Evaluating the Validity and Robustness of Instrumental-Variable Analyses

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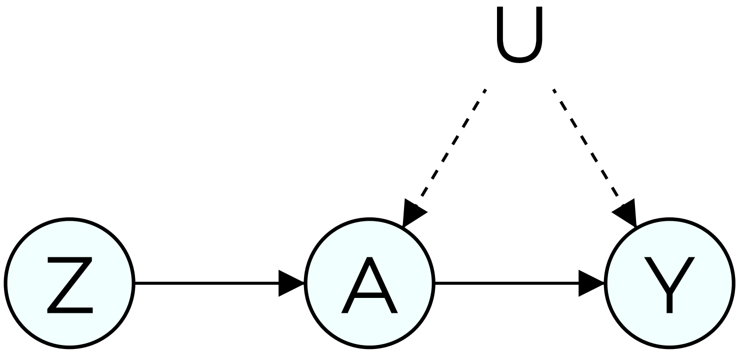
The Problem

- Is voting habit forming? (Davenport et al., 2010)
  - Z: instrument (encouragement to vote at $t = 1$)
  - A: treatment (voting at $t = 1$)
  - Y: outcome (voting at $t = 2$)
  - U: unobserved confounder (e.g. political interest)

- We want to investigate the effect of A on Y:



IV Theory Review

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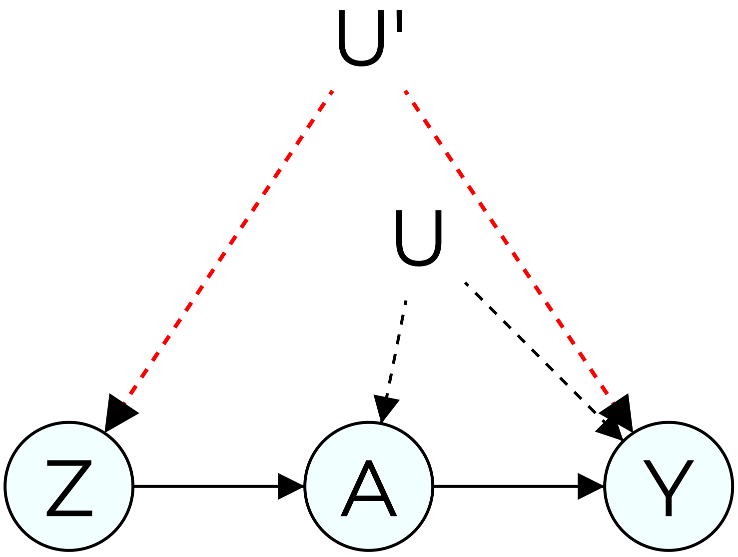
    - Violation: Confounding between Z and Y



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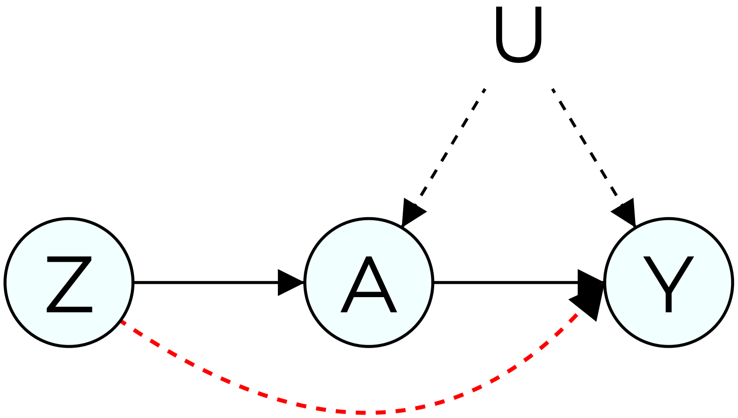

