Introduction to Probability
Part 1

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Describe the concept of probability

Use the rules of probability to perform basic probability calculations

Probability Definitions

- Probability is the chance that an event will or will not occur. The terms are typically expressed in fractions or decimals.
- An event is one or more of the possible outcomes of a situation or experiment

Probability Definitions

- An experiment is an activity which produces an event.
- Sample space is the set of all possible outcomes from an experiment

Probability Definitions

- Events are termed mutually exclusive when one and only one can take place at the same time.
- Collectively Exhaustive refers to lists containing all of the possible events which may result from an experiment.

Classical Probability

- The probability that an event will occur

where

- P = Probability of an event
- N = Number of outcomes where the event occurs
- S = Total Number of possible outcomes; and where each of the possible outcomes are equally likely

Rules / Conditions of Probability

Typical conditions of concern:

- The case where one event or another will occur
- The situation with two or more events where both may occur

Marginal or Unconditional Probability

$P(A)$ = the probability P of event A occurring

Where a single probability is involved, only one event can take place

Marginal or Unconditional Probability Example

A production lot of 100 parts contains one defective part. What is the P of selecting one part randomly from the lot, and drawing the defective?

Marginal or Unconditional Probability Example

$$P(D) = \frac{1}{100} = 0.01 = 1.0\%$$

Addition Rule for Mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B)$$

Addition Rule for Mutually Exclusive Events Example

- Suppose an investigator has planned to run an experiment, where they wish to select two machines randomly from the ten units on the floor for testing.

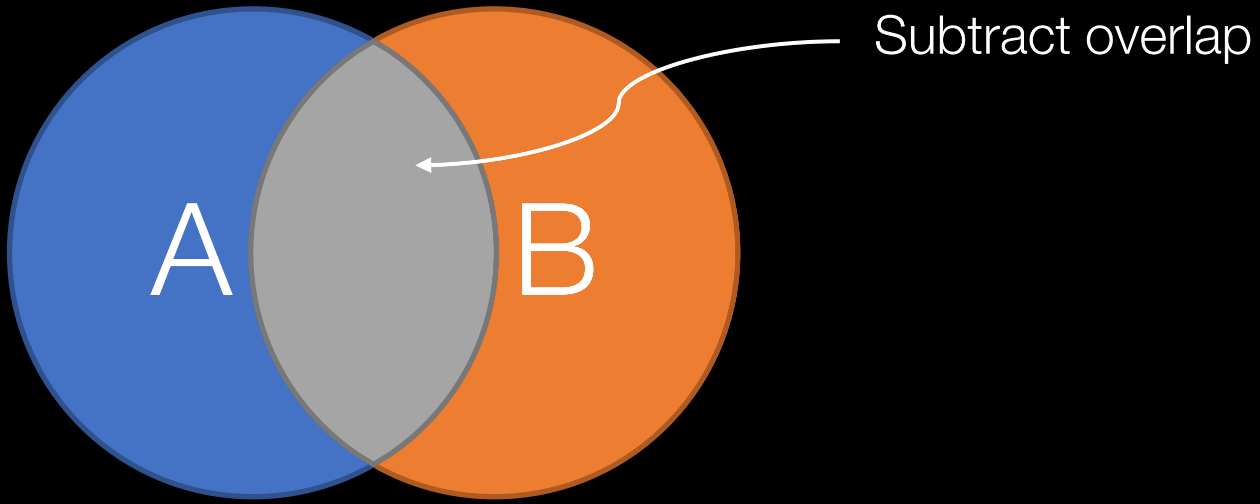
Addition Rule for Mutually Exclusive Events Example

- If each machine is numbered from 1 to 10, what is the probability that machine 4 or 8 will be selected on a single draw?
- P(4 or 8) = P(4) + P(8)

$$= \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = 0.2 \ or \ 20\%$$

Addition Rule for Non-Mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B) - P(A{+}B)$$



Addition Rule for Non-Mutually Exclusive Events Example

- Given a mixed lot with the following characteristics:

Vendor	# Defective	# Not Defective
Vendor A	15	85
Vendor B	10	55

Addition Rule for Non-Mutually Exclusive Events Example

- What is the probability, on a single random draw, of selecting a part from Vendor A or a defective part?

Addition Rule for Non-Mutually Exclusive Events Example

- If we were to simply use P(A) + P(B), then

Addition Rule for Non-Mutually Exclusive Events Example

- Note, however, that there are 15 more parts credited to the total than should be!

Vendor	# Defective	# Not Defective
Vendor A	15	85
Vendor B	10	55

Addition Rule for Non-Mutually Exclusive Events Example

- P(A and B) =
- So, P(A or B) = P(A) + P(B) – P(A+B)

Addition Rule for Non-Mutually Exclusive Events Example

- P(Vendor A or Defective) =

Sources

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Introduction to Probability
Part 2

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Discriminate between marginal, joint and conditional probabilities

Discriminate between independent and dependent events

Learning objectives:

Calculate marginal, joint, and conditional probability under independent and dependent conditions.

Statistical Independence and Dependence

- Events which are statistically independent are those where the outcome of one event has no effect on the outcome of the second event

Statistical Independence and Dependence

- Events which effect subsequent events are termed dependent.

Independent Conditions – Marginal Probability

- P(A) Independent Event (e.g. coin toss)

Independent Conditions – Joint Probability

- The probability of two or more events occurring together (or in succession) is the product of their marginal probabilities
- $P(AB) = P(A) \times P(B)$, where

Independent Conditions – Joint Probability

- P(AB) = probability of events A and B occurring together or in succession; joint probability
- P(A) = marginal probability of (A)
- P(B) = marginal probability of (B)

Independent Conditions – Joint Probability Example 1

- The probability of a machine operator producing a defective part at any point in time is 0.05. What is the probability that three bad parts will be produced in succession?

