Individuals and Moving Range Charts – Non Normal

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objectives:

Recall the 3 approaches for dealing with non-normal distributions

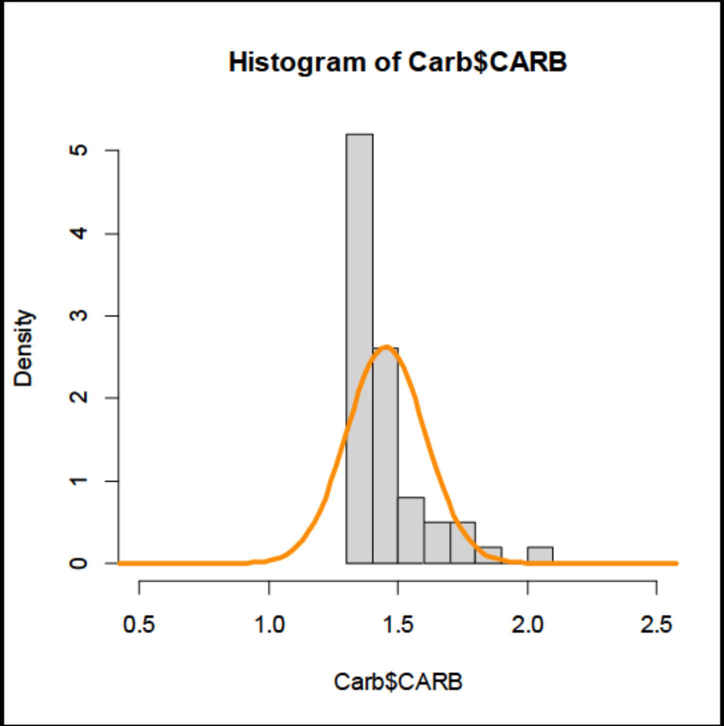
Test data for normality

Issues & Concerns Associated with X and MR Charts

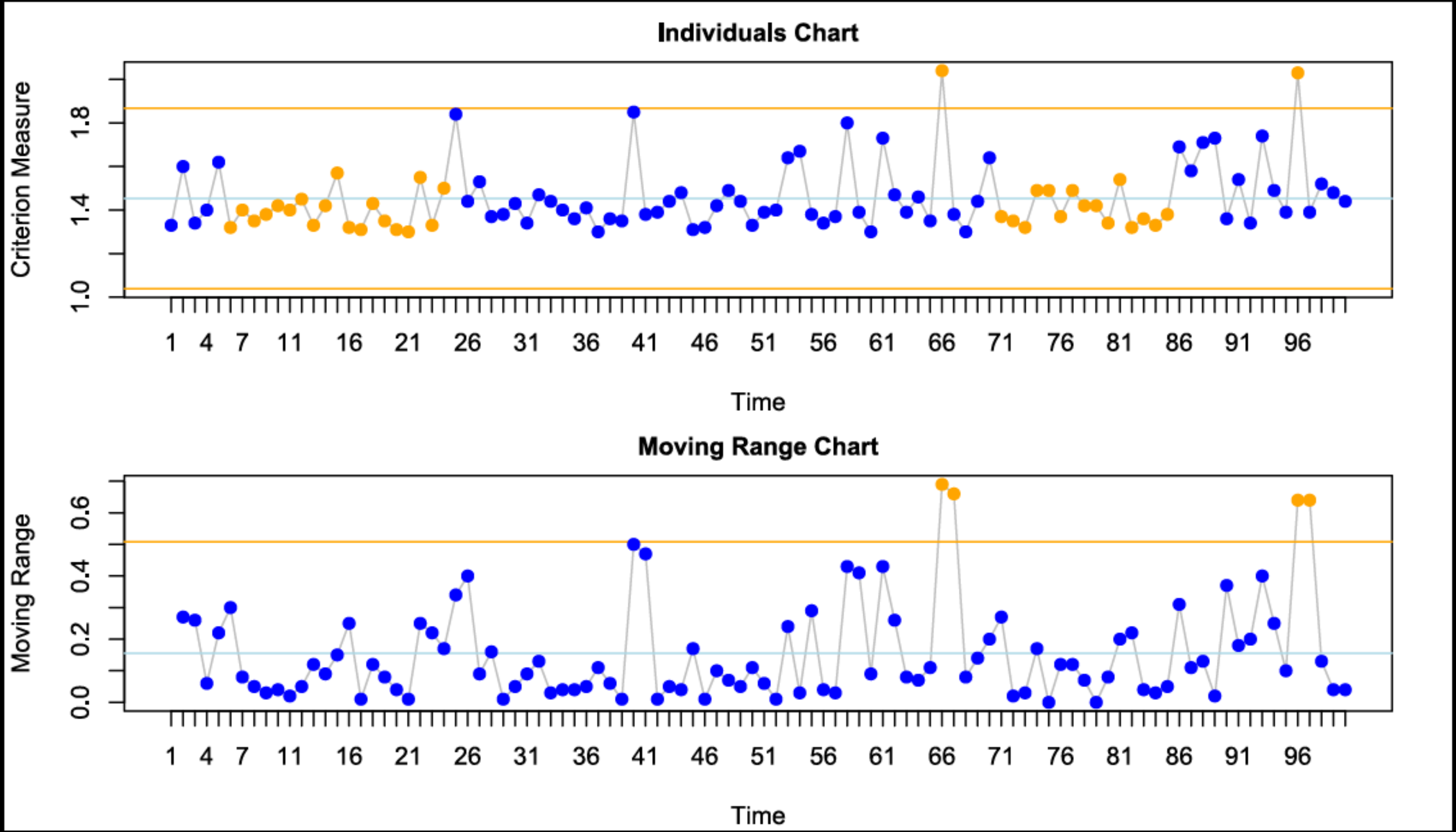
- The chart's sensitivity to changes in the process / population
- The relationship between successive points
- The effect of the shape of the process / population distribution

X and MR Charts - Distribution Shape

```
> nqtr(summary.continuous(Carb$CARB),4)
             1
dv.name      fx
n            100
missing      0
mean         1.453
var          0.0231
g3.skewness  1.8367
g3test.p     0
g4.kurtosis  3.6142
g4test.p     0.0002
```



X and MR Charts Distribution Shape



X and MR Charts

Non-Normal Distributions Approaches

1. The underlying distribution is non-normal, but can be transformed to a distribution which can be approximated by a normal distribution in order to obtain the control limits for the X chart (e.g. log-normal, Box Cox transformation)

X and MR Charts

Non-Normal Distributions Approaches

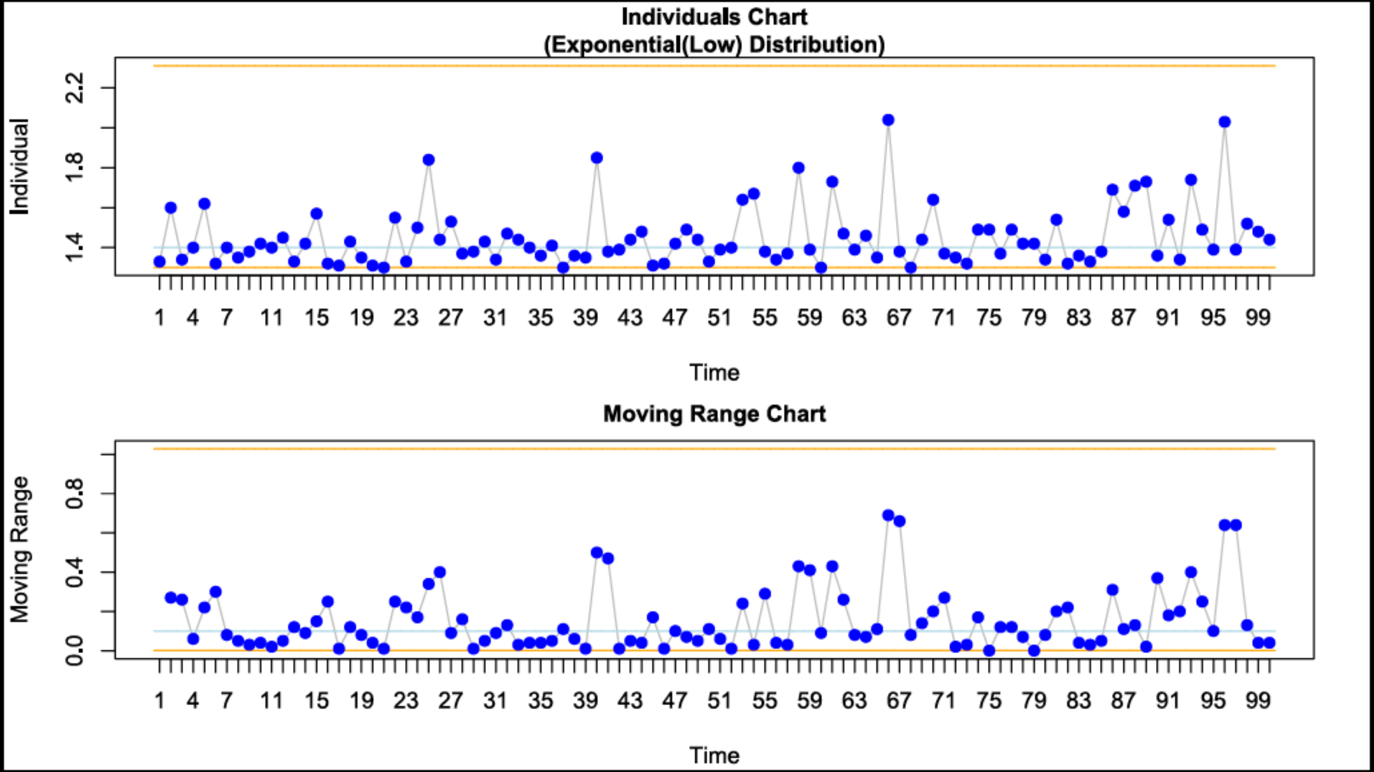
2. The underlying distribution is non-normal, but can be represented by an alternative, known mathematical model (e.g. exponential)

X and MR Charts

Non-Normal Distributions Approaches

3. The underlying distribution is non-normal, cannot be transformed to a normal distribution, and does not represent an alternative known mathematical model, so the data must be ‘fitted’ by software designed to apply a model associated with a family of distributions (e.g. Johnson, Weibull, Gamma, etc.)

X and MR Charts - Distribution Shape – Fitted Distribution



Testing for Normality

- Given a sample data set, is it reasonable to infer that the data were drawn from a normally distributed population ?

Testing for Normality

- If $n < 25$, use Anderson-Darling Test with Shapiro Wilk. If $p < 0.05$, Reject Hypothesis (Assumption) of Normality

- If $n \geq 25$, use Moment (Skewness & Kurtosis) Tests. Reject Hypothesis of Normality if either test yields p-value < 0.05.