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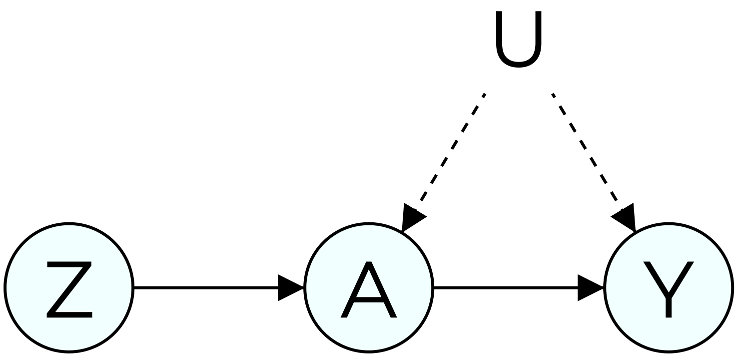

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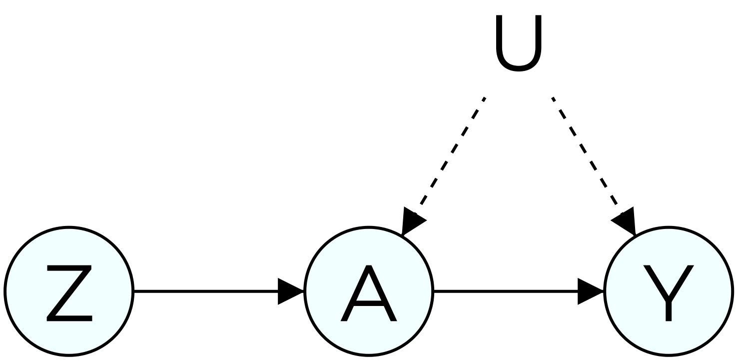
- LATE = $E[Y(a_1) - Y(a_0)|\text{compliers}]$ is identifiable if one assumes 1, 2, and 3 (Imbens & Angrist, '94)



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- Those assumptions indeed have observable implications (Pearl, '95; Balke & Pearl, '97)
    - These are useful for falsification tests

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  - We can evaluate bounds when assumptions are relaxed

Evaluating IV Assumptions

Testing

$$P(y_1, a_1|z_1) - P(y_1, a_1|z_0) \geq 0$$
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$$\theta \in [0, 0.2],$$

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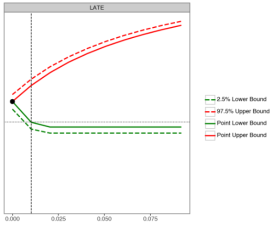
Applications



Testing Assumptions

Observable Implications

- In 1995, Pearl derived *Instrumental Inequalities* (Pearl, '95):

$$\max_a \sum_y [\max_z P(y, a|z)] \leq 1, \text{ under assumptions } 1, 2$$

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- In practice, falsification tests can detect large violations
- Smaller violations may go undetected
- Note: if exogeneity is satisfied (e.g. by random assignment), this is a test of the exclusion restriction

Generalized IV Falsification Test

- Kedagni and Mourifie (2020) proved that if a model satisfies exclusion restriction and exogeneity, then:

$$\max_z P(y_1, a|z) + \max_z P(y_1, a'|z) \leq 1$$

$$\max_z P(y_1, a|z) - \min_z P(y_1|z) - \min_z P(y_1, a|z) + P(y_0, a'|z) \leq 0$$

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- There are no other observable implications (sharpness)

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- The test is sharp (Kitagawa, 2015)

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  - One states a causal question, introduces data and assumptions, and gets sharp bounds on the estimand

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- **Sensitivity analysis**: optimize bounds over possible θ and ψ using *Autobounds*:

Empirical Applications

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- Sensitivity Analysis:
  - Proportion of violating units is at least 0.03
  - LATE can be signed in (0.03, 0.07)
  - LATE can't be signed above 0.07

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  - instrument Z: encouragement to vote
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- Just because we did not detect violations, it does not mean they are not there, so we proceed with sensitivity analysis

