## C1M1 peer reviewed

March 21, 2024

### 1 Module 1: Peer Reviewed Assignment

#### 1.0.1 Outline:

The objectives for this assignment:

- 1. Learn when and how simulated data is appropriate for statistical analysis.
- 2. Experiment with the processes involved in simulating linear data.
- 3. Observe how the variance of data effects the best-fit line, even for the same underlying population.
- 4. Recognize the effects of standardizing predictors.
- 5. Interpreting the coefficients of linear models on both original and standardized data scales.

### General tips:

- 1. Read the questions carefully to understand what is being asked.
- 2. This work will be reviewed by another human, so make sure that you are clear and concise in what your explanations and answers.

### A Quick Note On Peer-Reviewed Assignments

Welcome to your first peer reviewed assignment! These assignments will be a more open form than the auto-graded assignments, and will focus on interpretation and visualization rather than "do you get the right numbers?" These assignments will be graded by your fellow students (except in the specific cases where the work needs to be graded by a proctor) so please make your answers as clear and concise as possible.

```
[7]: # This cell loads the necesary libraries for this assignment library(tidyverse)
```

# 2 Problem 1: Simulating Data

We're going to let you in on a secret. The turtle data from the autograded assignment was simulated...fake data! Gasp! Importantly, simulating data, and applying statistical models to simulated data, are very important tools in data science.

Why do we use simulated data? Real data can be messy, noisy, and we almost never *really* know the underlying process that generated real data. Working with real data is always our ultimate end goal, so we will try to use as many real datasets in this course as possible. However, applying

models to simulated data can be very instructive: such applications help us understand how models work in ideal settings, how robust they are to changes in modeling assumptions, and a whole host of other contexts.

And in this problem, you are going to learn how to simulate your own data.

1. (a) A Simple Line Starting out, generate 10 to 20 data points for values along the x-axis. Then generate data points along the y-axis using the equation  $y_i = \beta_0 + \beta_1 x_i$ . Make it a straight line, nothing fancy.

Plot your data (using ggplot!) with your  $\mathbf{x}$  data along the x-axis and your  $\mathbf{y}$  data along the y-axis. In the Markdown cell below the R cell, describe what you see in the plot.

**Tip**: You can generate your x-data *deterministically*, e.g., using either a:b syntax or the seq() function, or *randomly* using something like runif() or rnorm(). In practice, it won't matter all that much which one you choose.

```
[8]: x <- seq(0, 50)

b_nought <- round(runif(1, min=1, max=5), 2)

b_one <- round(b_nought + runif(1, min=1, max=5), 2)

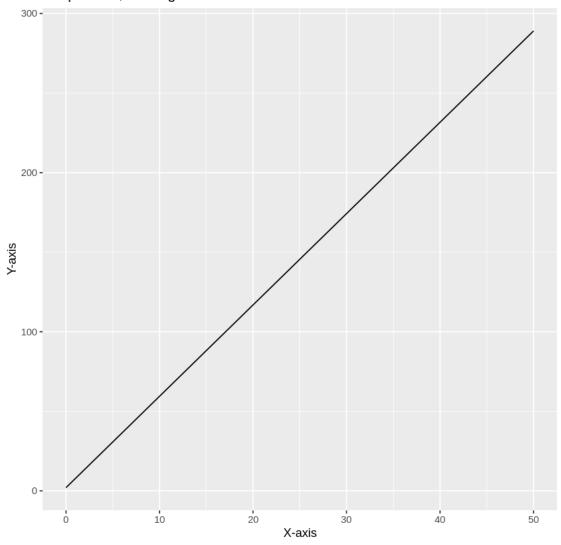
y <- b_nought + b_one * x

data <- data.frame(x = x, y = y)

plot <- ggplot(data, aes(x = x, y = y)) +
    geom_line() +
    labs(x = "X-axis", y = "Y-axis", title = paste0("Simple Line, B Nought:
    →",b_nought, " B One:", b_one))

print(plot)</pre>
```

Simple Line, B Nought: 2.05 B One: 5.74



the data follows the line  $y = b_nought + b_noex$ 

1. (b) The Error Component That is a perfect set of data points, but that is a problem in itself. In almost any real life situation, when we measure data, there will be some error in those measurements. Recall that our simple linear model is of the form:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

Add an error term to your y-data following the formula above. Plot at least three different plots (using ggplot!) with the different values of  $\sigma^2$ .