Sources

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Individuals and Moving Range Charts – Data Transformation

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Transform non-normal distributions using the Log Normal transformation

X and MR Charts

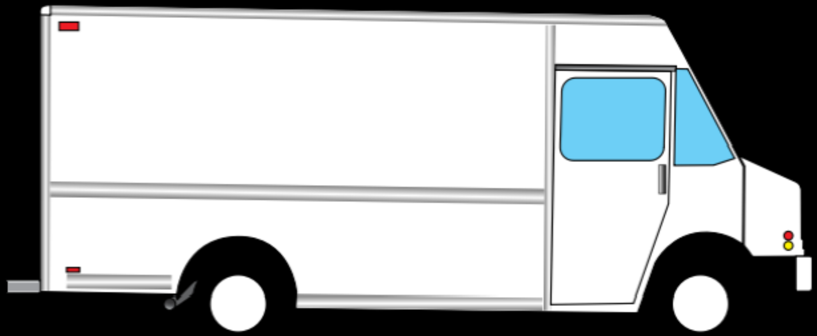
Lognormal Transformation

The Food Distributor Delivery Problem:

- Currently, for the food category in question, the temperature of the refrigeration unit upon delivery is supposed to be between 37 and 49 degrees, and ideally (Nominal) at 43 degrees.

X and MR Charts
Lognormal Transformation

- Each time a truck arrives with a temperature outside of these limits, the truck is rejected; the food is declared to be “spoilage”, and a claim filed against the Distributor.



X and MR Charts

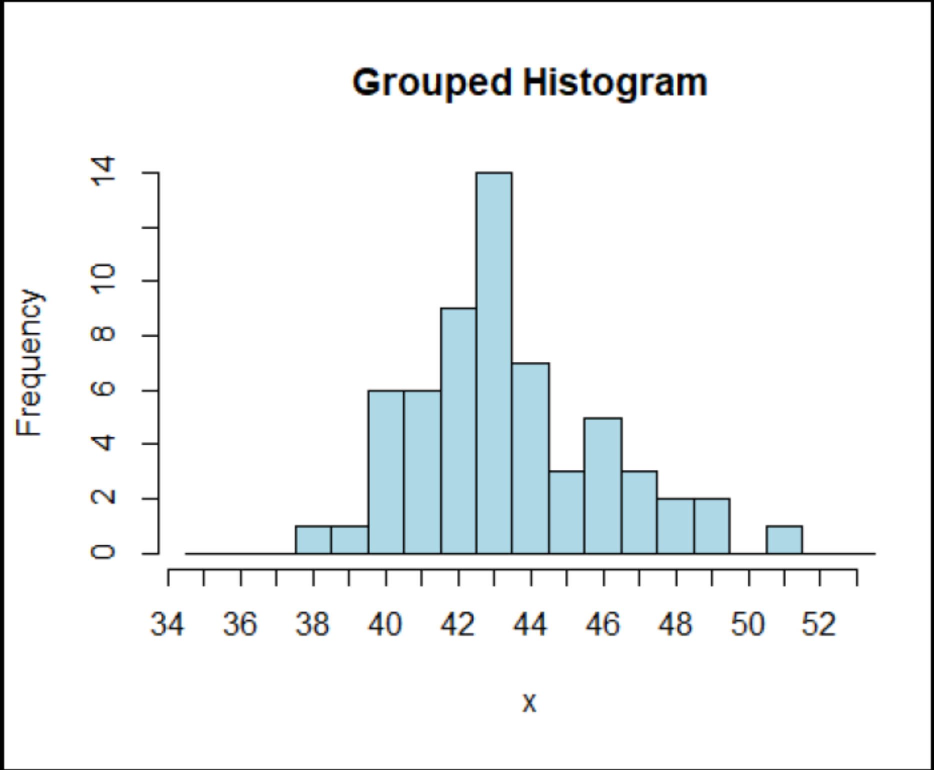
Lognormal Transformation

- For each truck that is rejected at the customer’s dock, it costs the Distributor approximately $550.00 in total losses.
- This could be a problem, in that the Distributor makes an average of 1000 deliveries per day for this type of food.

X and MR Charts
Lognormal Transformation

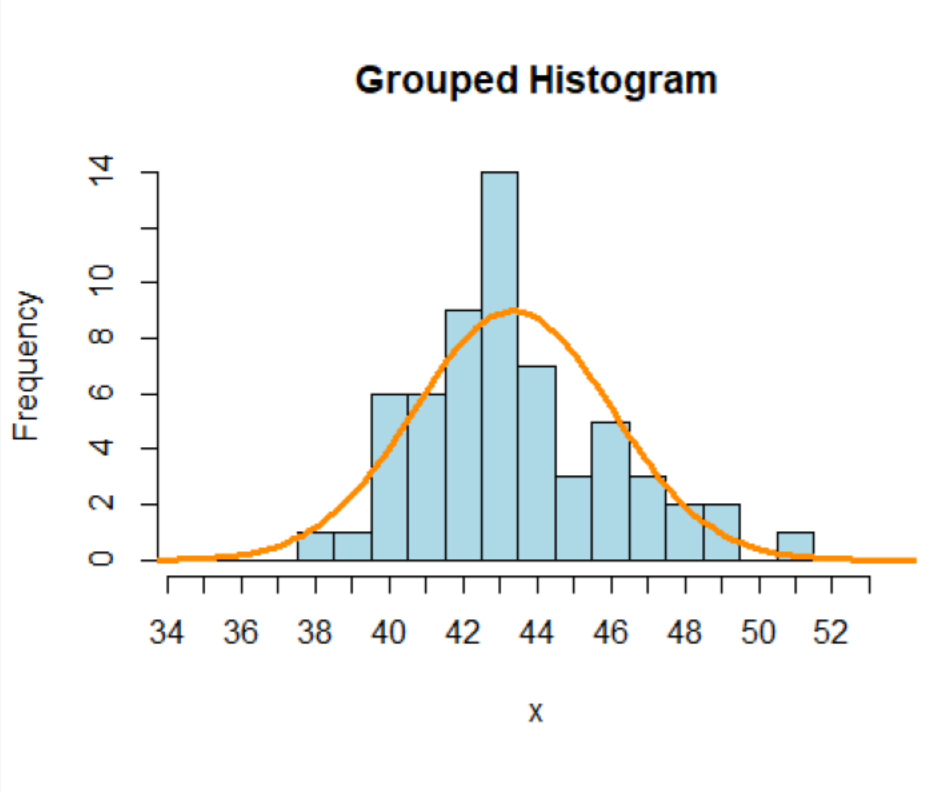
- A random sample of truck delivery records have been selected for review from the last few months of delivery data (Delivery.dat).
- When evaluating the data, we find that the Temperature data are non-normal, and that we are sampling from a moderately skewed, mesokurtic distribution:

Transforming Non-Normal Distributions




```
nqtr(summary.continuous(Delivery$Temp)
, 5)
n            60
mean         43.36667
var          7.08362
g3.skewness  0.63363
g3test.p     0.04312
g4.kurtosis  0.31915
g4test.p     0.47427
```

Transforming Non-Normal Distributions



This condition renders the use of s and $\hat{s}$ values questionable for the generation of control limits.

Transforming Non-Normal Distributions

- In many instances, these type of non-normal distributions may be transformed with mathematical functions to achieve a state of normality; specifically, the Natural-Log Transformation.

Transforming Non-Normal Distributions

- This transformation requires us to calculate the ln(X) values, re-test them for normality, and if we accept normality, generate the required control limits from these transformed values.
- All these operations may be executed within Rstudio.

Transforming Non-Normal Distributions

Filter

	Temperature
1	44
2	40
3	46
4	40
5	50
6	43
7	40
8	42
9	48
10	42
11	46

The raw data as it was gathered.

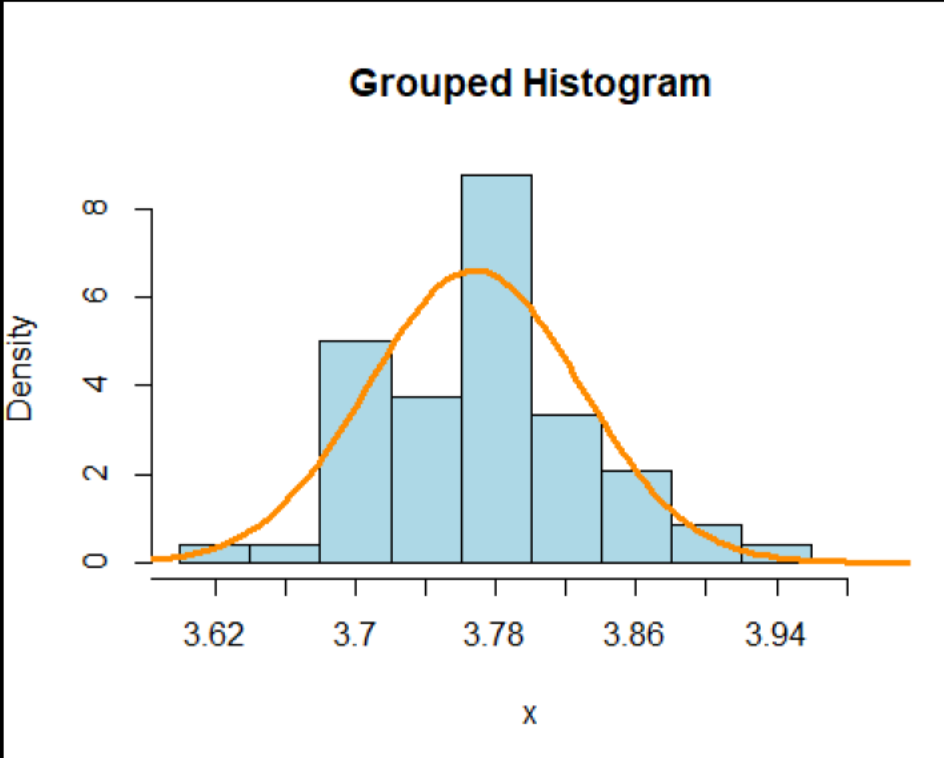
Transforming Non-Normal Distributions

```
> Delivery$lntemp<-log(Delivery$Temp)
```

Transforming Non-Normal Distributions

- Testing the transformed data for normality, we can now see that the log X values are normally distributed, so that the use of s (the standard deviation) is now justified.

Transforming Non-Normal Distributions




```
nqtr(summary.continuous(Delivery$lntemp)
,5)
n              60
mean           3.76788
var            0.00366
g3.skewness    0.46738
g3test.p       0.12596
g4.kurtosis    0.11273
g4test.p       0.68757
```

Transforming Non-Normal Distributions

When using the Lognormal Transformation, there are two considerations:

- If the data are all positive (no negative values) the log transformation can simply be applied to the original data values.

Transforming Non-Normal Distributions

- If there are negative values in the data set (which can occur with Interval scale data or when data are taken from a reference point), a constant must be added to each data value prior to performing the log transformation.

Transforming Non-Normal Distributions

- In this case, we will add 2 times the absolute value of the minimum value to each value prior to the lognormal transformation

Transforming Non-Normal Distributions

- This transformation helps to avoid taking the log of a negative number, which would result in a ‘NaN’ (not a number) in R.