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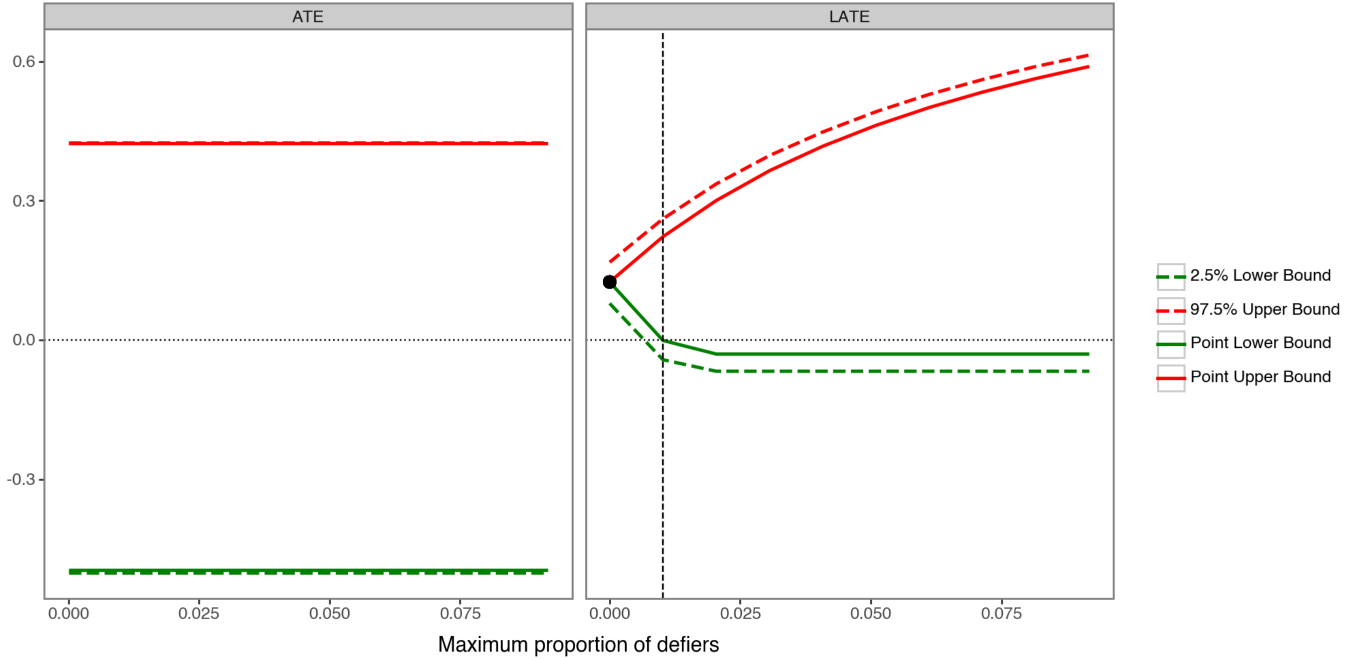
- Sensitivity Analysis:
  - How robust are those results to violations of no defiers?
  - ATE is not much affected
  - LATE is positive if the proportion of defiers is $< 1\%$
  - Violations cause nonlinear impact on the LATE