How does the value of  $\sigma^2$  affect the final data points? Type your answer in the Markdown cell below the R cell.

Tip: To randomly sample from a normal distribution, check out the rnorm() function.

```
[9]: sig_sq <- round(runif(3, min=1, max=150), 2)

plots <- list()
for (i in 1:length(sig_sq)) {

    y <- b_nought + b_one * x + rnorm(length(x), mean = 0, sd = sqrt(sig_sq[i]))

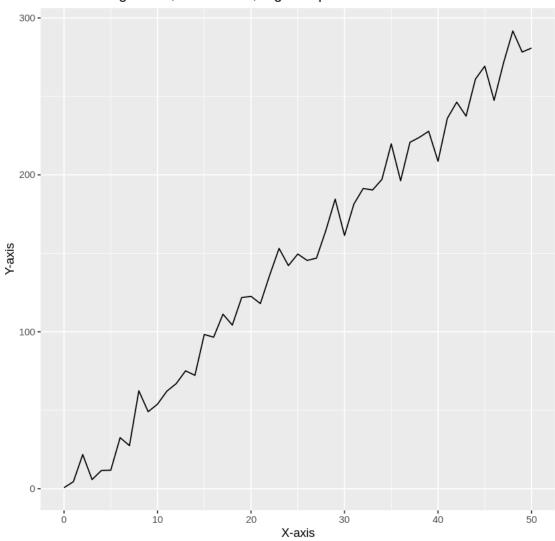
    data <- data.frame(x = x, y = y)

    plot <- ggplot(data, aes(x = x, y = y)) +
        geom_line() +
        labs(x = "X-axis", y = "Y-axis", title = paste0("Plot", i, " B Nought:
        -- ",b_nought, ", B One:", b_one,", Sigma squared:", sig_sq[i]))

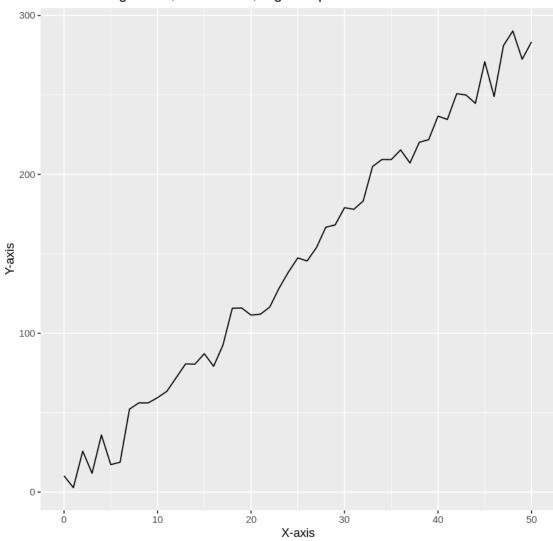
    plots[[i]] <- plot
}

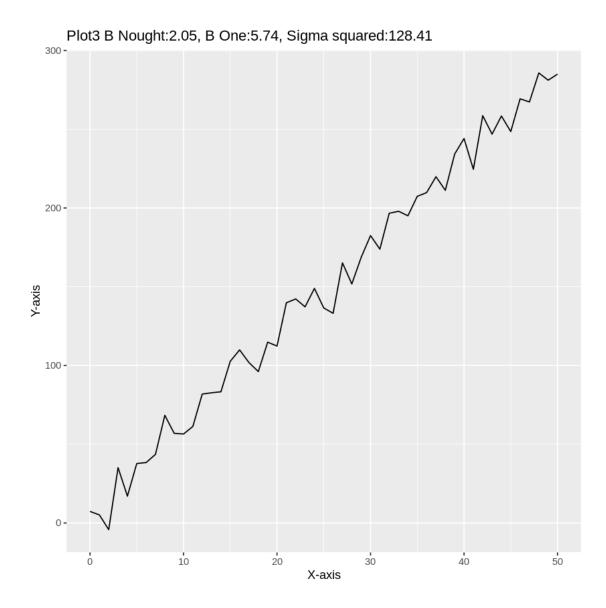
for (i in seq_along(plots)) {
    print(plots[[i]])
}</pre>
```

Plot1 B Nought:2.05, B One:5.74, Sigma squared:98.13



Plot2 B Nought:2.05, B One:5.74, Sigma squared:97.26





the larger the variance, the more "jagged" the line is

### 3 Problem 2: The Effects of Variance on Linear Models

Once you've completed **Problem 1**, you should have three different "datasets" from the same underlying data function but with different variances. Let's see how those variance affect a best fit line.

