

CRYPTOGRAPHY #03.2

RSA OAEP

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1. OAEP

Optimal Asymmetric Encryption Padding is a padding scheme used in RSA and defined in RFC 3447. OAEP allows to:

- add the element of randomness into the classical encryption scheme
- reduce risk related to the RSA homomorphism
- prevent partial decryption of a cryptogram

Padding – extending a message into the given size. There are many padding schemes, however, not all are considered as secure. Padding is used in symmetric block ciphers and in asymmetric block ciphers.

Accordingly to RFC 3447 let's define parameters for OAEP encoding:

hLen – length of the chosen hashing function in bytes

H – chosen hashing function

k – demanded length of the message after padding (can not exceed the size of modulus *n*) in bytes (**input parameter, however can be fixed if modulus *n* length is known**).

mLen – length of a message in bytes

mgf1 – mask function mentioned in chapter 2.

M – bytes of a message (**input parameter**).

NOTE: *k* parameter should be chosen carefully. It should be as big as possible. Optimal value is number of bytes of modulus *n* minus 1.

OAEP encoding scheme presented in RFC 3448:

- 1) if $mLen > k - 2 \cdot hLen - 2$, return error.
- 2) $lHash = H()$ ($lHash$ is a byte table returned by H . Hash is calculated from an empty string).
- 3) Generate an octet string PS consisting of $k - mLen - 2 \cdot hLen - 2$ zero octets. The length of PS may be zero.
- 4) $DB = lHash || PS || 0x01 || M$. Length of DB is equal to $k - hLen - 1$.
- 5) Generate a random octet string $seed$ of length $hLen$.
- 6) $dbMask = mgf1(seed, k - hLen - 1)$.
- 7) $maskedDB = DB \text{ \texttt{\textbackslash xor} } dbMask$.
- 8) $seedMask = mgf1(maskedDB, hLen)$.
- 9) $maskedSeed = seed \text{ \texttt{\textbackslash xor} } seedMask$.
- 10) $EM = 0x00 || maskedSeed || maskedDB$. The length of EM is k .
- 11) return EM .

OAEP decoding parameters:

$hLen$ – length of the chosen hashing function in bytes

H – chosen hashing function

k – length of EM

$mgf1$ – Mask function mentioned in chapter 2.

EM – bytes of an encoded message (**input parameter**)

OAEP decoding scheme presented in RFC 3448:

- 1) Separate the encoded message EM into a single octet Y , an octet string $maskedSeed$ of length $hLen$, and an octet string $maskedDB$ of length $k - hLen - 1$: $EM = Y || maskedSeed || maskedDB$.
- 2) $lHash = H()$ ($lHash$ is a byte table returned by H . Hash is calculated from an empty string).
- 3) $seedMask = mgf1(maskedDB, hLen)$.
- 4) $seed = maskedSeed \text{ \texttt{\textbackslash xor} } seedMask$.
- 5) $dbMask = mgf1(seed, k - hLen - 1)$.
- 6) $DB = maskedDB \text{ \texttt{\textbackslash xor} } dbMask$.
- 7) Separate DB into an octet string $lHash'$ of length $hLen$, a (possibly empty) padding string PS consisting of octets with hexadecimal value $0x00$, and a message M :
 $DB = lHash' || PS || 0x01 || M$.
- 8) If there is no octet with hexadecimal value $0x01$ to separate PS from M , return error.
- 9) if $lHash' \neq lHash$, return error.
- 10) if $Y \neq 0x00$, return error.
- 11) return M .

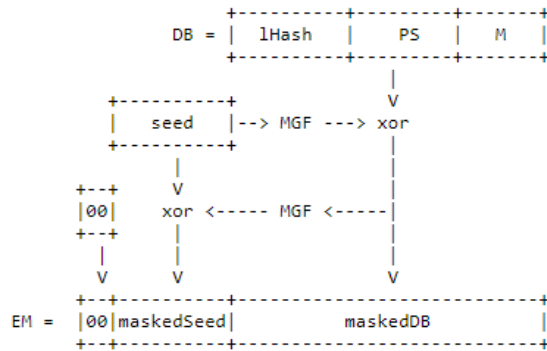


Fig. 1 OAEP coding scheme presented in RFC 3447

2. RSA – OAEP ciphering and deciphering scheme.

As we mentioned earlier, usage of simple RSA scheme is not 100% secure, that's why it involves OAEP. Before communication begins, RSA keys should be generated appropriately, and both sides should establish OAEP parameters.

RSA – OAEP ciphering scheme for a message m , and public key (n, e) :

- 1) $BEM = \text{OAEP_Encoding}(m)$
- 2) $C = \text{convertToNumber}(BEM)$
- 3) $C = C^e \bmod n$
- 4) $BC = \text{convertToBytes}(C)$
- 5) return BC

RSA – OAEP deciphering scheme for a cryptogram BC , and private key (d, e) :

- 1) $C = \text{convertToNumber}(BC)$
- 2) $C = C^d \bmod n$
- 3) $BC = \text{convertToBytes}(C)$
- 4) $m = \text{OAEP_Decoding}(BC)$
- 5) return m

Exercise 1 (warm up):

Execute the following source code:

```
RSA rsa = new RSA(); //here keys are generated
BigInteger c1 = new BigInteger("2").modPow(rsa.e, rsa.n);
BigInteger c2 = new BigInteger("12").modPow(rsa.e, rsa.n);
BigInteger c = c1.multiply(c2);
BigInteger m = c.modPow(rsa.d, rsa.n);
System.out.println(m);
```

What You noticed?

Execute the following source code:

```
RSA rsa = new RSA();
BigInteger c1 = new BigInteger("12").modPow(rsa.e, rsa.n);
BigInteger c2 = new
BigInteger("2").modInverse(rsa.n).modPow(rsa.e, rsa.n);
BigInteger c = c1.multiply(c2);
BigInteger m = c.modPow(rsa.d, rsa.n);
System.out.println(m);
```

What You noticed?

Exercise 2:

Write a function that will be coding and decoding messages with OAEP usage.
Assumptions for both functions:

- 1) H – is a SHA-256 hashing function. You can find it in *java.security.MessageDigest*.
- 2) Accordingly to the previous point, $hLen = 32$.
- 3) $k = 255$.

Functions structures:

- 1) `byte[] OAEPEncoding(byte[] message)`
- 2) `byte[] OAEPDecoding(byte[] cipher)`

Verification in *main* function:

```
String m = "message";

byte[] cipher = OAEPEncoding(m.getBytes());
```

```
byte[] decoded = OAEPDecoding(cipher);  
System.out.println(new String(decoded));
```

Exercise 3:

Improve RSA ciphering and deciphering functions done on previous classes. Both should combine RSA with OAEP as described in chapter 2. Verify your work in the same way as in Ex. 2.