# Mean and Range Charts

**Data Science for Quality Management:**
**Xbar and R / Xbar and S charts /**
**X and MR charts**

with **Wendy Martin**

**Learning objectives:**

Calculate Control Limits for the X Bar and R Chart

Estimate the standard deviation from the R chart

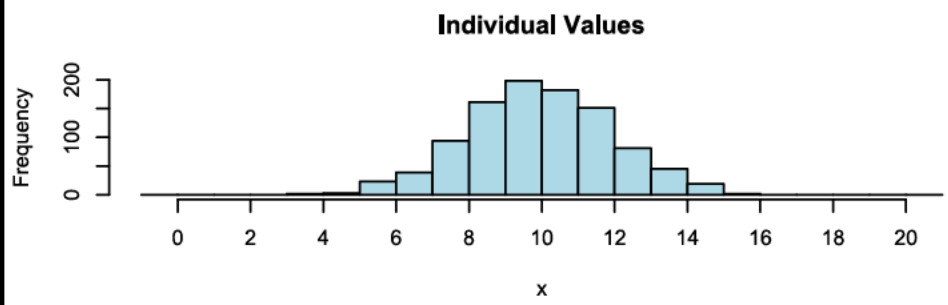
# Control Limit Formulas

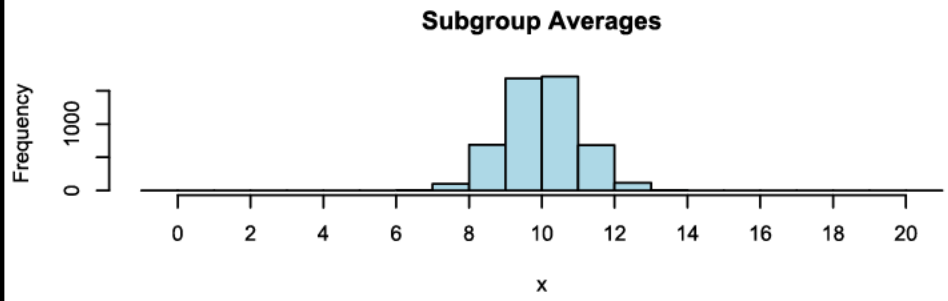
$$\mathrm{UCL}_R = D_4 \overline{R}$$
$$\mathrm{LCL}_R = D_3 \bar{R}$$

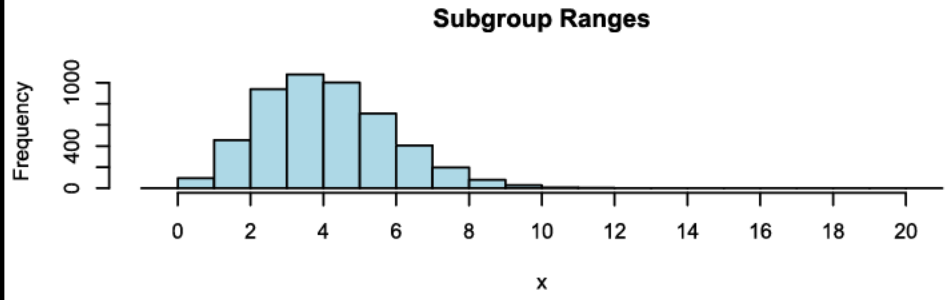
$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$
$$\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

# Control Chart Constants

- Derived from the random sampling distribution of ranges (specifically, the mean and standard deviation, $d_2$ and $d_3$) for a standard normal distribution and a specific subgroup sample size (n)




## Individual Values

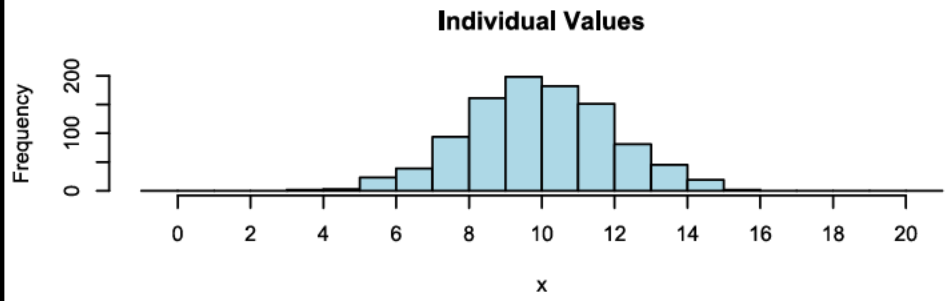
- Mean(X)
- SD(X)

