Duarte Salvado Rubio – PID Discretization

Starting from the expression of PID in the Laplace domain:

$$K_{PID}(s) = K_p + \frac{K_p}{T_i s} + s \cdot K_p T_d = K_p \cdot (1 + \frac{1}{T_i s} + T_d s)$$

The Tustin or bilinear discretization is used, where:

$$s \approx \frac{2}{T}\frac{(z-1)}{(z+1)}$$

When discretizing the derivative part, a problem arises: an unstable pole appears at Z = -1:

$$D_{\boldsymbol{d}}(s) = T_d s \xrightarrow[Tustin]{} \overline{D}_d(z) = T_d \cdot \frac{2}{T} \cdot \frac{z-1}{z+1}$$

To avoid this unstable pole, the backward difference method can be used:

$$s \approx \frac{z-1}{Tz}$$

Which applied to the derivative part of the regulator results in:

$$D_{\boldsymbol{d}}(s) = T_d s \xrightarrow[\substack{Backward\\ difference}]{} \overline{D}_d(z) = \frac{T_d}{T} \cdot \frac{z-1}{z}$$

For the integral part, the Tustin rule can be used:

$$D_{\boldsymbol{i}}(s) = \frac{1}{T_i s} \xrightarrow[Tustin]{} \overline{D}_i(z) = \frac{T}{2T_i} \cdot \frac{z+1}{z-1}$$

Therefore, the discretization of the regulator is obtained as a result:

$$K_{PID}(z) = K_p \cdot (1 + \frac{T}{2T_i}\frac{z+1}{z-1} + \frac{T_d}{T}\frac{z-1}{z})$$

To obtain the difference equation, the inverse Z transform is used, obtaining:

$$u_k = u_{k-1} + K_p(e_k - e_{k-1}) + \frac{K_p \cdot T}{T_i} e_k + \frac{K_p \cdot T_d}{T}(e_k - 2e_{k-1} + e_{k-2})$$