CONFOUNDING

$\mathbf{D}^{(1)}$: $n^{(1)}$ samples of $[X, Y]^\top$

$$[X, Y]^\top \sim \text{Categorical}(\vec{\theta}_{XY}), \text{ where } \vec{\theta}_{XY} = [\Pr(X = x, Y = y)]^\top$$

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$$p(\vec{\theta}_{XY}|\mathbf{D}^{(1)}) \sim \text{Dirichlet}\left(\#\{\mathbf{D}^{(1)}\} + \mathbf{1}\right)$$

$$\#\{\mathbf{C}^{(1)}\} \sim \text{Multinomial}\left(\vec{\theta}_{XY}, m^{(1)}\right)$$

$$\#\{\mathbf{C}^{(2)}\} \sim \text{Multinomial}\left(\vec{\theta}_{\mathrm{W}}, \mathrm{m}^{(2)}\right)$$

$$p\left(\vec{\theta}_{\vec{C}^{(t)}} \middle| \mathbf{D}^{(1)}, \ldots, \mathbf{D}^{(S)}\right)$$

$$\#\{\mathbf{C}^{(t)}\} \sim \text{Multinomial}\left(\vec{\theta}_{\vec{C}^{(t)}}, m^{(t)}\right)$$

$$[\underline{\tau}, \overline{\tau}]^\top = \text{Autobounds}(\vec{\theta}_{\text{XY}})$$

$$p(\underline{\tau}, \overline{\tau} \mid \mathbf{D}^{(1)}) = \int \delta \left\{ \text{Autobounds}(\vec{\theta}_{XY}) - [\underline{\tau}, \overline{\tau}]^\top \right\} p(\vec{\theta}_{XY} \mid \mathbf{D}^{(1)}) d\vec{\theta}_{XY}$$

$$\mathbb{E}_{\mathbf{C}^{(1)}}\left[\text{width of Autobounds}_{\alpha=95\%}\left(\mathbf{D}^{(1)} \cup \mathbf{C}^{(1)}\right) \mid \mathbf{D}^{(1)}\right]$$

$$\arg\min_t \mathbb{E}_{\mathbf{C}^{(t)}} \left[\text{width of Autobounds}_{\alpha=95\%} \left(\mathbf{D}^{(1)} \cup \mathbf{C}^{(t)} \right) \mid \mathbf{D}^{(1)} \right]$$

$$\arg\min_t \mathbb{E}_{\mathbf{C}^{(t)}} \left[\text{width of Autobounds}_{\alpha=95\%} \left(\mathbf{D}^{(1)} \cup \ldots \cup \mathbf{D}^{(S)} \cup \mathbf{C}^{(t)} \right) \mid \mathbf{D}^{(1)}, \ldots, \mathbf{D}^{(S)} \right]$$

$$\iint \left| \text{Autobounds}_{\alpha=95\%}\left(\mathbf{D}^{(1)} \cup \mathbf{C}^{(1)}\right) \right| \mathrm{p}(\mathbf{c}^{(1)} | \vec{\theta}_{XY})\, \mathrm{p}(\vec{\theta}_{XY} \mid \mathbf{D}^{(1)})\, \mathrm{d}\mathbf{c}^{(1)} \mathrm{d}\vec{\theta}_{XY}$$

$$\iint \left|\text{Autobounds}_{a=95\%}\left(\mathbf{D}^{(1)} \cup \ldots \mathbf{D}^{(S)} \cup \mathbf{C}^{(t)}\right)\right| \mathrm{p}(\mathbf{c}^{(t)} | \vec{\theta}_{\vec{C}^{(t)}})\, \mathrm{p}(\vec{\theta}_{\vec{C}^{(t)}} \mid \mathbf{D}^{(1)}, \ldots, \mathbf{D}^{(S)})\, \mathrm{d}\mathbf{c}^{(t)} \mathrm{d}\vec{\theta}_{\vec{C}^{(t)}}$$