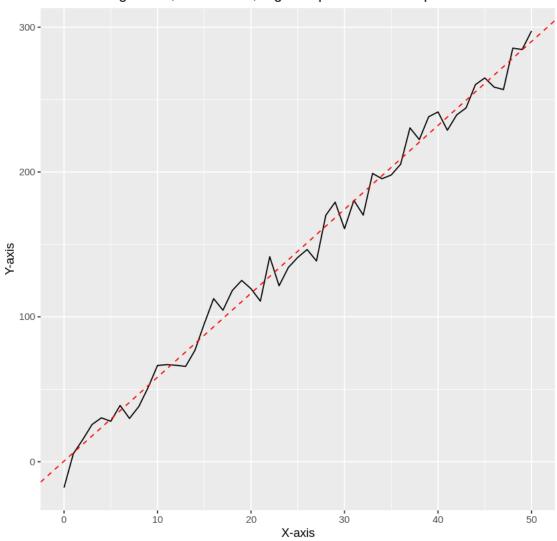
Use the lm() function to fit a best-fit line to each of those three datasets. Add that best fit line to each of the plots and report the slopes of each of these lines.

Do the slopes of the best-fit lines change as  $\sigma^2$  changes? Type your answer in the Markdown cell below the R cell.

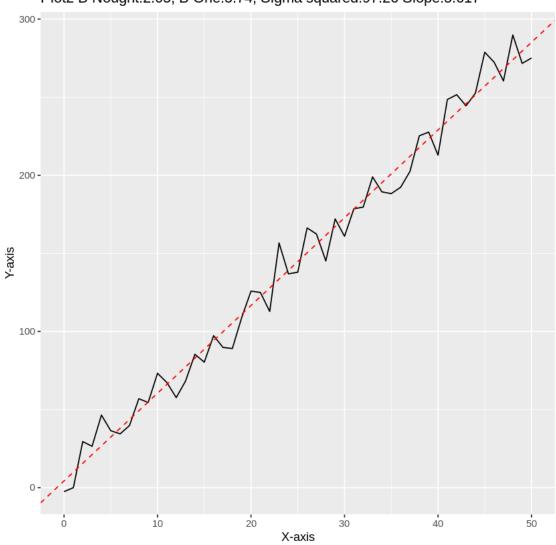
**Tip**: The lm() function requires the syntax lm(y~x).

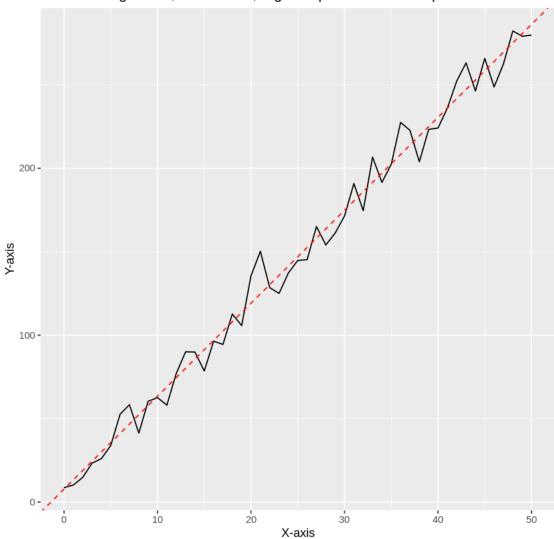
```
[10]: plots <- list()
      for (i in 1:length(sig_sq)) {
          y <- b_nought + b_one * x + rnorm(length(x), mean = 0, sd = sqrt(sig_sq[i]))
          data \leftarrow data.frame(x = x, y = y)
          model \leftarrow lm(y \sim x, data = data)
        slope <- coef(model)[2]</pre>
        intercept = coef(model)[1]
        plot <- ggplot(data, aes(x = x, y = y)) +
          geom_line() +
          geom_abline(intercept = intercept, slope = slope, linetype = "dashed", __
       ⇔color = "red") +
          labs(x = "X-axis", y = "Y-axis", title = paste0("Plot", i, " B Nought:
       →",b_nought, ", B One:", b_one,", Sigma squared:", sig_sq[i], " Slope:", □
       →round(slope, 3)))
        plots[[i]] <- plot</pre>
      for (i in seq_along(plots)) {
        print(plots[[i]])
```