Independent Conditions – Joint Probability Example 1

- P(ABC) = P(A) x P(B) x P(C)
- P(3 Defectives) = P(Def) x P(Def) x P(Def)
- P(3 Def) = 0.05 x 0.05 x 0.05
- P(3 Def) = 0.000125

Independent Conditions – Conditional Probability

- $P(B|A)$ = Probability of event B occurring, given that A has occurred.
- $P(B|A) = P(B)$…because A and B are independent!

Dependent Conditions – Conditional Probability

- Note that this is equivalent to calculating the probability of the part being defective, given a sample space of B, after A has been drawn.

Dependent Conditions – Conditional Probability Example

- Assuming a randomly selected part is from Vendor A, what is the P that it is also defective?

Vendor	# Defective	# Not Defective	Total
Vendor A	15	85	100
Vendor B	10	55	65
Total	25	140	**165**

Dependent Conditions – Conditional Probability Example

Dependent Conditions – Conditional Probability Example

Vendor	# Defective	# Not Defective	Total
Vendor A	15	85	100
Vendor B	10	55	65
Total	25	140	**165**

- Note: This is the same as observing that given the 15 defectives out of 100 Vendor A parts, then

Dependent Conditions

Note also that the P(Defective and Vendor A) constitutes a joint probability under statistical dependence. Creating a table of joint P values for the sample space:

Event	P	Fraction
Vendor A and Defective	0.0909	$^{15}/_{165}$
Vendor A and Not Defective	0.5151	$^{85}/_{165}$
Vendor B and Defective	0.0606	$^{10}/_{165}$
Vendor B and Not Defective	0.3333	$^{55}/_{165}$

Dependent Conditions – Conditional Probability Example

- As a second example, assume that a non-defective part has been drawn. What is the P that it is from Vendor B?

Dependent Conditions – Conditional Probability Example

- Note that should a non-defective part have been selected, the P of it being a part from Vendor B is

Joint Probabilities Under Statistical Dependence

- The formula for joint probabilities under statistical dependence is a variation of the conditional probability formula

Joint Probabilities Under Statistical Dependence

$$P(B|A) = \frac{P(BA)}{P(A)} \Rightarrow P(BA) = P(B|A) \, x \, P(A)$$

Noting that $\Rightarrow$ $P(BA) = P(AB)$

And that P(BA) = P of events B and A happening together or in succession

Joint Probabilities Under Statistical Dependence Example

- As an example, we can check any of the joint probability calculations from the joint P table; for example,
- P(A and Def) = 0.0909 or ${}^{15}/_{165}$

Joint Probabilities Under Statistical Dependence Example

$$P(BA) = P(B|A)\ x\ P(A)$$

$$P(BA) = \frac{P(BA)}{P(A)}\ x\ P(A)$$

$$\frac{P(A\ and\ Def)}{P(Def)}\ x\ P(Def) = \frac{0.0909}{0.1515}\ x\ 0.1515 = 0.0909$$

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Probability Distributions Part1

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objective:

Describe the concept of a probability distribution

Probability Distributions

- Probability distributions are theoretical frequency distributions which are collectively exhaustive.

Probability Distributions

- For example, suppose we have historical evidence to show that a particular vendor will provide a defective part to us 20 times out of 100. Therefore, the P of receiving a Defective part (D) is:

Probability Distributions

- Let us determine the probabilities associated with any two parts randomly drawn from a large production lot. Given:

1st Part	2nd Part	# Def. @ 2 parts	P
D (0.20)	ND (0.80)	1	0.16
D (0.20)	D (0.20)	2	0.04
ND (0.80)	D (0.20)	1	0.16
ND (0.80)	ND (0.80)	0	0.64
		Total	1.00

Probability Distributions

- We can now create a probability distribution conforming to our theoretical expectation for two parts so that:

# of Defectives	Draws	P(D)
0	(ND, ND)	0.64
1	(ND, D) + (D, ND)	0.32
2	(D,D)	0.04

Probability Distributions

In R / Rstudio

```
> table.dist.binomial(n, p)
```

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Probability Distributions
Part 2

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Discriminate between discrete and continuous probability distributions

Identify the probability distributions most commonly used in decision making

Types of Probability Distributions

- Discrete – A discrete probability distribution is one where there are a limited number of possible values

Types of Probability Distributions

- Continuous – A continuous probability distribution has relatively unlimited possibilities for variable values

Random Variables

A random variable is one which can take on different values as a result of the outcomes of a random experiment.

Random variables, further, can be either discrete or continuous.

Probability Distribution for Discrete Random Variable

- Assume that an automated process produces between 50 and 60 parts per day. During a two month production period, daily production levels (DP) were noted and the following data were generated:

Daily Production (DP)	# of Days	P(DP)
50	1	0.027
51	2	0.054
52	2	0.054
53	3	0.081
54	5	0.135
55	7	0.189
56	6	0.162
57	4	0.108
58	4	0.108
59	2	0.054
60	1	0.027
	$\sum f = 37$	1.000

Probability Distribution for Discrete Random Variable

R / Rstudio

```
> frequency.dist.grouped( )
```

Expected Value of a Discrete Random Variable

- One of the most important factors related to any probability distribution is the ability to define the expected value of a random variable.

Expected Value of a Discrete Random Variable

- The expected value of a discrete random variable is the weighted average of the expected outcomes.