Transforming Non-Normal Distributions

```
> Delivery$Temp2<-2*abs(min(Delivery$Temp))
+ Delivery$Temp


> Delivery$lntemp2<-log(Delivery$Temp2)
```

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Individuals and Moving Range Charts – Data Transformation

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Calculate the natural tolerance of the transformed distribution and transform it back into the original distribution

Transforming Non-Normal Distributions

- Once we've transformed the data, we can calculate the:

- Upper Natural Process Limit at + 3σ
- Lower Natural Process Limit at – 3σ.

Transforming Non-Normal Distributions

In R Studio

```
natural.tolerance.normal(x = Delivery$lntemp)

Delivery.ln<-natural.tolerance.normal(x =
Delivery$lntemp)

(LNPL.ln<-Delivery.ln$lower.limit) = 3.586387

(UNPL.ln<-Delivery.ln$upper.limit) = 3.949363
```

Transforming Non-Normal Distributions

Once the UNPL and LNPL have been determined in the transformed distribution, we would then use these values to generate our control limits for our raw data chart by taking the inverse of the log values.

Transforming Non-Normal Distributions

In R Studio

`(LNPL<-exp(LNPL.ln))` = $e^{3.586387} = 36.1034$

`(UNPL<-exp(UNPL.ln))` = $e^{3.949363} = 51.9023$

Transforming Non-Normal Distributions

- These new control limits for the X chart would be placed into the upper and lower control limits.
- The centerline would be changed to the median, and the Moving Range chart values would be identical to what was previously employed.

Transforming Non-Normal Distributions

```
spc.chart.variables.individual.and.movingrange.
generic.simple(individuals = Delivery$Temp,
      chart1.center.line = median(Delivery$Temp),
      chart1.control.limits.ucl = UNPL,
      chart1.control.limits.lcl = LNPL)
```

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Individuals and Moving Range Charts – Known Mathematical Model

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Recognize and test data for exponentiality

X and MR Charts Non-Normal Distributions

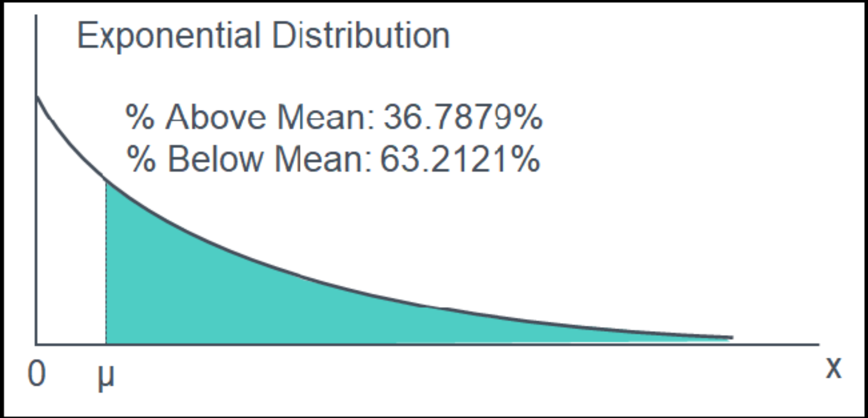
- The underlying distribution is non-normal, but can be represented by an alternative, known mathematical model (e.g. exponential)

The Exponential Distribution

- Although the Normal Distribution is commonly associated with many types of data sets, it is not the only continuous function which appears with great frequency in industry.

The Exponential Distribution

- The Exponential Distribution is one example of a frequently-occurring continuous function found in business and industrial situations.



X and MR Charts Non-Normal Distributions

The Request For Proposal (RFP) Cycle Time Problem:

- A Brand Marketing Agency is currently the second largest agency in the country.
- Their services includes branding, website design, ecommerce solutions, graphic design, and digital marketing.

X and MR Charts Non-Normal Distributions

- They serve industries from startup companies to large software and technology firms.

X and MR Charts Non-Normal Distributions

- Periodically, the company receives an RFP, which must be processed and answered within 48 hours, or they lose the opportunity to acquire the business. Ideally (i.e. nominally), they would like to process a response in 36 hours.

X and MR Charts Non-Normal Distributions

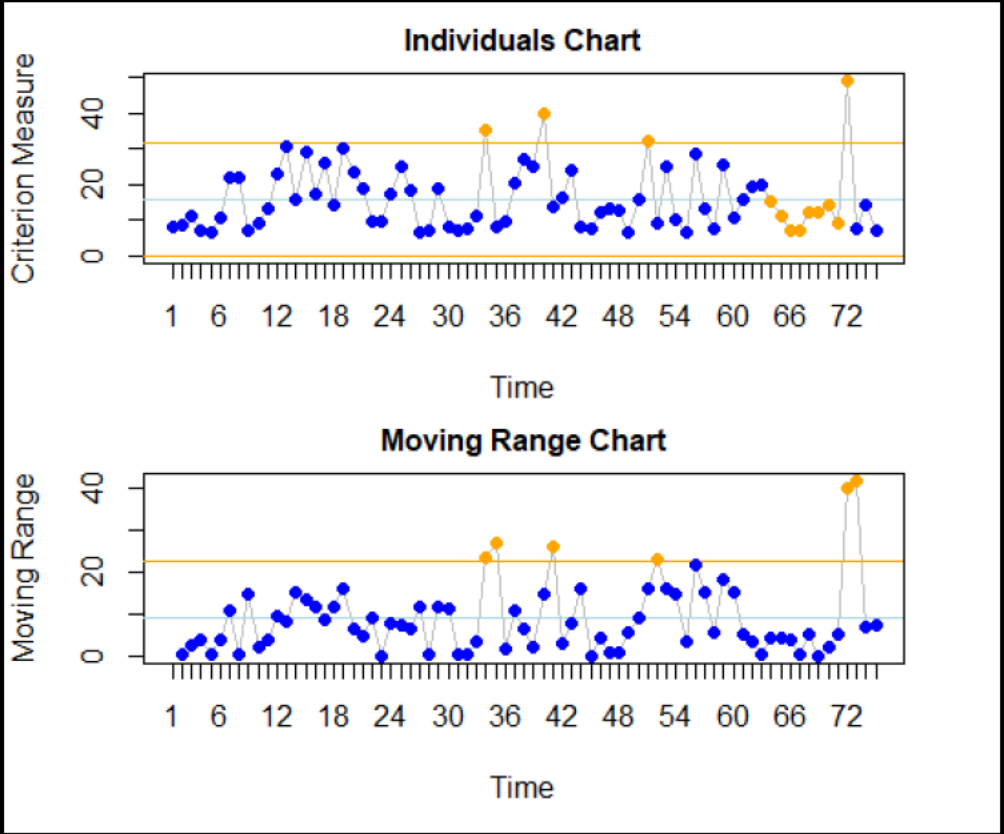
- Data has been collected for the last 75 RFP Responses processed, with the number of hours required for completion recorded (RFP_Response_Time.dat).

X and MR Charts Non-Normal Distributions

- In the case of these data, the worst possible decision would be to plot the data on a standard X and Moving R chart, assuming normality:

X and MR Charts Non-Normal Distributions

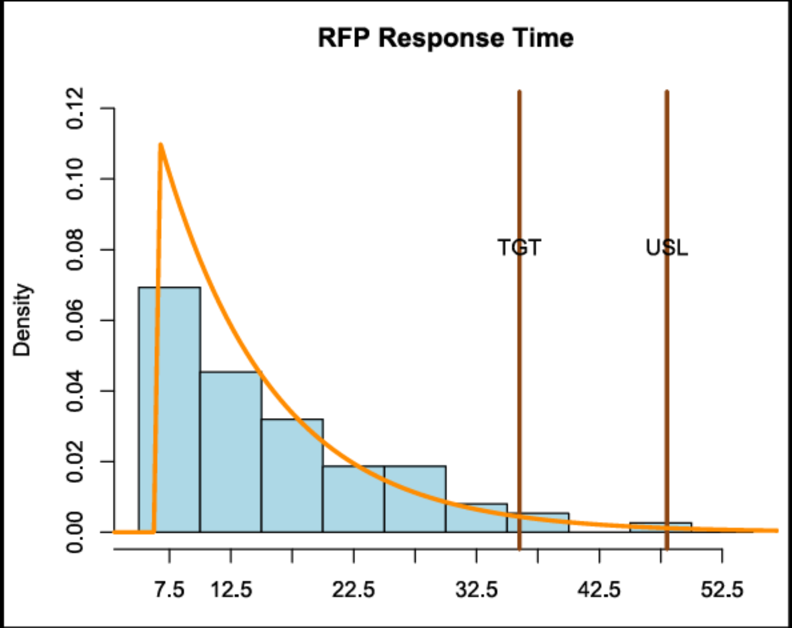
- ….especially after testing for normality



X and MR Charts Non-Normal Distributions: Approaches

- Conducting the moment tests for normality (n > 25)

```
nqtr(summary.continuous(RFB_Response_Time$Time, stat.min=T),5)
n 75
mean         15.84275
var          79.7072
min          6.76
g3.skewness  1.28236
g3test.p     0.00008
g4.kurtosis  1.72653
g4test.p     0.02277
```



X and MR Charts Non-Normal Distributions : Approaches

- Noting the data appears to be an Exponential function we could run the Shapiro-Wilks Test for Exponentiality:

```
shapiro.wilk.exponentiality.test(x = RFB_Response_Time$Time)
data:  input data
W = 0.014175, null hypothesis W statistic = 1,
p-value = 0.9287
alternative hypothesis: true W statistic is not equal to 1
sample estimates:
          W sample.size
 0.01417539 75.00000000
```

X and MR Charts Non-Normal Distributions

- This would cause us to infer that the RFP Response Time data was drawn from a population which could be evaluated as an Exponential Distribution.

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Individuals and Moving Range Charts – Known Mathematical Model

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Calculate Control Limits for data that is distributed exponentially

X and MR Charts Non-Normal Distributions

- To generate control limits, we would use the following procedure.
- For 3 standard error limits, we need to find the values associated with the center 99.73% (UNPL and LNPL) of the distribution.

X and MR Charts Non-Normal Distributions

- To accomplish this, we first generate the mean and lowest observed value associated with the data, so that we can obtain an estimate of Omicron from X_L (the minimum value).

