{k+1} + (1-a)w_k$, where $a \neq 1$ is constant and $\{w_k\}$ is a white gaussian sequence with zero mean and variance σ^2 , and x_0 and x_1 are null constants.
 - (a) Rewrite the process in a vector form, taking $X_k = [x_k \quad x_{k+1}]^T$.
 - (b) Conclude that X_k has zero mean.
 - (c) Obtain the equation for the evolution of the covariance of X_k .
 - (d) Determine the steady state covariance matrix.
 - (e) Use MATLAB/Python to obtain realizations of the process X_k for $a = 0.1$ and for $a = 0.95$, when $\sigma^2 = 0.01$. Compare in each case the corresponding realizations of x_k and w_k .

- 3 Consider the process defined in the previous problem, with $a = 0.9$.
 - (a) Compute the covariance matrices of X_{50}, X_{100}, X_{500} . Also obtain the steady state covariance.
 - (b) Implement a Monte Carlo simulation for the evolution of the process.
 - (c) Perform a set of simulations to estimate the covariance matrices of X_{50}, X_{100}, X_{500} . Consider different numbers of runs and present the results.

- 4 Consider the linear motion of a point mass given by $\ddot{x} = a$
 - (a) Taking the position x and the velocity \dot{x} of the point mass as state variables and the acceleration a as the input, obtain a state space model.
 - (b) Assume the acceleration is constant for intervals of width T . Obtain a discrete time equivalent model, taking T as the sampling period.
 - (c) Assume now that the acceleration measurement in each interval is corrupted by an error that follows a normal distribution with zero mean and variance σ_a^2 , and that such errors for different intervals are independent.
 - i. Obtain a linear model in discrete time with a random input that describes such behavior.
 - ii. Obtain the equations that give the mean and the covariance of the state.
 - iii. Taking $T = 0.1$, $a_k = 0$ and $\sigma_a^2 = 0.01$ and assuming the system is initially at rest, use MATLAB/Python to obtain several realizations of the system evolution. Use Monte Carlo simulation estimate the expected value and covariance of x_{100} . Compare the results with the exact value of such covariance.