Daily Production (DP)	P	Weighted P Value (DP x P)
50	0.027	1.351
51	0.054	2.757
52	0.054	2.811
53	0.081	4.297
54	0.135	7.297
55	0.189	10.405
56	0.162	9.081
57	0.108	6.162
58	0.108	6.270
59	0.054	3.189
60	0.027	1.621
	Sum	55.243

Expected Value of a Discrete Random Variable

- Therefore, $E(DP) = 55.243$

Expected Value of a Discrete Random Variable

R / R Studio

```
> weighted.mean(x,y)
```

OR

```
> mean( )
```

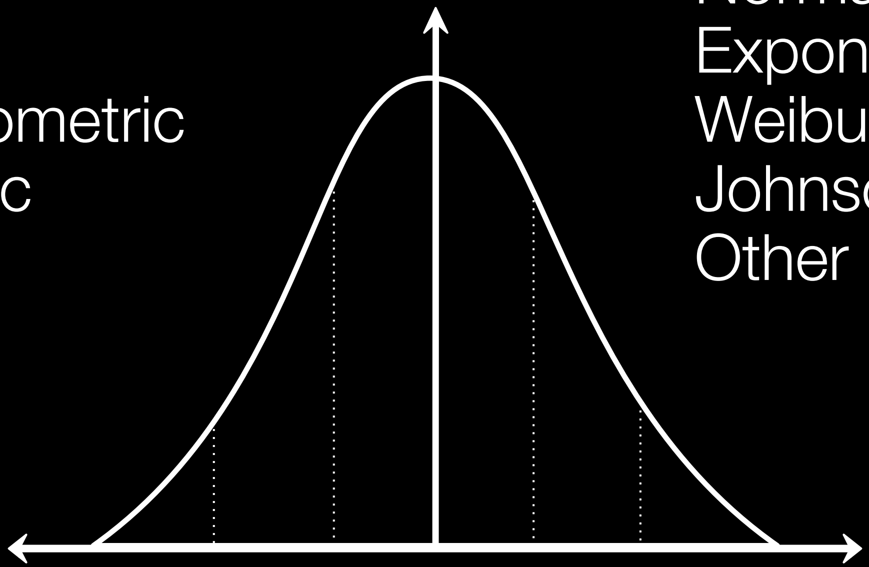
Some Commonly-Employed Probability Distributions

Discrete

- Binomial
- Poisson
- Hypergeometric
- Geometric

Continuous

- Normal
- Exponential
- Weibull Family
- Johnson Family
- Other Distributions



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The Binomial Distribution

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Describe the Binomial probability distribution

Calculate probabilities using the Binomial distribution

The Binomial Distribution

- The Binomial distribution relates to a discrete random variable (nominal data).

- The basis of this distribution is the Bernoulli process.

The Bernoulli Process

- Each trial or experiment has only two possible outcomes
- The P of any and all outcomes remains fixed over time
- The trials or experiments are statistically independent

The Binomial Formula

$$P(r\ in\ n\ trials) = \left[\frac{n!}{r!\,(n-r)!}\right][p^r][q^{n-r}]$$

where

p = probability of occurrence

q = 1-p = probability of failure

r = number of occurrences desired

n = number of trials

Binomial Example

- A vendor frequently ships 2 bad parts out of 10.
- Suppose the vendor ships our company 50 parts. If we tell them that at least 9 parts out of 10 must be good, and nothing in their manufacturing process has changed, what is the P that we will receive what we asked for?

Binomial Example

- p = 0.80, q = 0.20, r = 45, n = 50

$$P(45\ in\ 50) = \left[\frac{50!}{45!\,(50-45)!}\right][0.8^{45}][0.2^{5}]$$

= 0.02953

Binomial Example

- What if we wanted to know the probability of getting at least 9 out of 10 good parts in the shipment of 50? $P \geq 45$?
- We would sum the following:
- $P(45) + P(46) + P(47) + P(48) + P(49) + P(50)$

Probability Distributions

In R / Rstudio

```
> table.dist.binomial(n, p)
> pbinom( )
```

The Poisson Distribution

- This probability distribution is for discrete random variables which can take integer (whole) values (ordinal data).

Poisson Data Examples

- The number of parts produced during a 10 minute period
- The number of breakdowns per shift
- The number of failures per 100 cycles

The Poisson Formula

$$P(X) = \frac{\lambda^X}{X!} e^{-\lambda}$$

where
P(X) = probability exactly X occurrences
λ = Mean number of occurrences per time interval (or unit)
e = 2.71828

Poisson Example

- λ = 25 parts produced per hour
- X = 10 parts produced in one hour

$$P(10) = \frac{25^{10}}{10!} e^{-25}$$

= 0.0000365

Probability Distributions

In R / Rstudio

```
> table.dist.poisson(λ)
> ppois( )
```

Test for Poisson Distribution

In R / Rstudio

```
> poisson.dist.test( )
```

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982

The Poisson Distribution

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Describe the Poisson probability distribution

Calculate probabilities using the Poisson distribution

The Poisson Distribution

- This probability distribution is for discrete random variables which can take integer (whole) values (ordinal data).