X and MR Charts Non-Normal Distributions

```
nqtr(summary.continuous(RFP_Response_Time$Time, stat.min=T),5)
n 75
mean          15.84275
var           79.7072
min           6.76
g3.skewness 1.28236
g3test.p    0.00008
g4.kurtosis 1.72653
g4test.p    0.02277
```

X and MR Charts Non-Normal Distributions

- Next, using the natural tolerance function for the Exponential Low Distribution function in lolcat, obtain the LNPL and UNPL

```
> natural.tolerance.exp.low( )
```

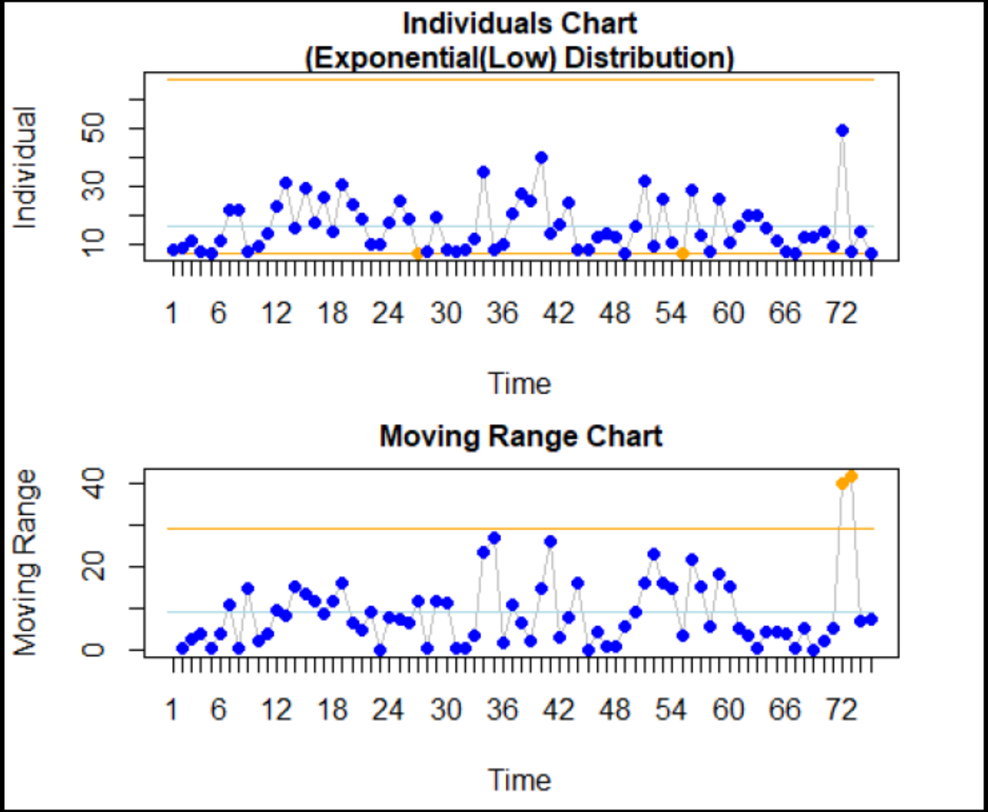
natural.tolerance	lower.limit	upper.limit	lower.area	upper.area
60.00338	6.77227	66.77565	0.00135	0.00135

X and MR Charts Non-Normal Distributions

- Next, using the variables chart for the Exponential Low Distribution function in lolcat, generate the chart

```
spc.chart.variables.individual.and.movi
ngrange.exponential.low.simple(individu
als =, low =)
```

X and MR Charts Non-Normal Distributions



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Individuals and Moving Range Charts – Known Mathematical Model

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objectives:

Calculate Control Limits for data that is distributed exponentially

Generate the X and MR chart using R software for exponential data

X and MR Charts Non-Normal Distributions

- The difficulty associated with mathematical distributions as skewed as the Exponential function relates not so much to the X chart, but to the control limits to be employed for the Moving Ranges.

X and MR Charts Non-Normal Distributions

- The constants associated with the Moving Range chart simply do not accommodate the expected distribution of Moving Ranges at n=2 that one would anticipate from an Exponential function.

X and MR Charts Non-Normal Distributions

- Review the following distribution of Moving Ranges generated from a Monte Carlo simulation of values anticipated from a standard exponential function with an Omicron of 6.76 and a mean of 15.84:

X and MR Charts Non-Normal Distributions

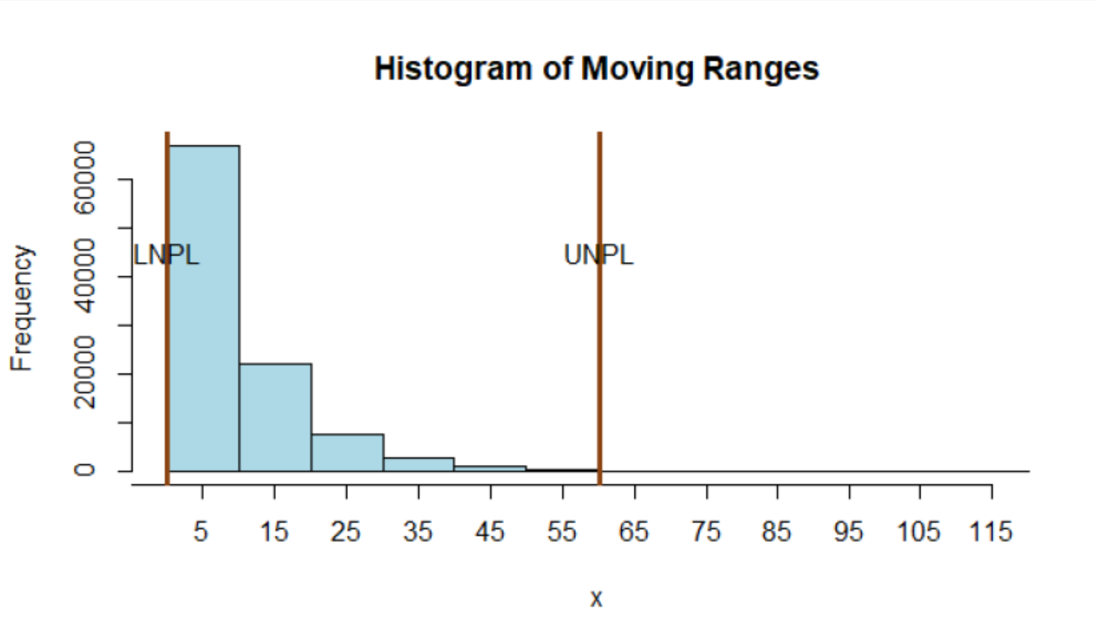
```
rexp.low(n = 100000, low = 6.76, mean = mean(RFP_Response_Time$Time))

mr.exp.low<- c(abs(diff(mc)))
```

Shape Test indicates the data are distributed exponentially

X and MR Charts - Expected Moving Range Values

```
nqtr(natural.tolerance.exp(x =
mr.exp.low),5)
natural.tolerance 60.2142
lower.limit       0.01231
upper.limit       60.22655
lower.area        0.00135
upper.area        0.00135
```



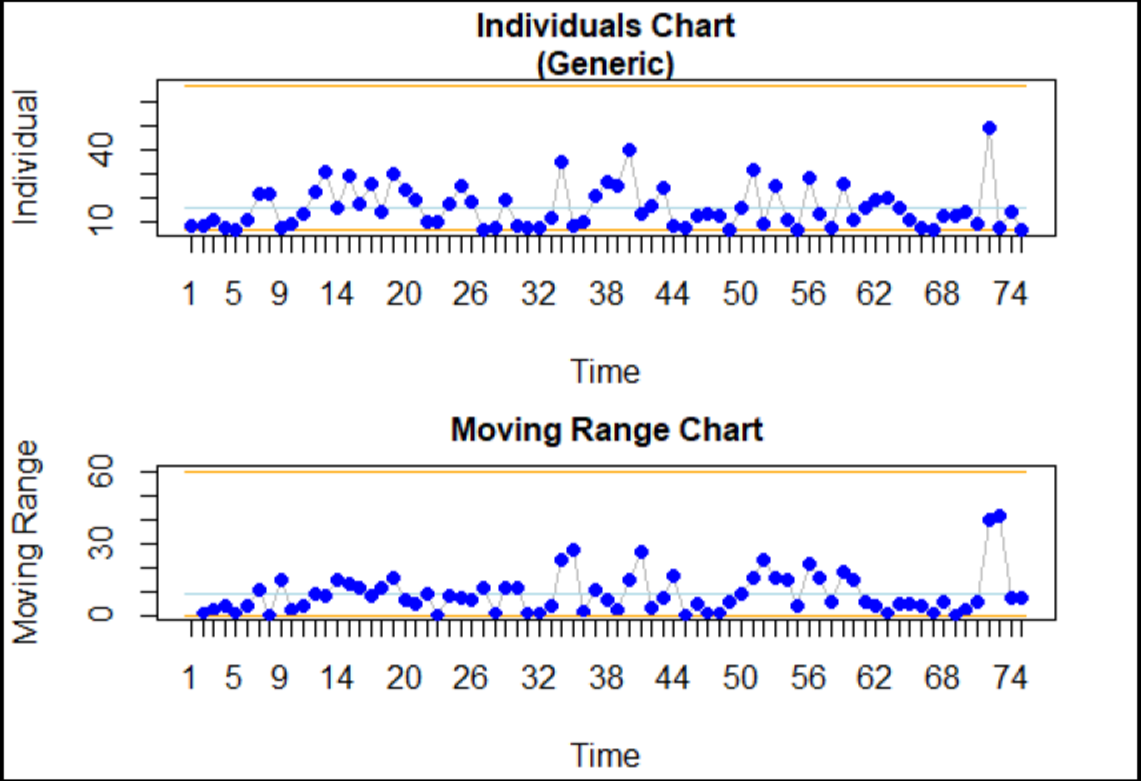
X and MR Charts

Standard Exponential Formula-Based MR Chart

- Using the estimates of control limits generated for the X chart from the Exponential distribution, but with the standard exponential Moving Range values, we would obtain:

X and MR Charts

Standard Exponential Formula-Based MR Chart



X and MR Charts Non-Normal Distributions

- What would you use as a centerline?
  - Mean?
  - Gives you real information from all points
  - Need to recalculate the run rules
  - How would you do this?

X and MR Charts Non-Normal Distributions

- $0.6321^x = 0.0027$ below the mean
  - The lowest value is the mode – you expect to see it a lot!
  - Run below $= \frac{\ln .0027}{\ln .6321} = 12.894$ and

- $0.3679^x = 0.0027$ above the mean
  - Run above$= \frac{\ln .0027}{\ln .3679} = 5.914$

X and MR Charts Non-Normal Distributions

Define new rules

```
# Changing rules
rules <-
spc.rulesets.nelson.1984.test.1.2.3.4()

# Turn off the lower control limit rule
rules$outside.limits <-
spc.controlviolation.nelson.1984.test1.outsid
e.zone.a.upper
```

X and MR Charts Non-Normal Distributions

Define new rules

```
# If using the mean for the X chart, adjust the run rules

rules$runs        <- NULL

rules$runs.above <- spc.controlviolation.nelson.1984.test2.runs.above.create(point.count = 6)

rules$runs.below <- spc.controlviolation.nelson.1984.test2.runs.below.create(point.count = 13)
```

X and MR Charts Non-Normal Distributions

Define new rules

```
# Test for run rules

runs.overall <-
unique(exp.chart$chart1.is.control.violation$
rule.results$runs.above |
exp.chart$chart1.is.control.violation$rule.re
sults$runs.below)
```

X and MR Charts Non-Normal Distributions

- Median?
  - Allows you to use the traditional run rules of 8 above or below
  - Less precise estimate of the location

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Explain why data is fit with R software

X and MR Charts Non-Normal Distributions

- The underlying distribution is non-normal, cannot be transformed to a normal distribution, and does not represent an alternative known mathematical model, so the data must be ‘fitted’ by software designed to apply a model associated with a family of distributions (e.g. Johnson, Weibull, Gamma, etc.)