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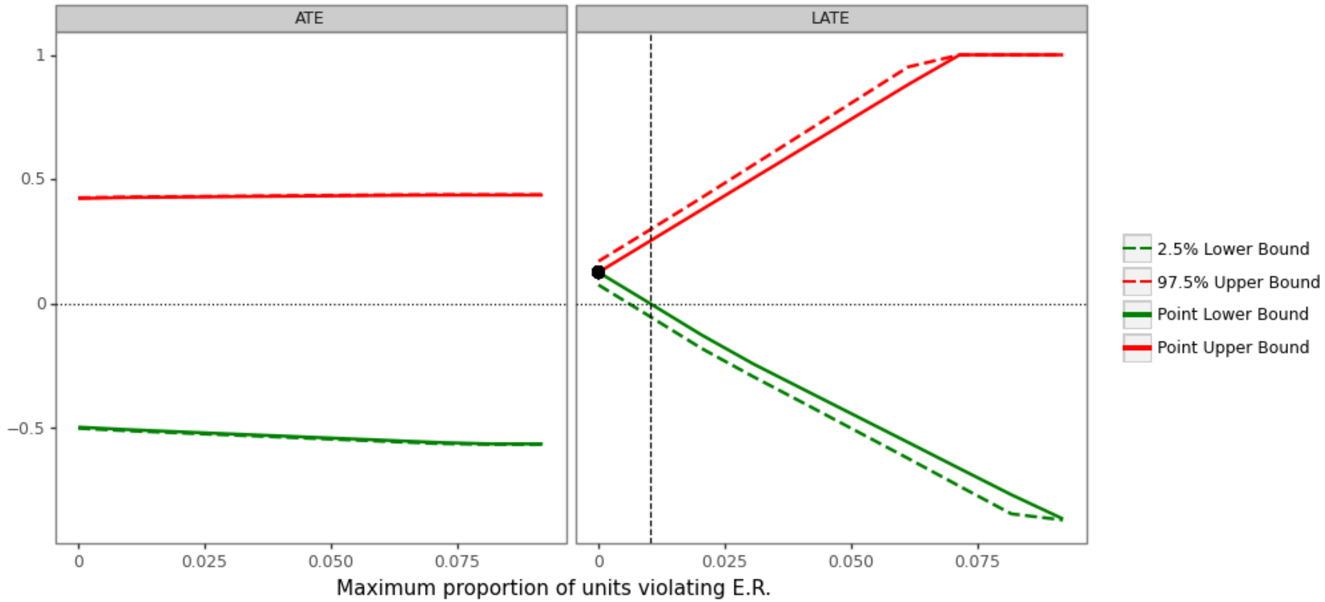

More Complex Scenarios

Judge IV Design and Issues

- Use judge random assignment as natural experiment:
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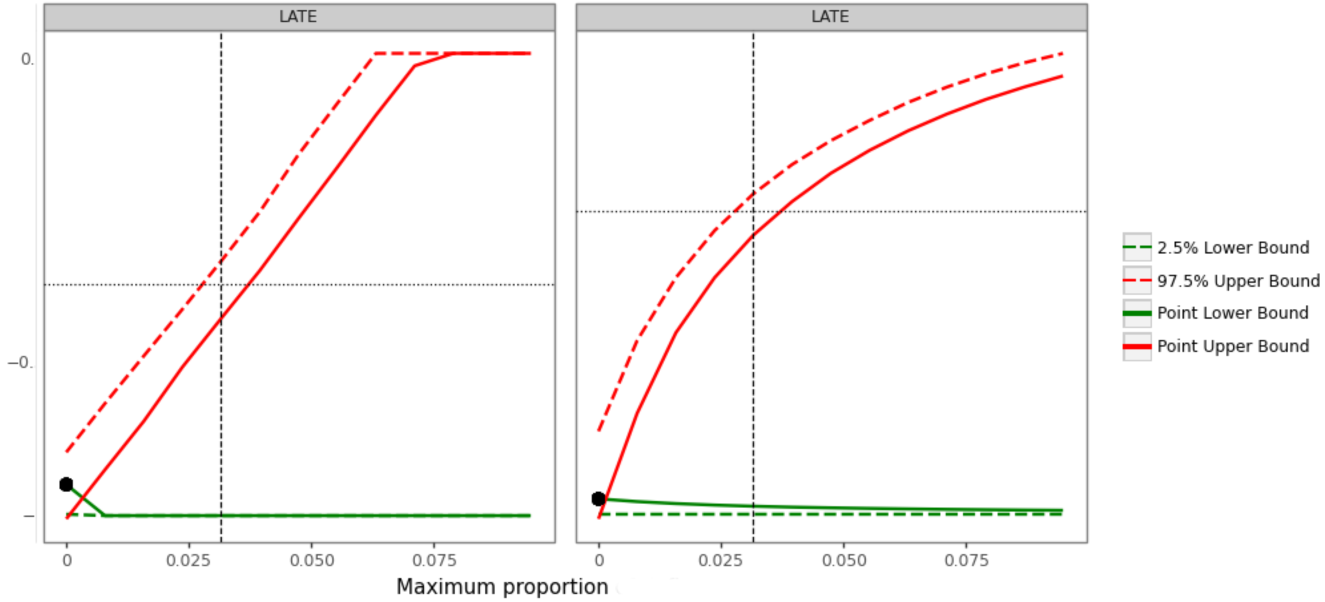
- Reanalysis of Stevenson (2018): positive effect of 0.13
- Our paper: derive results for many-valued instrument
- Today: compare two judges at a time (more severe to more lenient)