Poisson Data Examples

- The number of parts produced during a 10 minute period
- The number of breakdowns per shift
- The number of failures per 100 cycles

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$$P(X) = \frac{\lambda^X}{X!} e^{-\lambda}$$

where

P(X) = probability exactly X occurrences

λ = Mean number of occurrences per time interval (or unit)

e = 2.71828

Poisson Example

- λ = 25 parts produced per hour
- X = 10 parts produced in one hour

$$P(10) = \frac{25^{10}}{10!} e^{-25}$$

= 0.0000365

The Poisson Distribution in R

In R / Rstudio

```
> table.dist.poisson(λ)
> ppois( )
```

Test for Poisson Distribution

In R / Rstudio

> poisson.dist.test( )

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982

The Normal Distribution

Data Science for Quality Management: Probability and Probability Distributions

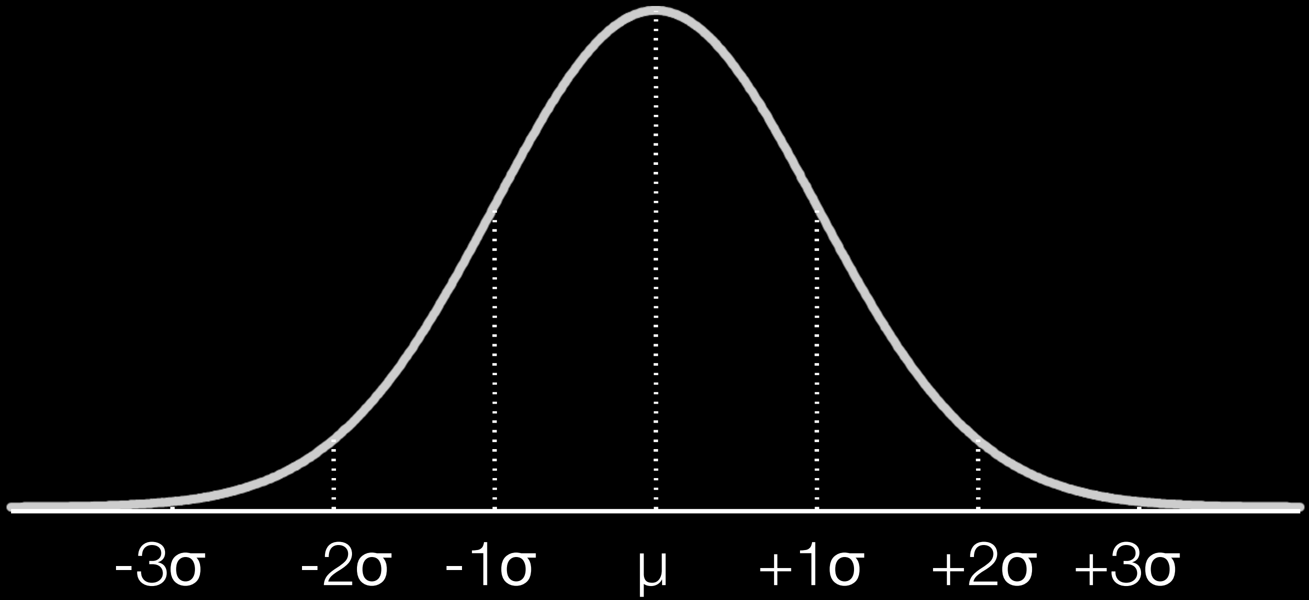
with **Wendy Martin**

Learning objectives:

Describe the Normal probability distribution

Calculate probabilities using the standard normal distribution

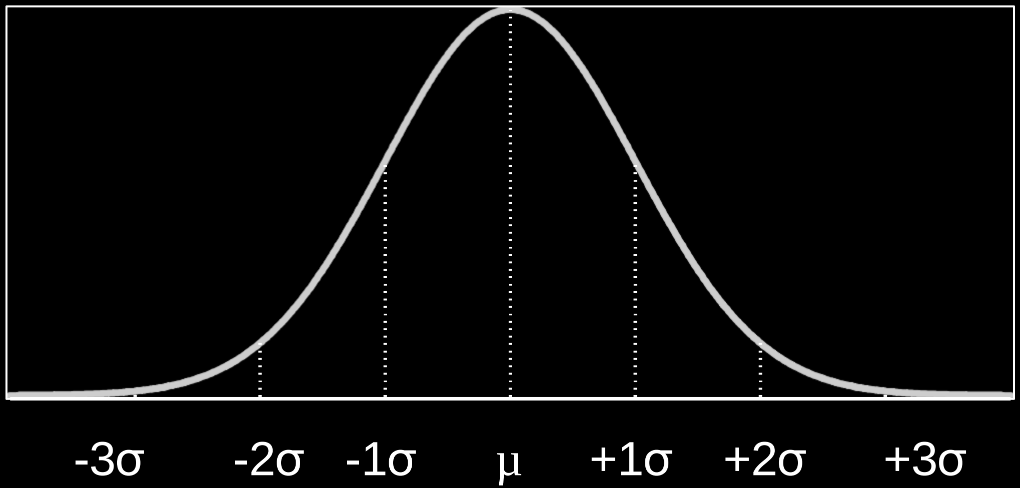
The Normal Distribution



The Normal Distribution

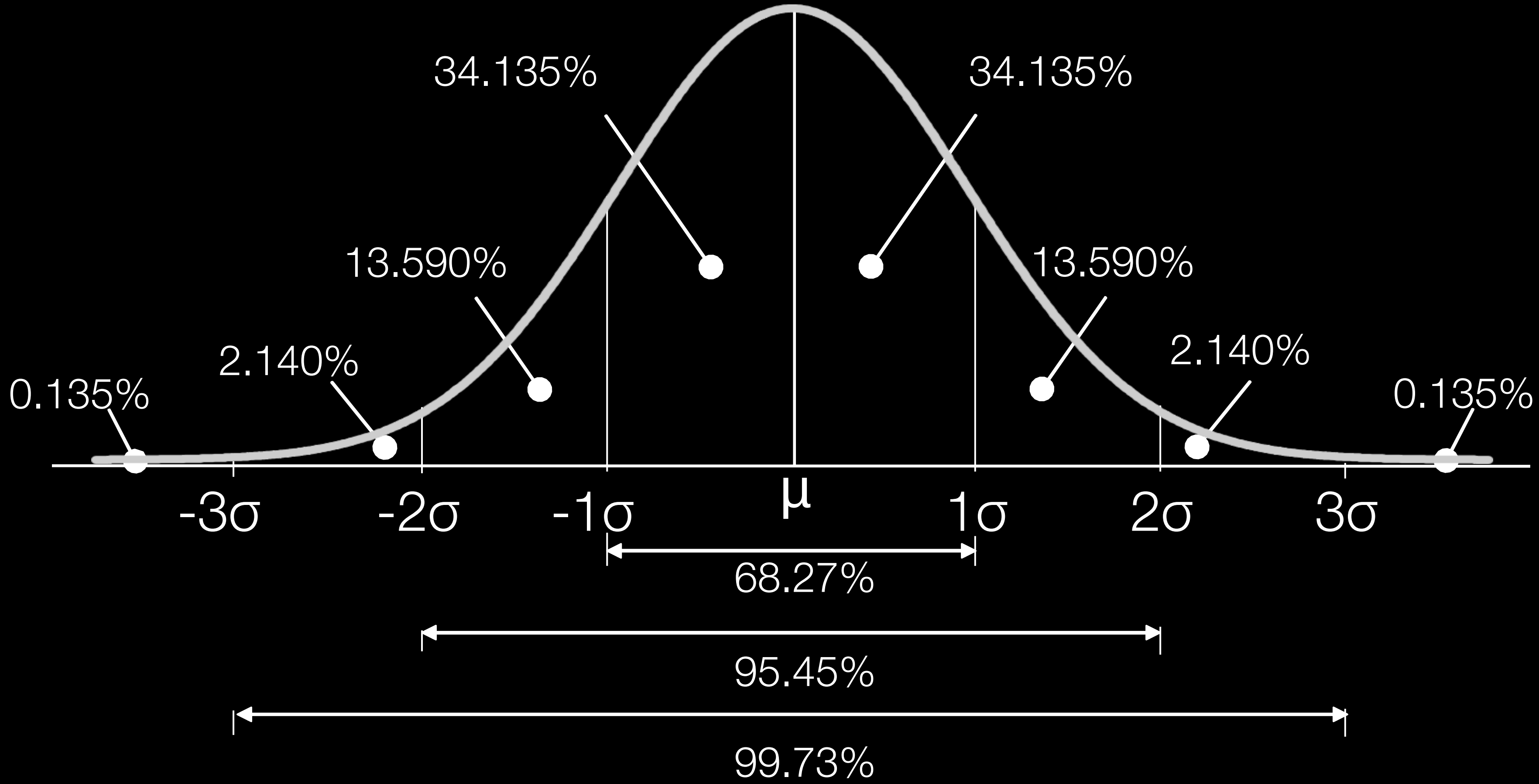
- A theoretical probability distribution for a continuous random variable
- Sometimes (inappropriately) referred to as the bell-shaped curve or distribution
- One of the most important distributions because of its wide range of practical applications

The Normal Distribution

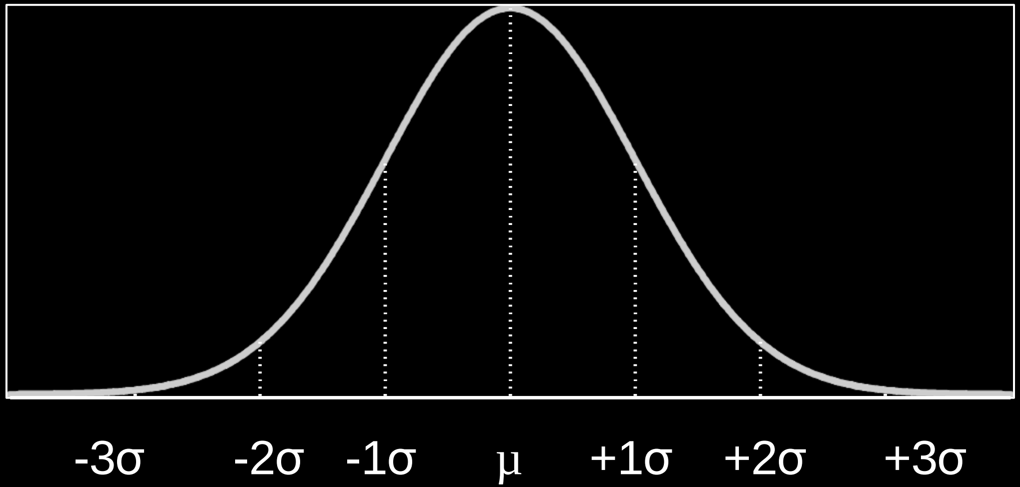


1. Mean = Median = Mode
2. Symmetrical around μ
3. Tails extend to ∞
   but never touch the horizontal axis
4. $\gamma_3 = 0.00$
5. $\gamma_4 = 0.00$
6. Areas under the curve are predictable
   based upon standard deviation values.

Areas Under the Normal Curve



The Normal Distribution



$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(X-\mu)^2}{2\sigma^2}\right]$$

Area Calculations

- The area corresponding to any score value may be found using a z-score, where

$$z = \frac{X - \mu}{\sigma}$$

- Z is the number of standard deviation units from X to μ

Example

- To date, tooling used on a particular drilling process has lasted an average of 180 hours before requiring replacement, with a standard deviation of 5 hours.