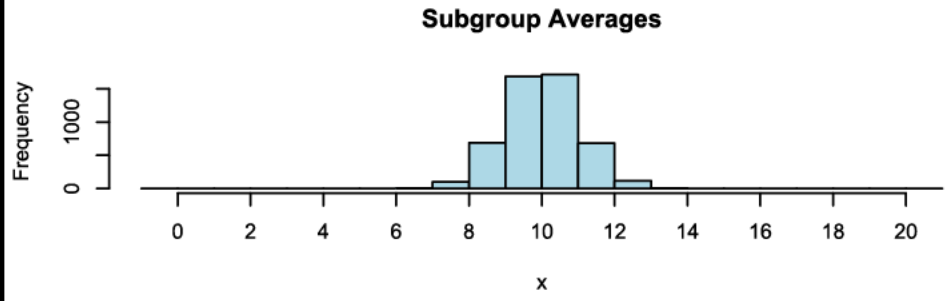
## Subgroup Averages

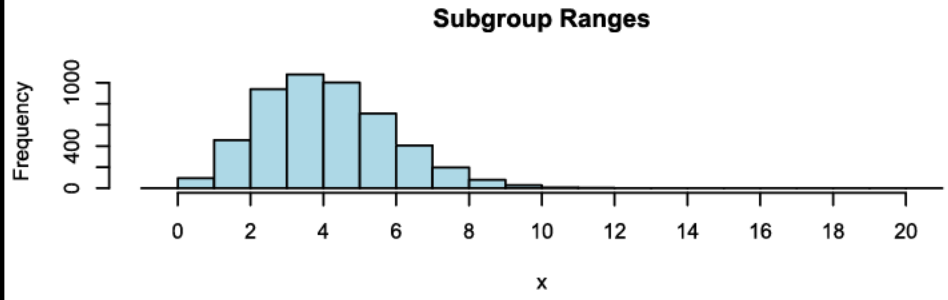
- Mean($\bar{X}$)
- SD($\bar{X}$)

## Subgroup Ranges

- Mean(R)
- SD(R)




## Individual Values

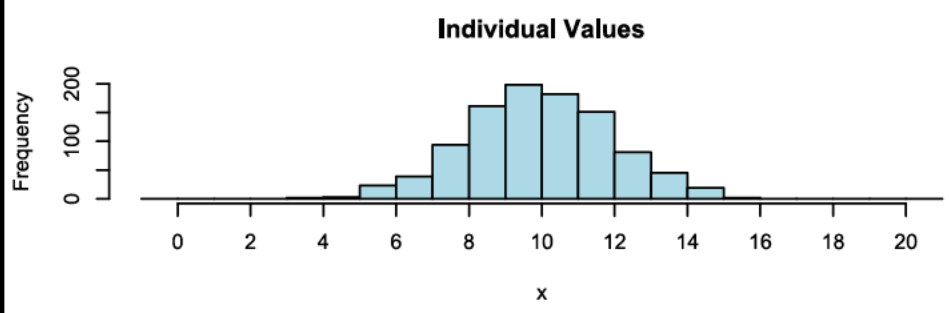
- Mean(X)
- SD(X)