Plot1 B Nought:2.05, B One:5.74, Sigma squared:98.13 Slope:5.794



Plot2 B Nought:2.05, B One:5.74, Sigma squared:97.26 Slope:5.617





Plot3 B Nought: 2.05, B One: 5.74, Sigma squared: 128.41 Slope: 5.57

The slopes do appear to change, especially for the higer variabilities

# 4 Problem 3: Interpreting the Linear Model

Choose one of the above three models and write out the actual equation of that model. Then in words, in the Markdown cell below the R cell, describe how a 1 unit increase in your predictor affects your response. Does this relationship make sense?

```
[11]: # Your Code Here

paste0("my second graph shows that y = ", b_nought, ' + ', b_one, "*", x[2], '.

→So any 1 unit increase in X will results in a ', b_one, ' increase in Y')
```

'my second graph shows that y = 2.05 + 5.74\*1. So any 1 unit increase in X will results in a 5.74 increase in Y'

## 5 Problem 4: The Effects of Standardizing Data

We spent some time standardizing data in the autograded assignment. Let's do that again with your simulated data.

Using the same model from **Problem 3**, standardize your simulated predictor. Then, using the lm() function, fit a best fit line to the standardized data. Using ggplot, create a scatter plot of the standardized data and add the best fit line to that figure.

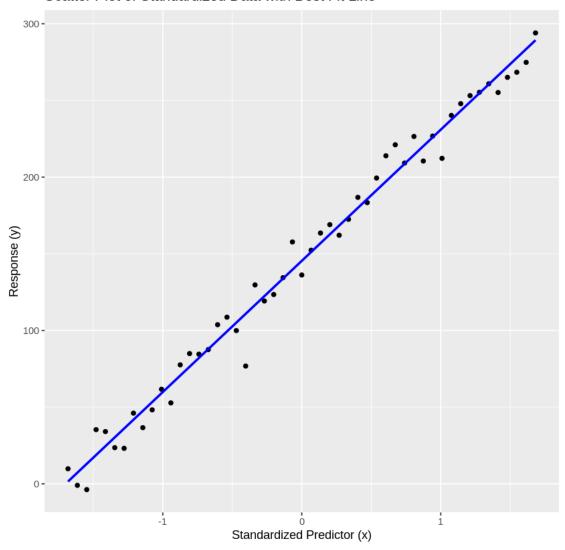
```
[12]: y <- b_nought + b_one * x + rnorm(length(x), mean = 0, sd = sqrt(sig_sq[2]))

z <- (x - mean(x)) / sd(x)

# Create a data frame with z and y
data <- data.frame(x = z, y = y)

ggplot(data, aes(x = z, y = y)) +
    geom_point() +
    geom_smooth(method = "lm", formula = y ~ x, se = FALSE, color = "blue") +
    labs(x = "Standardized Predictor (x)", y = "Response (y)") +
    ggtitle("Scatter Plot of Standardized Data with Best Fit Line")</pre>
```

### Scatter Plot of Standardized Data with Best Fit Line



### 6 Problem 5: Interpreting the Standardized Model

Write out the expression for your standardized model. In words, in the *Markdown* cell below the R cell, describe how a 1 unit increase in your standardized predictor affects the response. Is this value different from the original model? If yes, then what can you conclude about interpretation of standardized predictors vs. unstandardized predictors.

```
[13]: model <- lm(y ~ z, data = data)
slope <- round(coef(model)[2], 2)
intercept = round(coef(model)[1], 2)
# Your Code Here</pre>
```

```
paste0("my standardized graph shows that y = ", intercept, ' + ', slope, '. So_{\sqcup} _{\hookrightarrow} any 1 unit increase in X will results in a ', slope, ' increase in Y')
```

'my standardized graph shows that y=145.4+85.55. So any 1 unit increase in X will results in a 85.55 increase in Y'