Example

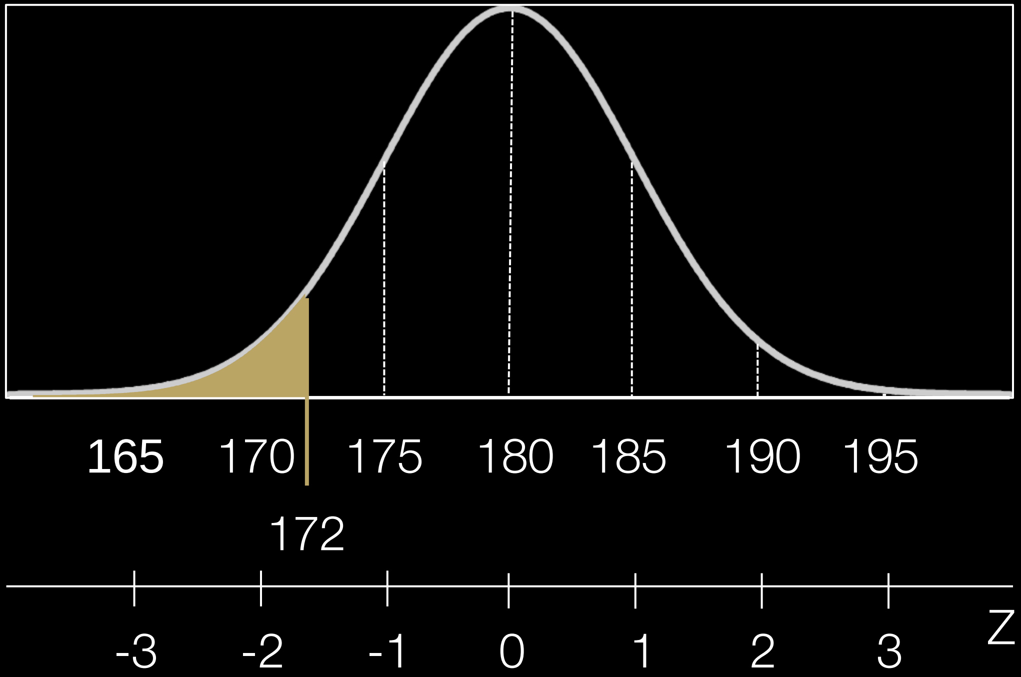
- What is the probability that a tool selected at random from the tool crib will last less than 172 hours before replacement is required?

Example

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{172 - 180}{5}$$

$z =$ -1.60



Normal Distribution in RStudio

In R / Rstudio

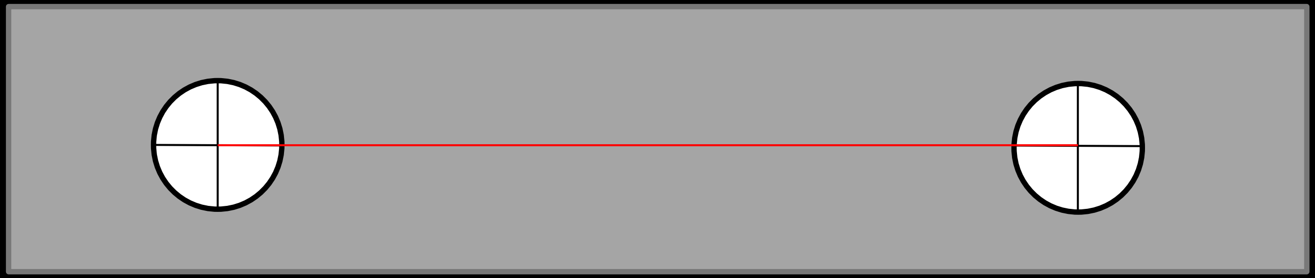
```
> pnorm(q, mean, sd, lower.tail)
```

Example 2

- A stamping operation has been running consistently, punching two holes in sheet metal.

Example 2

- The center-to-center distance between the two holes has been an average (μ) of 5.20mm, with a standard deviation (σ) of 0.05mm.



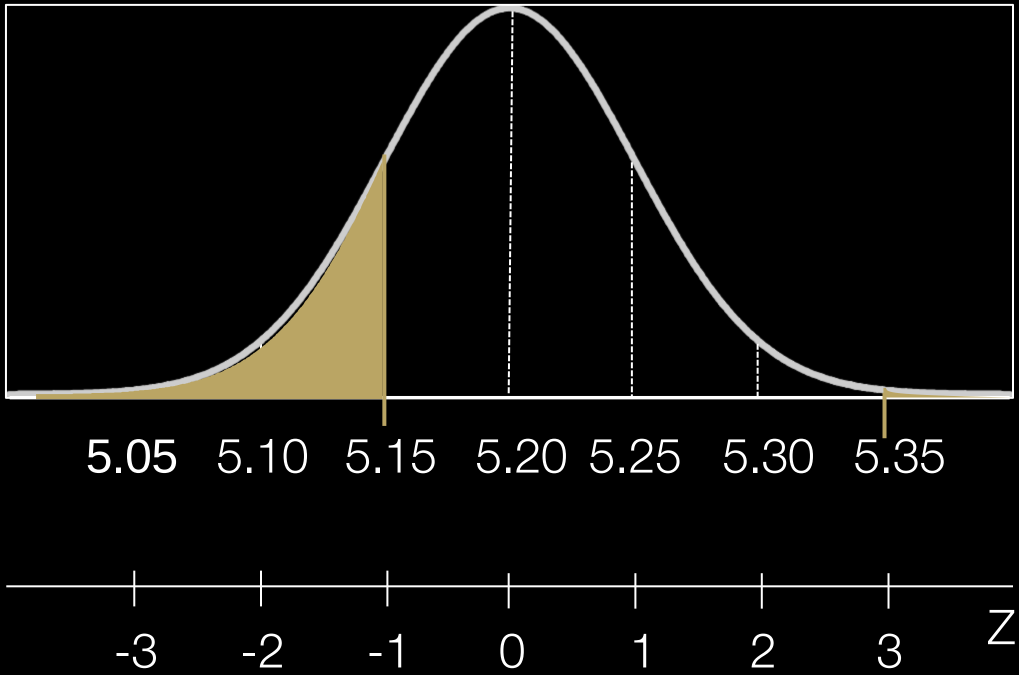
Example 2

- The process produces center-to-center distances that can be modeled with a normal distribution.