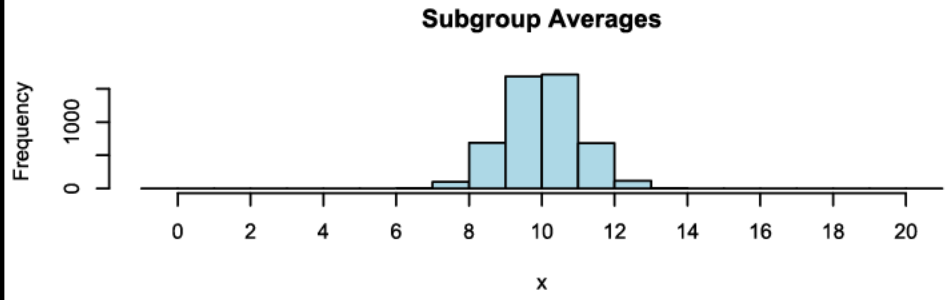
## Subgroup Averages

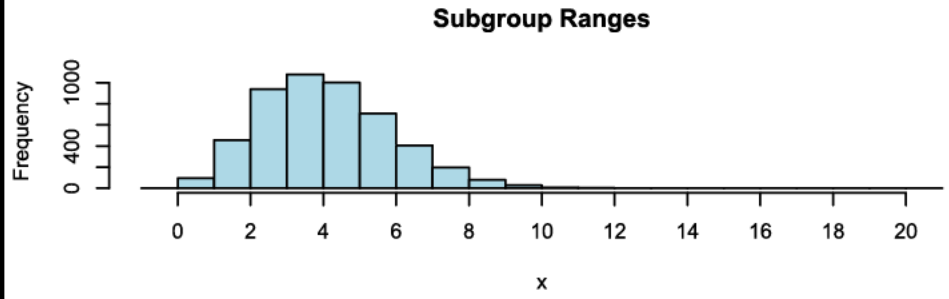
- Mean(X) = Mean ($\overline{X}$)
- SD($\overline{X}$) = $\frac{SD(X)}{\sqrt{n}}$

## Subgroup Ranges

- Mean(R) = $d_2$ SD(X)
- SD(R) = $d_3$ SD(X)




## Individual Values

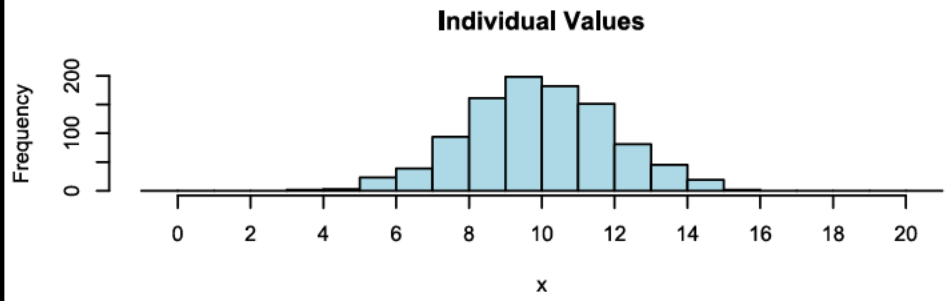
$\overline{X} = 10$ $\quad SD(X) = \frac{\overline{R}}{d_2}$

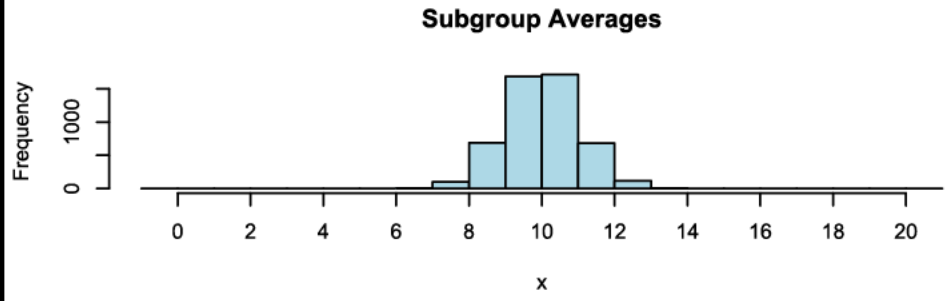
## Subgroup Averages

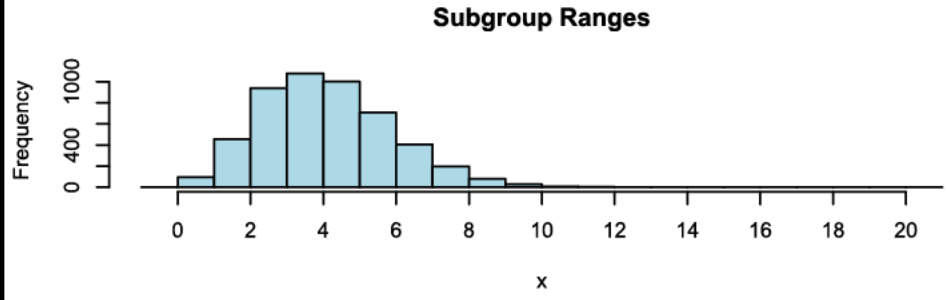
$\overline{\overline{X}} \cong 10$ $\quad SD(\overline{X}) = \frac{\overline{R}}{d_2\sqrt{n}}$