X and MR Charts Non-Normal Distributions

The MAP Sensor Problem:

- A major automobile manufacturer produces a Manifold Absolute Pressure Sensor (MAP) Sensor, an electronic device that links the Powertrain Control Module with the engine in all its automobiles.

X and MR Charts Non-Normal Distributions

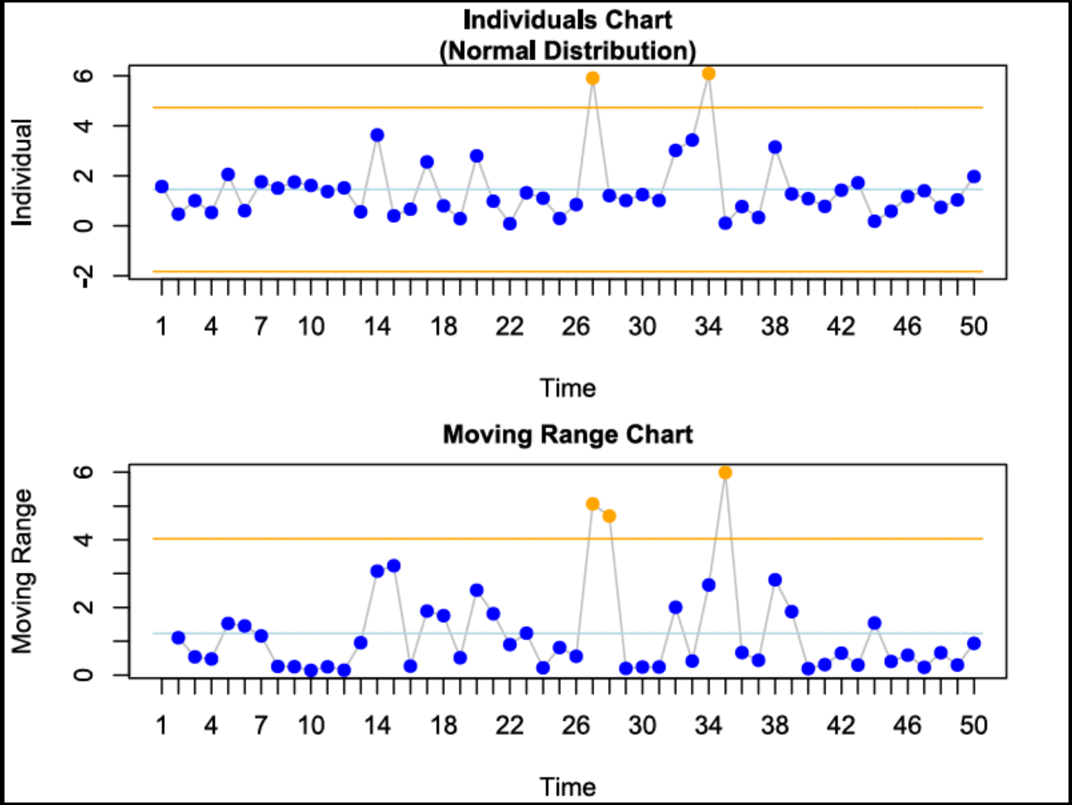
- Inside this sensor is a ceramic substrate, with surface mounted components.
- The placement of these components is critical, and their location is measured from datum reference points in the X, Y, and Z axes.

X and MR Charts Non-Normal Distributions

- The data file mapsensor.dat contains the z-axis values for one of the critical components; from 50 consecutive production lots.
- The specification for this component is 0.9500+/-0.4000 (coded data in thousandths of an inch)

X and MR Charts Non-Normal Distributions

- The initial X and Moving R chart appeared as follows:

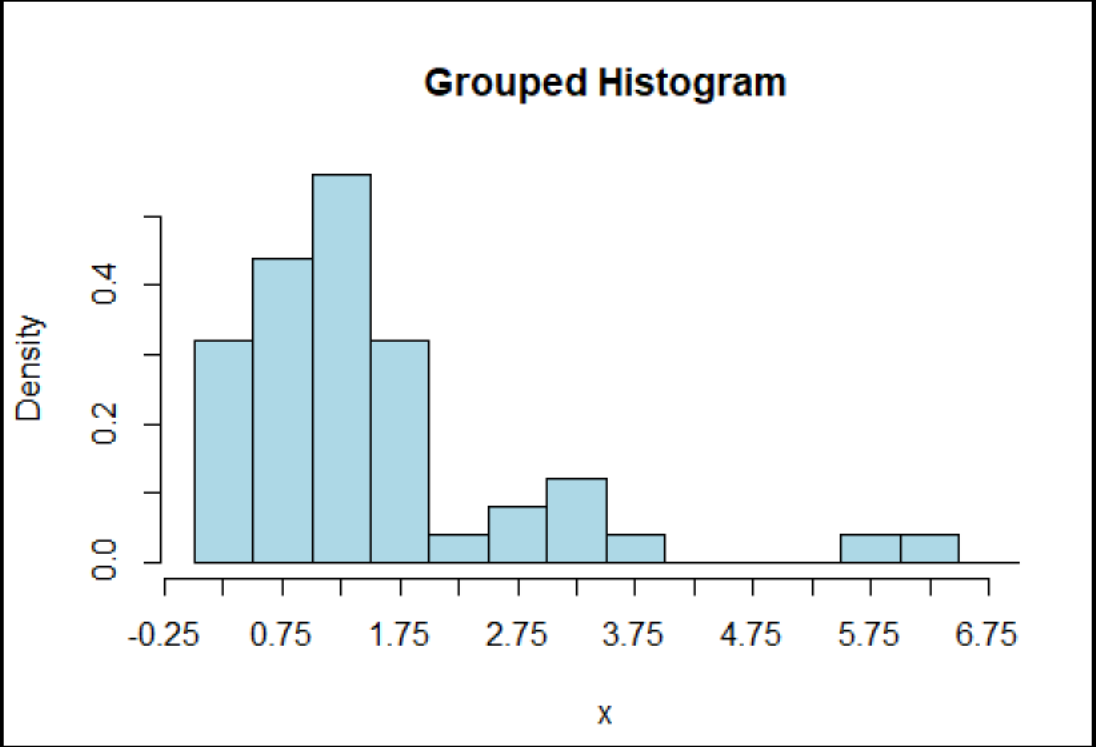


X and MR Charts Non-Normal Distributions

- A cursory view of the X chart showed a clearly suspicious LCL, as compared to the observed data. Generating a normality analysis and frequency histogram for the observed data, the reason became obvious:

X and MR Charts Non-Normal Distributions

```
nqtr(summary.continuous(mapsensor$z_axis, stat.min=T),5)
n 50
mean      1.45956
var       1.59556
min       0.0878
g3.skew   2.11091
g3test.p  0
g4.kurt   5.28718
g4test.p  0.00039
```



X and MR Charts Non-Normal Distributions

- If no adequate transformations proved satisfactory, it would be at this point that the distribution-fitting approach might be employed to find the equivalent standard error values which can be employed with the data set.

X and MR Charts Non-Normal Distributions

- In some ways, this is a somewhat iterative process.
- Additionally, different software packages, even when fitting the identical distributions, will not necessarily yield the same ‘best fit’ result.

X and MR Charts Non-Normal Distributions

- In fact, sometimes no totally satisfactory fit can be found, in which case one chooses between sufficient fits at the extreme versus central portion of the distribution.

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

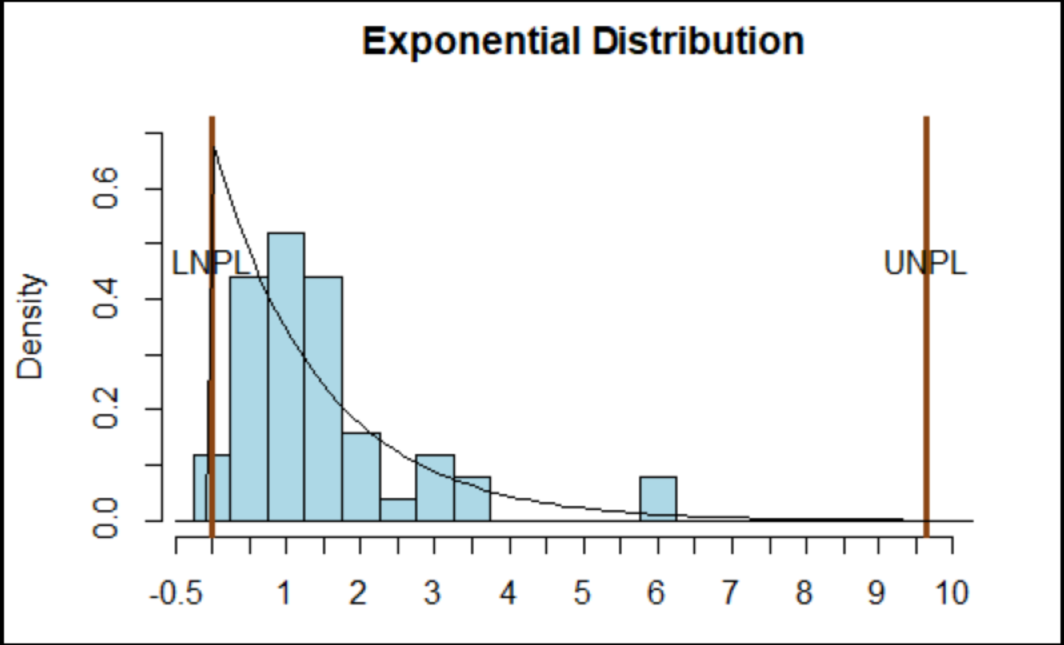
Learning objective:

Calculate Control Limits for data using a fitted distribution

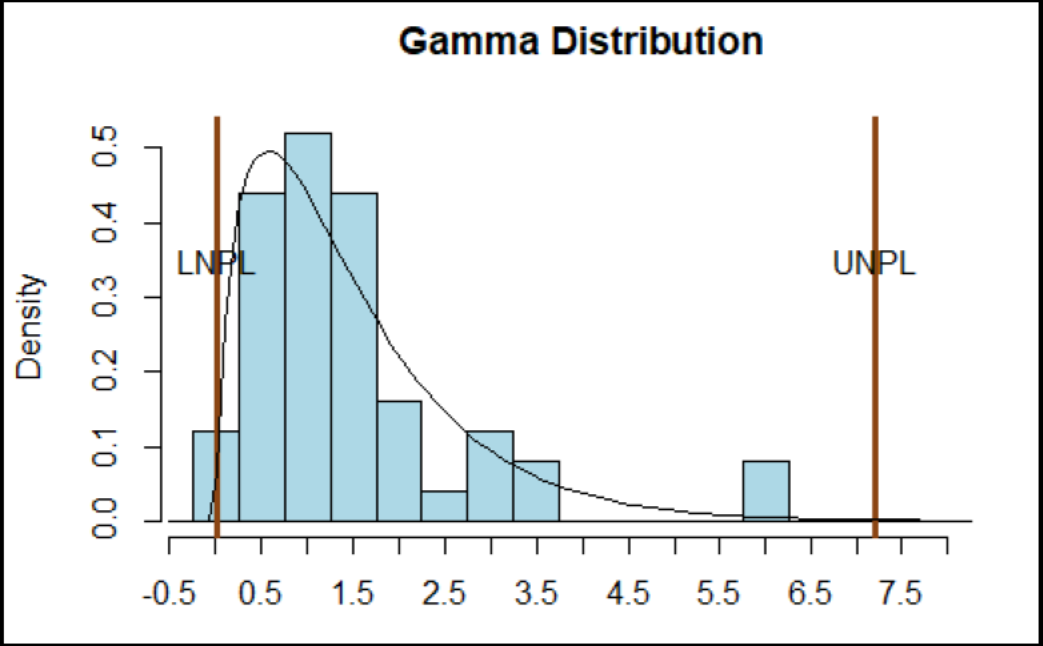
X and MR Charts Non-Normal Distributions

