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P1: $y = Cx + n$

$\Leftrightarrow C^TQ^{-1}y = C^TQ^{-1}Cx + C^TQ^{-1}n$

$\Leftrightarrow x = (C^TQ^{-1}C)^{-1}C^TQ^{-1}y - (C^TQ^{-1}C)^{-1}C^TQ^{-1}n$

let $\bar{C} = (C^TQ^{-1}C)^{-1}C^TQ^{-1}$ then $x = \bar{C}y - \bar{C}n$ ($\Rightarrow$ optimal estimator)

$\Rightarrow \mu_{x|y} = \bar{C}y = (C^TQ^{-1}C)^{-1}C^TQ^{-1}y$

$\Sigma_{x|y} = \bar{C}Q\bar{C}^T = (C^TQ^{-1}C)^{-1}C^TQ^{-1}Q[(C^TQ^{-1}C)^{-1}C^TQ^{-1}]^T = (C^TQ^{-1}C)^{-1}$

P2: $\mu_{x_t} = A\mu_{x_{t-1}} + \mu_{m_t} = A\mu_{t-1}$

$\Sigma_{x_t} = A\Sigma_{x_{t-1}}A^T + R = A\Sigma_{t-1}A^T + R$

$x(t)|_{x(t-1)} \sim \text{Gauss}(A\mu_{t-1}, A\Sigma_{t-1}A^T + R)$

P3: method 1: $P(x_t|x_{t-1}, y(t)) = \eta P(z_t|x_t)P(x_t|x_{t-1})$

since $y(t) = Cx(t) + n(t)$ $\quad y(t)|x(t) \sim G(Cx(t), Q)$

by the procedure in P1:

we can get: $\mu_{x_t|y_t} = (C^TQ^{-1}C)^{-1}C^TQ^{-1}y_t$

$\Sigma_{x_t|y_t} = (C^TQ^{-1}C)^{-1}$

For $P(x_t|x_{t-1})$, by procedure in P2:

we can get: $\mu_{x_t|x_{t-1}} = A\mu_{t-1}$

$\Sigma_{x_t|x_{t-1}} = A\Sigma_{t-1}A^T + R$

Totally: $\Sigma_{x_t|x_{t-1},y_t}^{-1} = C^TQ^{-1}C + (A\Sigma_{t-1}A^T + R)^{-1}$

$$\mu_{x_t} = \Sigma_{x_t}\frac{\mu_{x_t|x_{t-1}}}{\Sigma_{x_t|x_{t-1}}} + \Sigma_{x_t}\frac{\mu_{x_t|y_t}}{\Sigma_{x_t|y_t}} = \frac{C^TQ^{-1}y_t + \frac{A\mu_{t-1}}{A\Sigma_{t-1}A^T+R}}{C^TQ^{-1}C + (A\Sigma_{t-1}A^T+R)^{-1}}$$

$$= \frac{(A\Sigma_{t-1}A^T+R)C^TQ^{-1}y_t + A\mu_{t-1}}{(A\Sigma_{t-1}A^T+R)(C^TQ^{-1}C) + I}$$

method2: $\bar{\mu}_t = A\mu_{t-1}$

$$\bar{\Sigma}_t = A\Sigma_{t-1}A^T + R$$

$$K_t = (A\Sigma_{t-1}A^T+R)[C(A\Sigma_{t-1}A^T+R)C^R + Q]^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(y_t - C\bar{\mu}_t)$$

$$= A\mu_{t-1} + (A\Sigma_{t-1}A^T+R)(C(A\Sigma_{t-1}A^T+R)C^R+Q)^{-1}(y_t - CA\mu$$

$$\Sigma_t = (I - (A\Sigma_{t-1}A^T+R)[C(A\Sigma_{t-1}A^T+R)C^R+Q]^{-1})(A\Sigma_{t-1}A^T+R$$