## Subgroup Ranges

$\overline{R} \cong 4.10$ $\quad SD(R) = \frac{d_3\overline{R}}{d_2}$




## Natural Process Limits

$$\bar{X} \pm 3\frac{\bar{R}}{d_2}$$

## Control Limits

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2\sqrt{n}}$$

## Control Limits

$$\bar{R} \pm 3\frac{d_3\bar{R}}{d_2}$$

# Control Limit Formulas - Range

$$\mathrm{UCL}_R = D_4\overline{R} \quad \Rightarrow \quad D_4 = 1 + 3\frac{d_3}{d_2}$$

$$\mathrm{LCL}_R = D_3\bar{R} \quad \Rightarrow \quad D_3 = 1 - 3\frac{d_3}{d_2}$$

$$\bar{R} \pm 3\frac{d_3\bar{R}}{d_2}$$

# Control Limit Formulas - Mean

$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$

$$\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2\sqrt{n}}$$

# Control Chart Constants

| n | $A_2$ | $D_3$ | $D_4$ | $d_2$ | $d_3$ | $c_4$ |
|---|---|---|---|---|---|---|
| 2 | 1.880 | None | 3.267 | 1.128 | 0.853 | 0.7979 |
| 3 | 1.023 | None | 2.574 | 1.693 | 0.888 | 0.8862 |
| 4 | 0.729 | None | 2.282 | 2.059 | 0.880 | 0.9213 |
| 5 | 0.577 | None | 2.115 | 2.326 | 0.864 | 0.9400 |
| 6 | 0.483 | None | 2.004 | 2.534 | 0.848 | 0.9515 |
| 7 | 0.419 | 0.076 | 1.924 | 2.704 | 0.833 | 0.9594 |
| 8 | 0.373 | 0.136 | 1.864 | 2.847 | 0.820 | 0.9650 |
| 9 | 0.337 | 0.184 | 1.816 | 2.970 | 0.808 | 0.9693 |
| 10 | 0.308 | 0.223 | 1.777 | 3.078 | 0.797 | 0.9727 |
| 11 | 0.285 | 0.256 | 1.744 | 3.173 | 0.787 | 0.9754 |
| 12 | 0.266 | 0.283 | 1.717 | 3.258 | 0.778 | 0.9776 |
| 13 | 0.249 | 0.307 | 1.693 | 3.336 | 0.770 | 0.9794 |
| 14 | 0.235 | 0.328 | 1.672 | 3.407 | 0.763 | 0.9810 |
| 15 | 0.223 | 0.347 | 1.653 | 3.472 | 0.756 | 0.9823 |

# Control Limit Calculations

$$\mathrm{UCL}_R = 2.115(0.0076) = 0.0161$$
$$\mathrm{LCL}_R = none$$

$$\mathrm{UCL}_{\bar{X}} = 1.0410 + 0.577(0.0076) = 1.0454$$
$$\mathrm{LCL}_{\bar{X}} = 1.0410 - 0.577(0.0076) = 1.0366$$

# Sources

The material used in the PowerPoint presentations associated with this course was drawn from a number of sources. Specifically, much of the content included was adopted or adapted from the following previously-published material:


- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982
- Luftig, J. Advanced Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1984.
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- Luftig, J. Guidelines for Reporting the Capability of Critical Product Characteristics. Anheuser-Busch Companies, St. Louis, MO. 1994
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- Luftig, J. and Petrovich, M. Quality with Confidence in Manufacturing. SPSS, Inc. Chicago, IL 1997
- Littlejohn, R., Ouellette, S., & Petrovich, M. Black Belt Business Improvement Specialist Training, Luftig & Warren International, 2000
- Ouellette, S. Six Sigma Champion Training, ROI Alliance, LLC & Luftig & Warren, International, Southfield, MI 2005