Example 2

- The specifications for these parts require a maximum, or upper (USL), limit of 5.35mm and a minimum, or lower (LSL), limit of 5.15mm.
- What percentage of the manufactured parts are likely to fall outside of the specifications?

Example



$$z = \frac{X - \mu}{\sigma}$$

Normal Distribution in RStudio

In R / Rstudio

```
> pnorm(q, mean, sd, lower.tail)
```

Testing for Normality

- When $n < 25$, use the Anderson-Darling test for normality (double check with Shapiro-Wilk test).

- When $n \geq 25$, use the skewness and kurtosis tests (D'Agostino).

Testing for Normality in RStudio

In R / Rstudio

> anderson.darling.normality.test( )
shapiro.wilk.normality.test( ) or
summary.continuous( )

> dagostino.normality.omnibus.test( ) or
summary.continuous( )

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The Exponential Distribution

Data Science for Quality Management: Probability and Probability Distributions

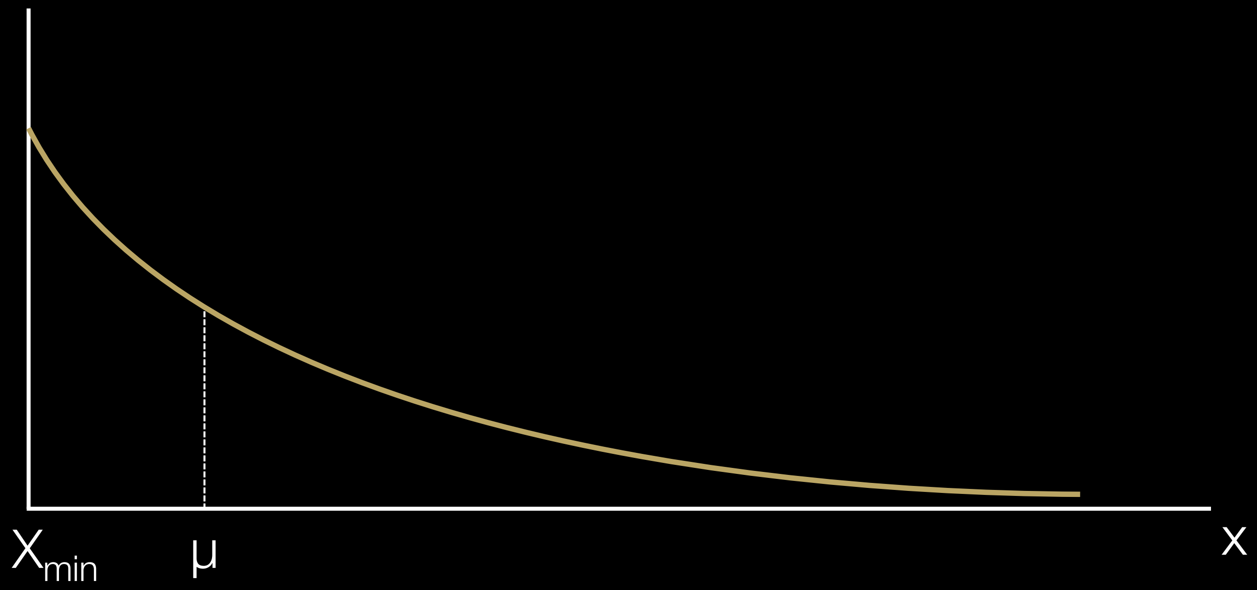
with **Wendy Martin**

Learning objectives:

Describe the Exponential probability distribution

Calculate probabilities using the Exponential distribution

The Exponential Distribution



The Exponential Distribution

- The exponential distribution occurs in a number of situations in the industrial environment.
- Time to failure often follows an exponential distribution.

The Exponential Distribution

- Measurement from a physical process that has a restraint, such as the location of a hole from a reference edge, where the reference edge is pressed against a fixture, may follow an exponential distribution.

The Exponential Distribution

- Roundness of shaft, measured by total indicator reading, may also follow this type of distribution.

The Exponential Distribution

- The exponential distribution is a continuous random variable probability distribution with the form:

$$y = \frac{1}{\mu - x_{min}} e^{\left[-\frac{x - x_{min}}{\mu - x_{min}}\right]}$$

The Exponential Distribution

- When $x_{min} = 0$, the equation reduces to:

$$y = \frac{1}{\mu} e^{\left[-\frac{x}{\mu}\right]}$$

The Exponential Distribution

- The normal distribution contains an area of 50% above and 50% below μ.
- With the exponential distribution, 36.8% of the area under the curve is above the average (μ) and 63.2% is below.

Applications / Observations

- Predictions based on an exponentially distributed process often only require the μ (and sometimes x_{min}) of the process.

Applications / Observations

- For prediction purposes, finding the area under the curve beyond the time period of concern is generally the point of interest.

- These prediction often relate to reliability issues or time between failure analyses.