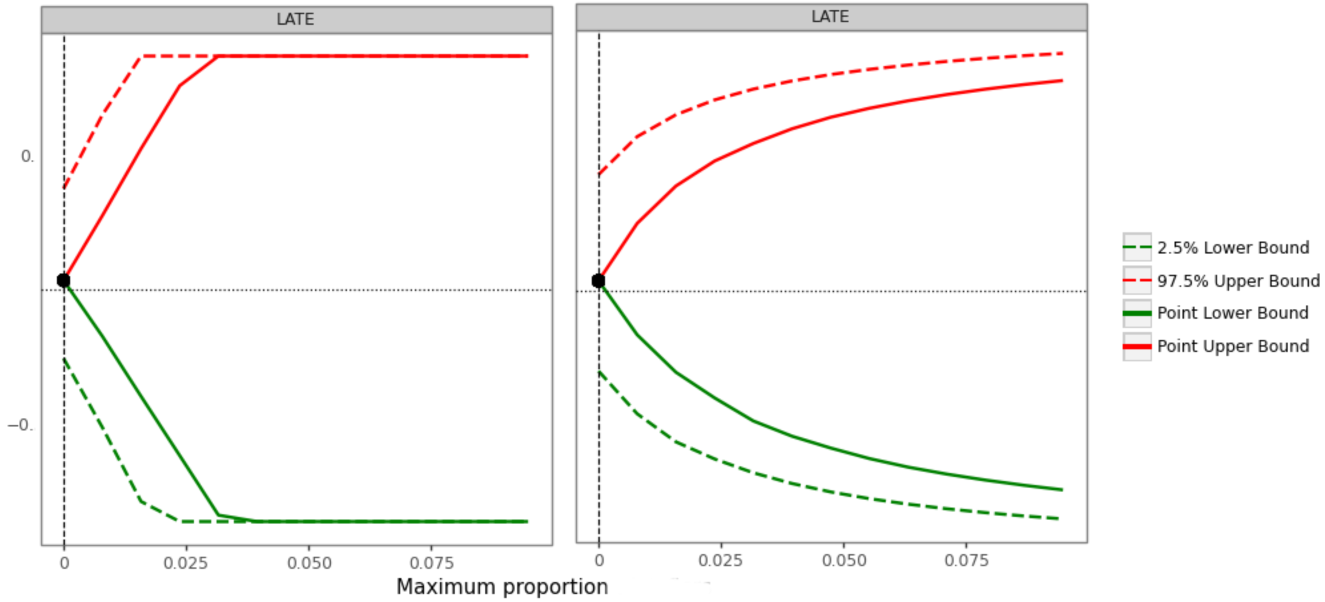
Reanalysis of Stevenson (2018)

- Comparison between the most extreme judges (z_0 / z_n):
  - "No defiers" assumption is rejected (p-value < 0.01)
  - The bounds cross in the region where θ is 0.0015, with LATE equal to -0.84
  - High negative LATE contradicts the main result of the paper, suggesting small violations cause high bias to the LATE estimate, even when evaluated at the minimum θ



Reanalysis of Stevenson (2018)

- Comparison between somewhat extreme judges (z_2 / z_n):
  - "No defiers" is not rejected for this case
  - No pre-existent violation, so LATE is identifiable at 0.044
  - LATE is unsigned if we allow for small θ deviation (close to 0)



Conclusions

- IV assumptions often characterized as untestable
- We *can* empirically evaluate key assumptions
  - Falsify monotonicity/exclusion restriction
  - Sensitivity analysis for defiers and E.R. violations
- Show we can reject assumptions in practice
- In applications, IV results extremely sensitive to minor violations
- Extensions:
  - Characterize robustness in the IV literature
  - Use framework for other models, e.g. factorial experiments

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