- Let’s look at some possibilities for the Map Sensor data

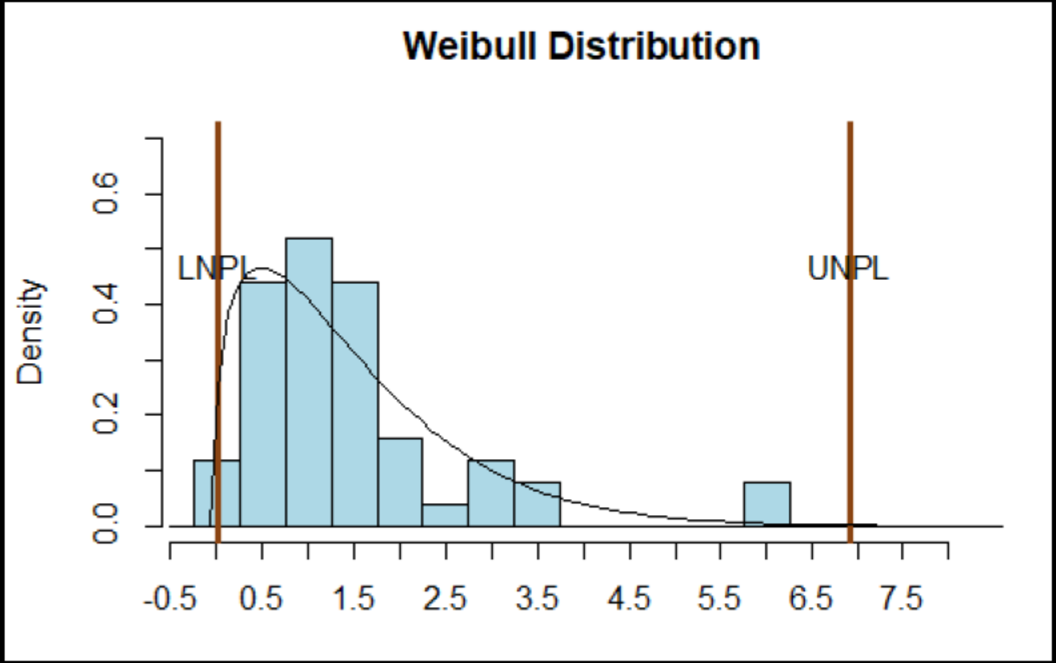
X and MR Charts Non-Normal Distributions



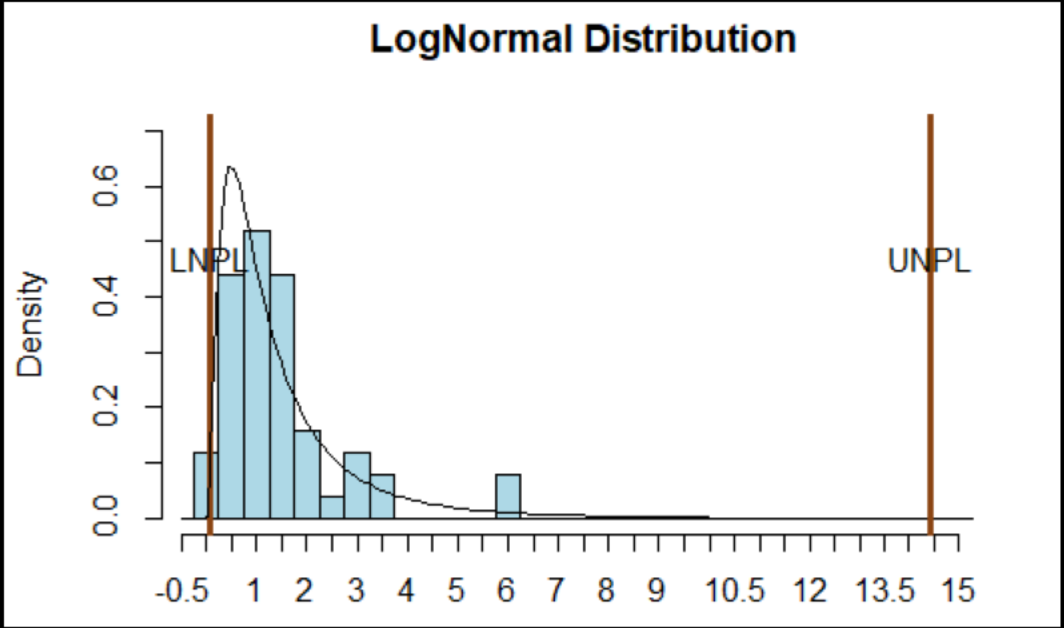
X and MR Charts Non-Normal Distributions



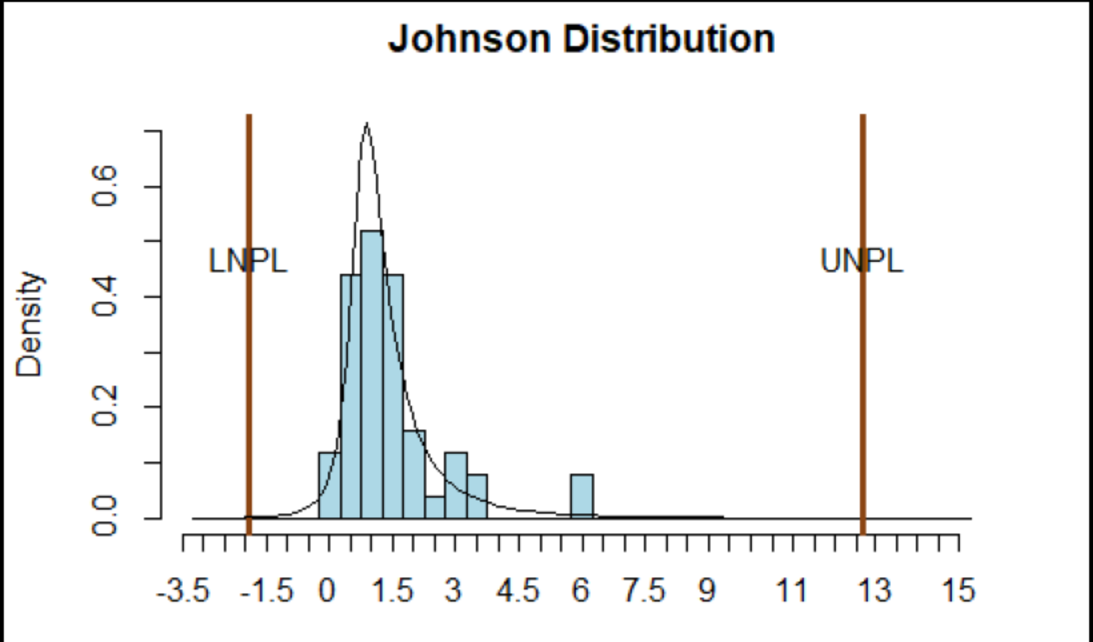
X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions

- Best fit from available distributions is the distribution with:
  - Lowest AIC value
  - Best fit in the tail regions in plots

X and MR Charts Non-Normal Distributions

Distribution	LPL	UPL	AIC	Other
Exponential	0.002	9.644	139.81	
Weibull	0.009	6.913	136.89	
Gamma	**0.021**	**7.213**	**135.31**	
Log Normal	0.076	14.418	136.99	
Johnson	-1.908	12.687	147.71	