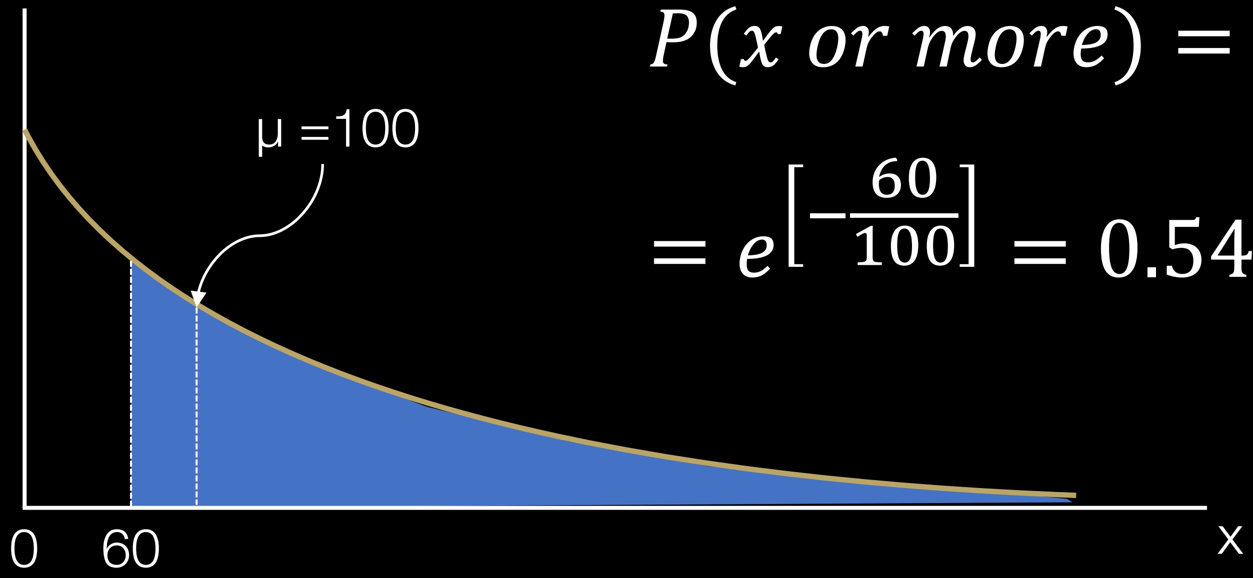
Example 1

- An in-plant study has shown that an engine control module laboratory tester is capable of operating on an average of 100 hours between breakdowns (MTBF).

Example 1

- What is the probability that the tester will run for at least 60 successive hours without a breakdown (assuming that the time to failure pattern is distributed exponentially)?

Example 1



$$P(x \text{ or more}) = e^{\left[-\frac{x}{\mu}\right]}$$

$$= e^{\left[-\frac{60}{100}\right]} = 0.5488$$

Exponential Distribution in RStudio

- pexp(q, rate, lower.tail)

Example 2

- The distribution of time for a particular grinding machine is characterized by the exponential distribution.
- The mean time between breakdowns has been established at 50 minutes.

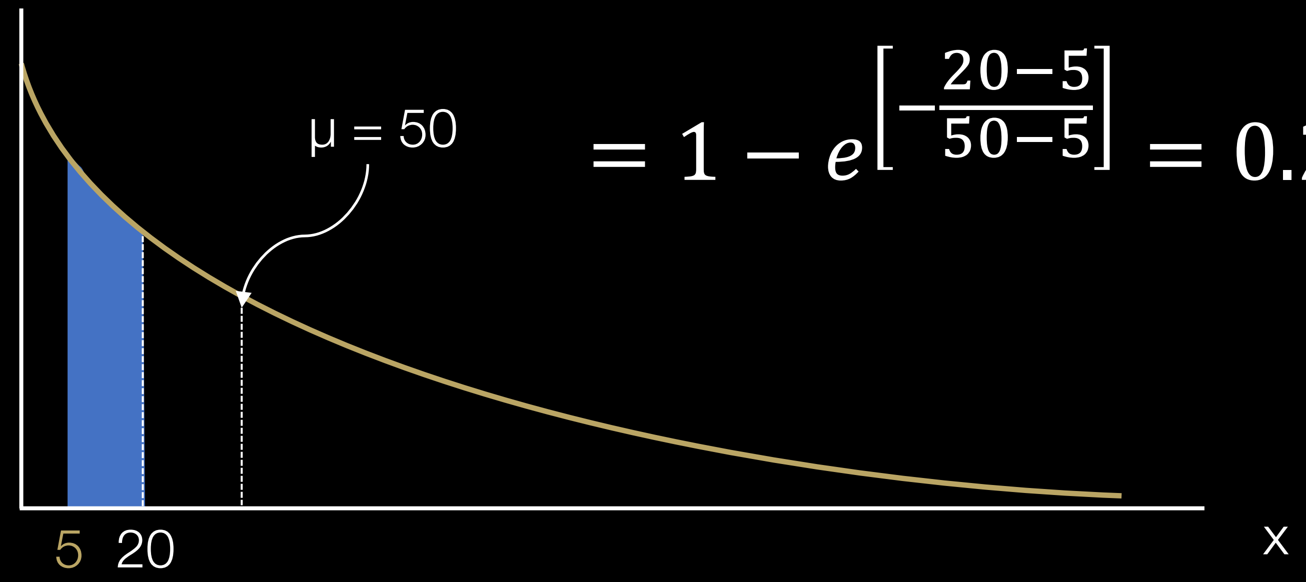
Example 2

- The origin parameter (x_{min}) is estimated to be 5 minutes.
- What is the probability of this machine running 20 minutes or less before a breakdown?

Example 2

$$P(x\ or\ less) = 1 - e^{\left[-\frac{x - x_{min}}{\mu - x_{min}}\right]}$$

$$= 1 - e^{\left[-\frac{20-5}{50-5}\right]} = 0.2835$$



Exponential Distribution in RStudio

- pexp(q, rate, lower.tail)

Testing for Exponentiality

- When $n \leq 100$, use the Shapiro-Wilk test
- When $n > 100$, use the Epps and Pulley test

Testing for Exponentiality

In R / Rstudio

```
> shapiro.wilk.exponentiality.test( )

> shapetest.exp.epps.pulley.1986( )
```

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982