X and MR Charts & Non-Normal Distributions

Control Chart with Gamma distribution for the individuals, Exponential distribution for moving range

```
spc.chart.variables.individual.and.movingrange.
generic.simple(individuals = mapsensor$z_axis
,chart1.center.line = median(mapsensor$z_axis)
,chart1.control.limits.lcl = LNPL.gamma
,chart1.control.limits.ucl = UNPL.gamma
,chart2.control.limits.lcl = LNPL.mr.exp
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```

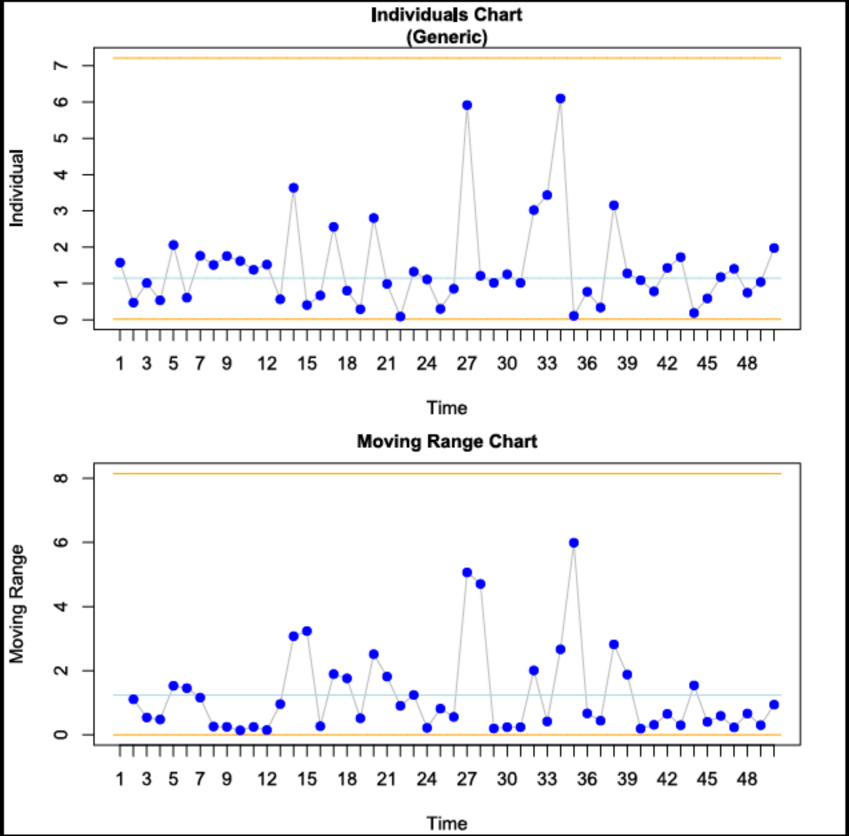

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

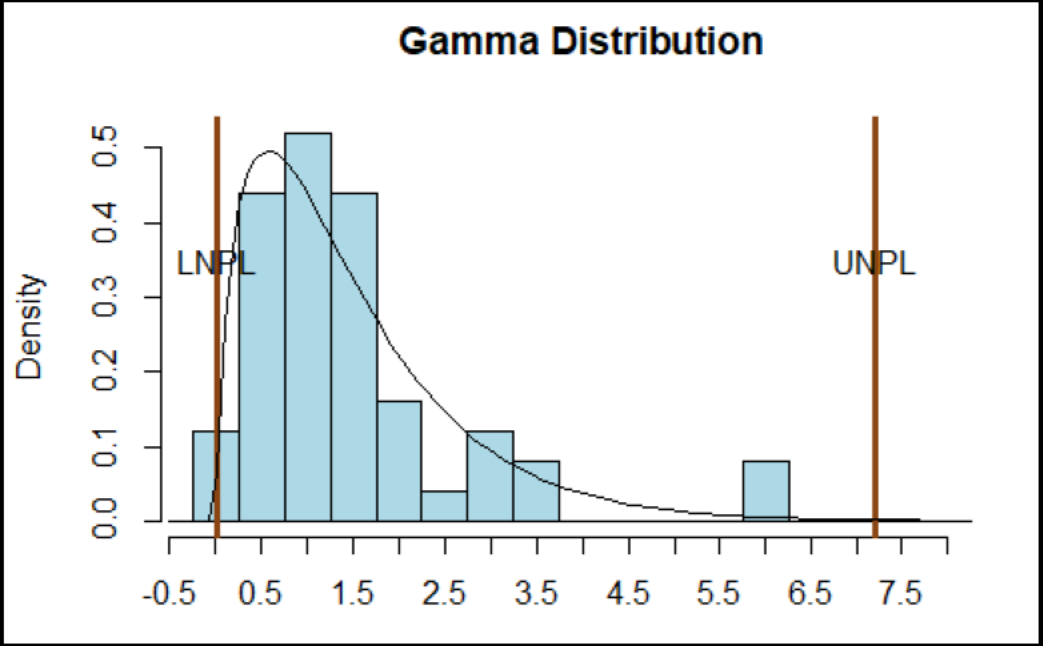
Learning objective:

Perform a goodness of fit test for multiple distributions

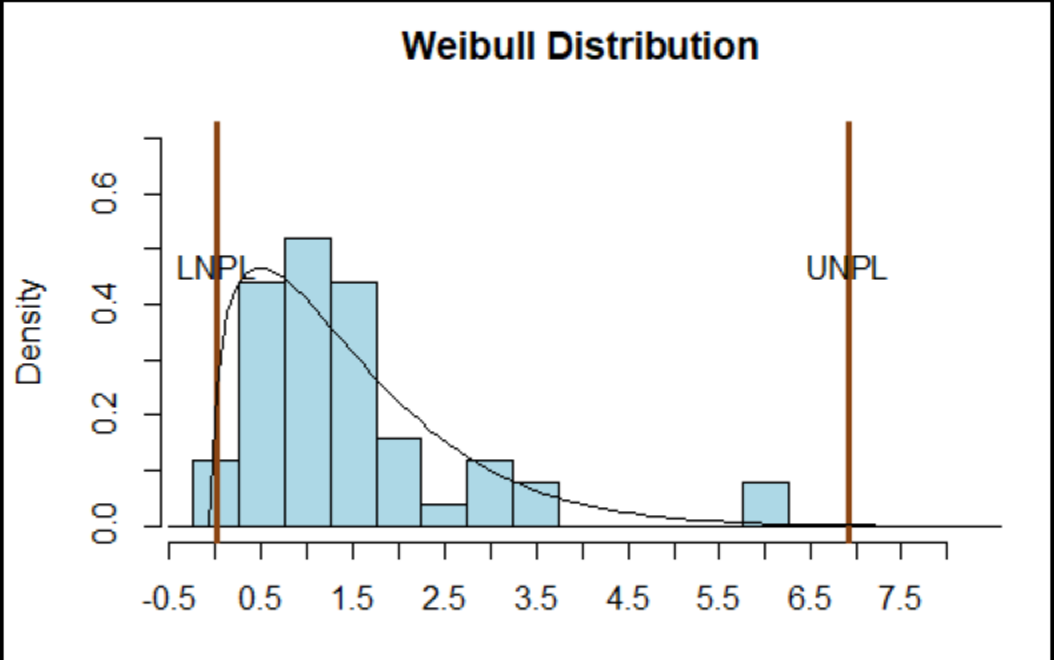
X and MR Charts Non-Normal Distributions

- Let’s look at some more possibilities for the Map Sensor data

X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions



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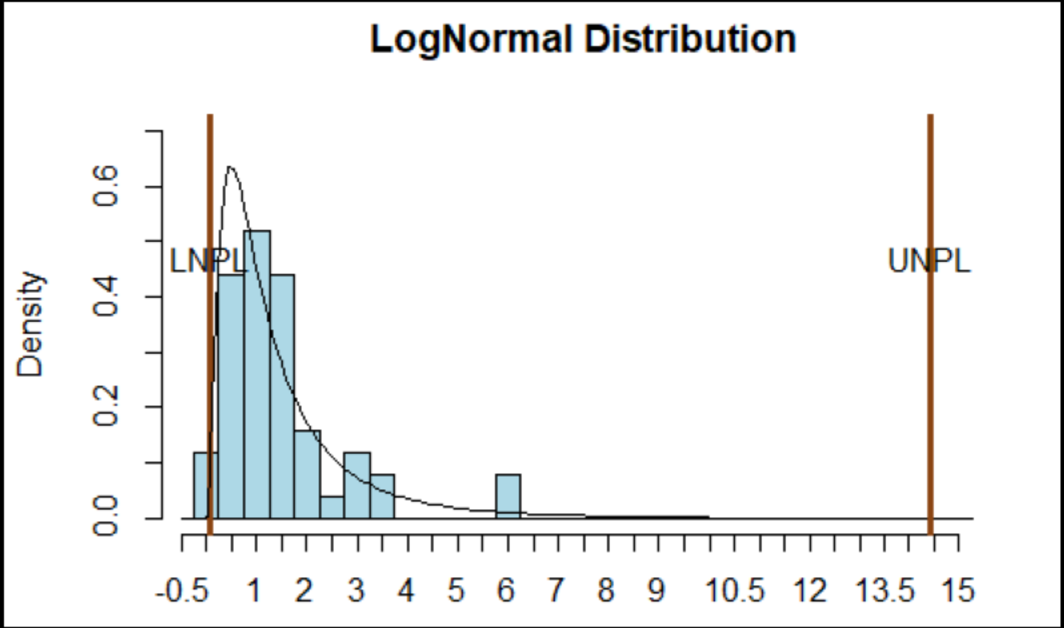
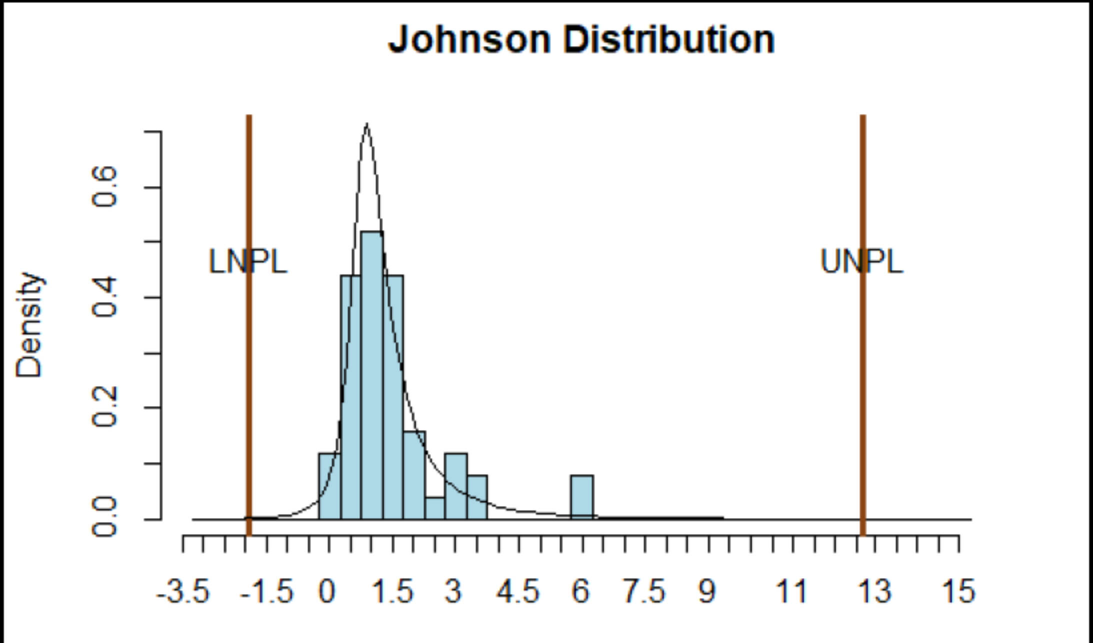


X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions

- Best fit from available distributions is the distribution with:
  - Lowest AIC value
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X and MR Charts Non-Normal Distributions

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X and MR Charts & Non-Normal Distributions

Control Chart with Gamma distribution for the individuals, Exponential distribution for moving range

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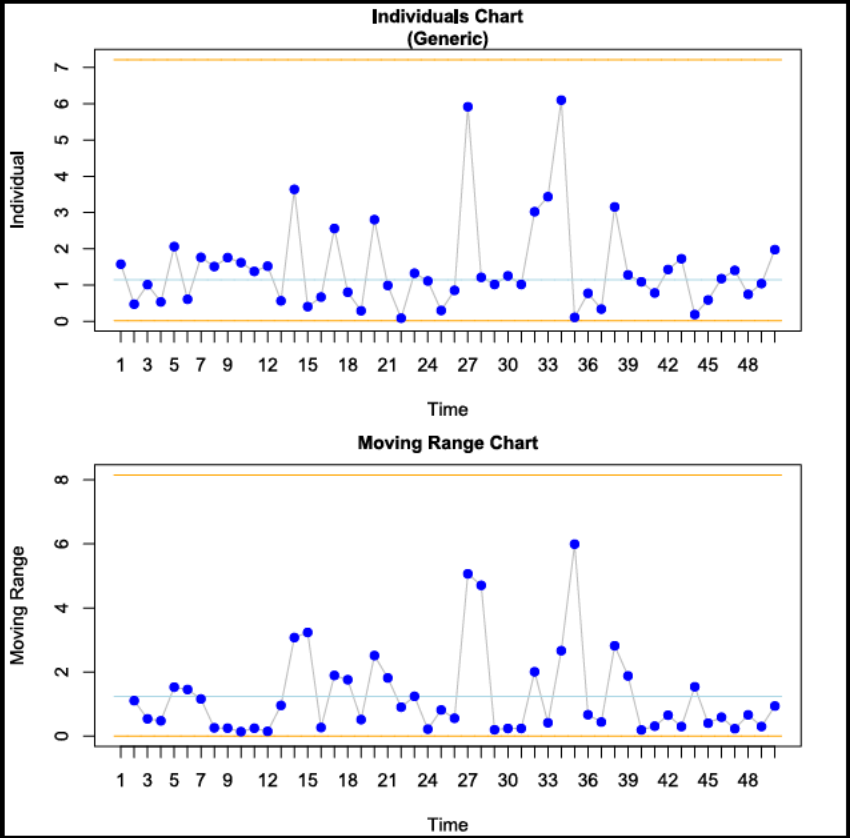

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

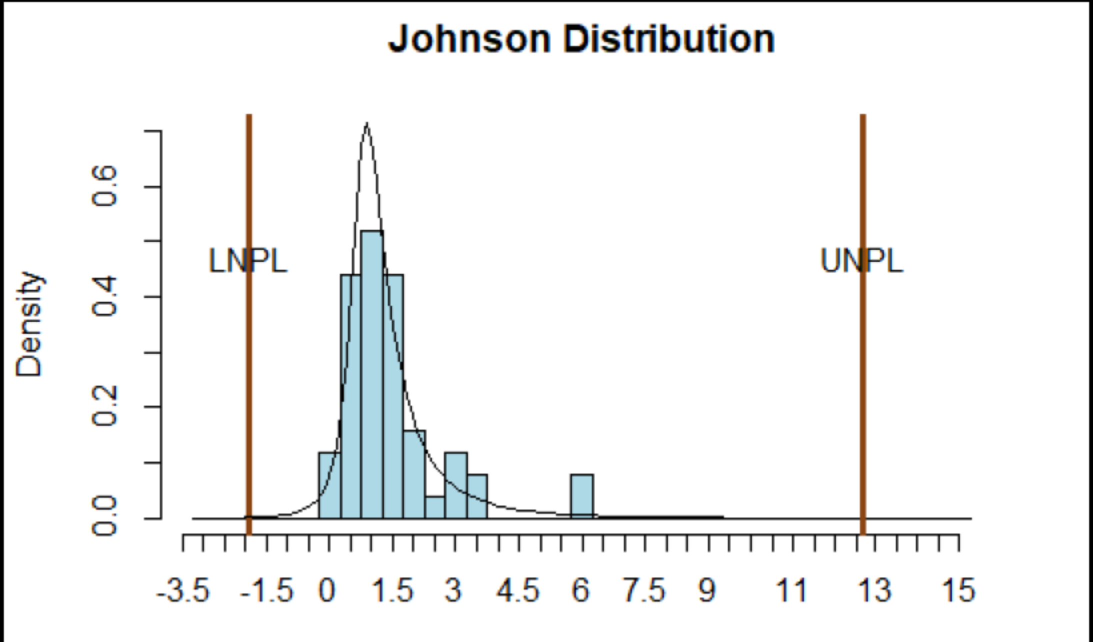
Learning objective:

Perform a goodness of fit test for the Johnson Distribution

X and MR Charts Non-Normal Distributions

- The Johnson family of distributions is highly flexible, fitting a wide variety of curves:
  - Johnson S_U
  - Johnson S_B
  - Lognormal
  - Normal

X and MR Charts Non-Normal Distributions



X and MR Charts Non-Normal Distributions

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X and MR Charts Non-Normal Distributions

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X and MR Charts & Non-Normal Distributions

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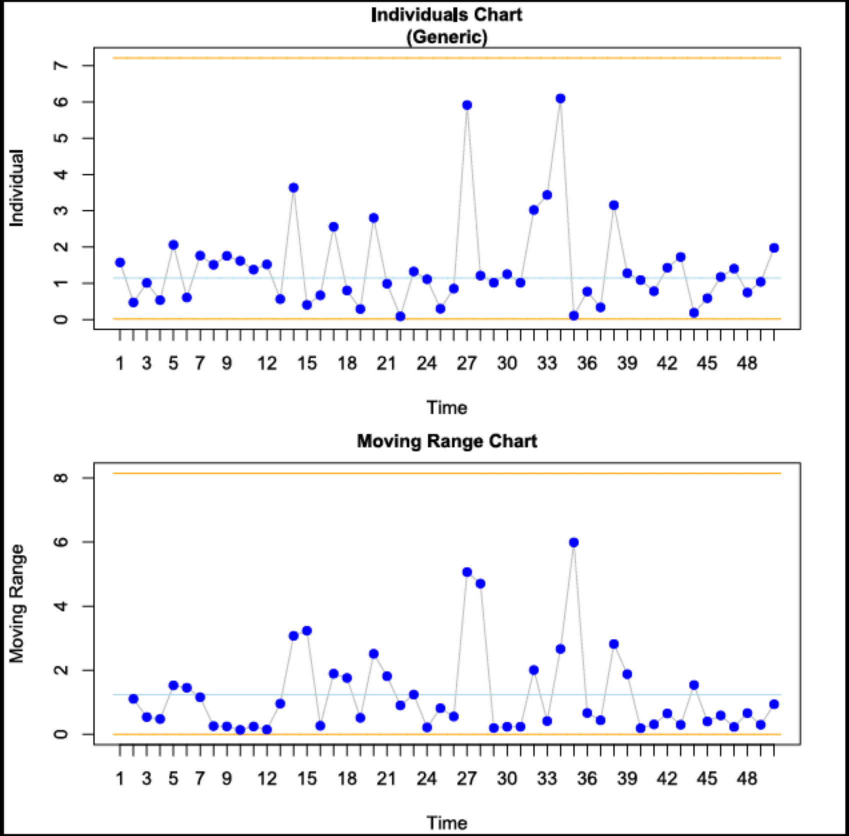

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Calculate Control Limits for data using a fitted distribution

X and MR Charts Non-Normal Distributions

- Best fit from available distributions is the distribution with:
  - Lowest AIC value
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X and MR Charts Non-Normal Distributions

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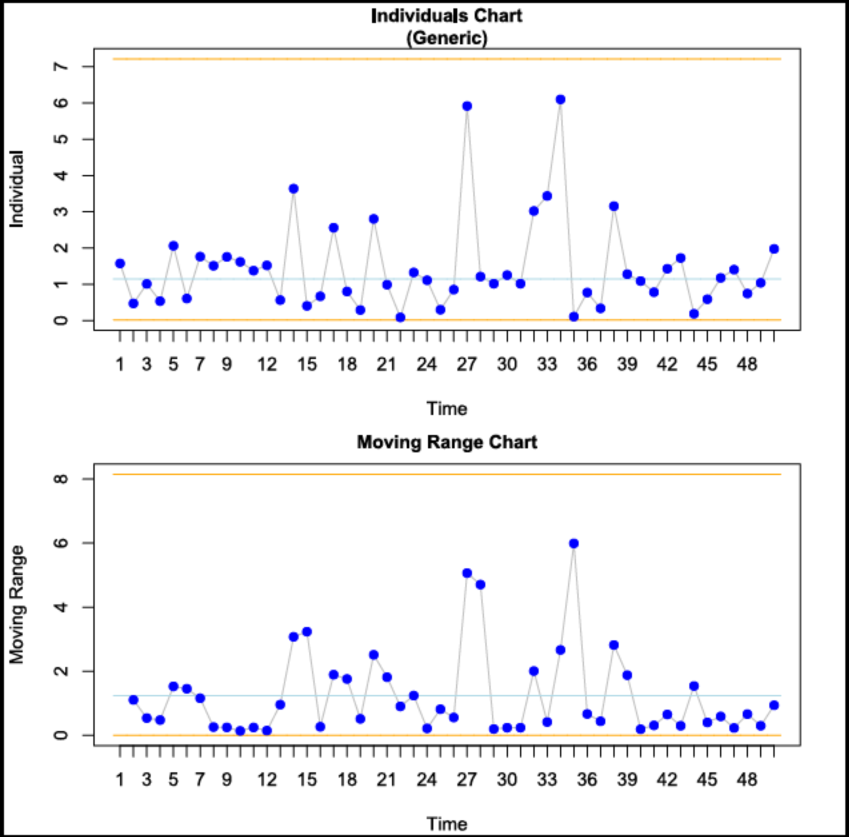

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Individuals and Moving Range Charts – Distribution Fitting

Data Science for Quality Management: X and Moving Range Charts for Non-Normally Distributed Data

with **Wendy Martin**

Learning objective:

Generate the X and MR chart using R software for fitted data

X and MR Charts Non-Normal Distributions

- Best fit from available distributions is the distribution with:
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X and MR Charts Non-Normal Distributions

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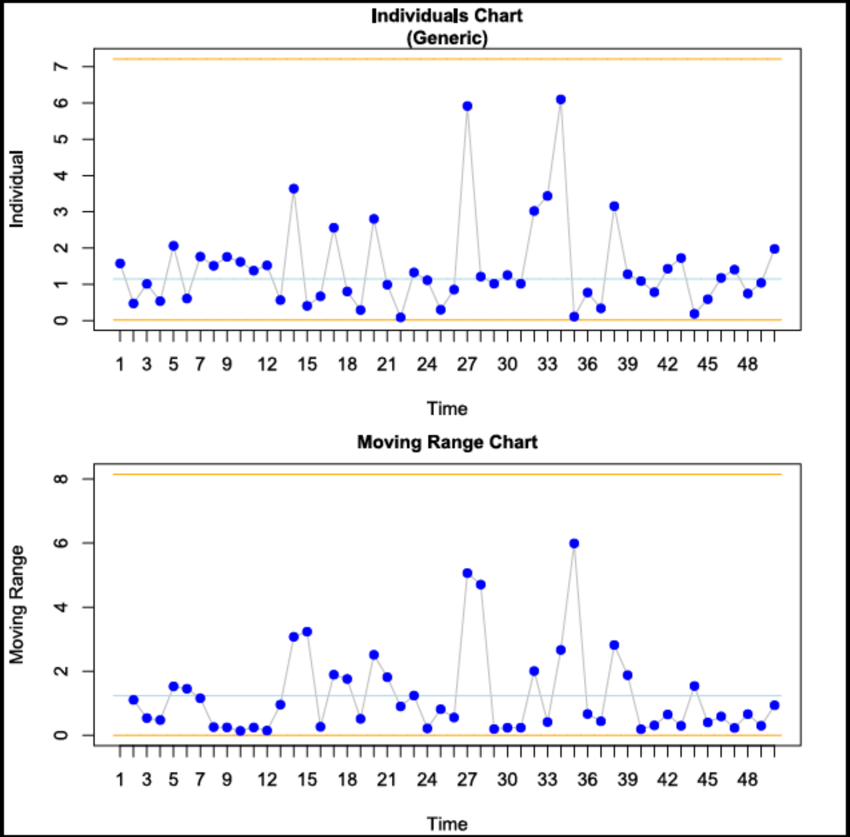

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