Assessing Capability with Non-Normal Data – Exponential Distribution

Data Science for Quality Management: Process Capability

with **Wendy Martin**

Learning objective:

Assess capability / performance from a non normal distribution with exponentially distributed data

Assess Potential Capability Due to Spread - Cp

For unilateral specifications, normal

$$C_p = \frac{2\left|\bar{X} - SL\right|}{6\hat{\sigma}}$$

For unilateral specifications, non-normal

$$C_p = \frac{2\left|\tilde{X} - SL\right|}{NT}$$

Assess Potential Capability Due to Spread - Cp

```
# Cp Only
cap.exp[1,1:4] = 1.158435
```

$$C_p = \frac{2|\tilde{X} - SL|}{NT} = \frac{2|13.245 - 48|}{60} = 1.158$$

Assess Capability to Produce Within Specification – Cpk

Determine % OOS using Exponential Low curve:

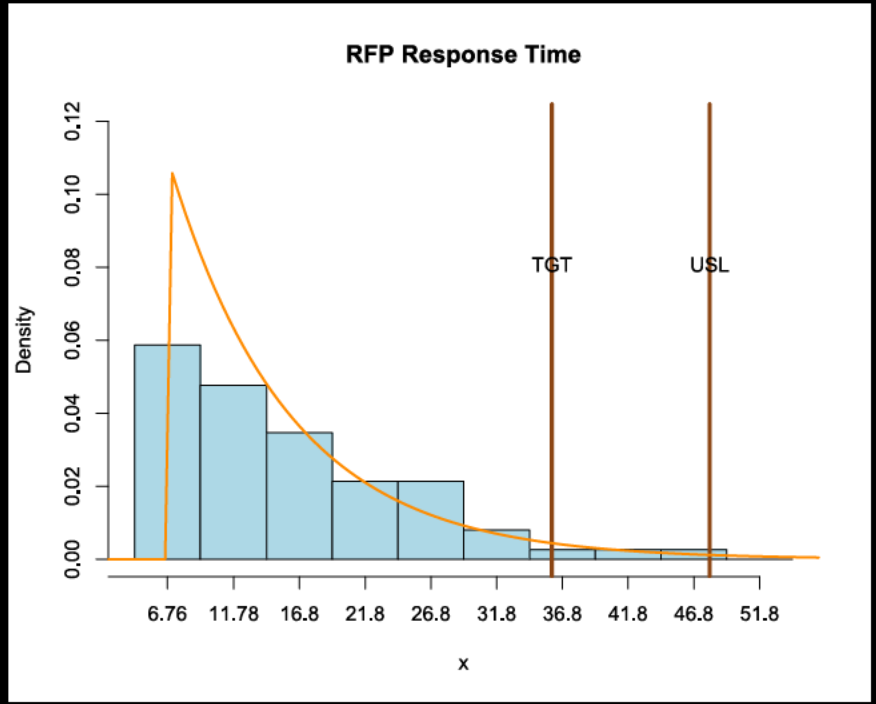
```
l.out<- pexp.low(q = LSL
, low = min(data)
, mean = mean, lower.tail = T) = NA


u.out<- pexp.low(q = USL
, low = min(data)
, mean = mean, lower.tail = F) = 0.01066834
```

Make a Visualization

```
hist.grouped(RFP$Time
    ,xlim=c(min(RFP$Time)-1
    ,max(RFP$Time))
    ,ylim=c(0,0.12)
    ,anchor.value = min(RFP$Time)
    ,stat.target = 36
    ,stat.usl = 48
    ,main = "RFP Response Time"
    ,freq=F)

hist.add.distribution.curve.exp.low
(x = RFP$Time, low =
min(RFP$Time),lwd=2,freq = F)
```



Assess Capability to Produce Within Specification – Cpk

Use the % OOS to calculate a Z Score and divide by 3 to get Cpk (eq)

```
Zu<-qnorm(0.01066834, lower.tail=F)
Cpk.upper<-Zu/3 = 0.7673244
```

Assess Capability to Produce Within Specification- Cpk

```
# Cpk Only
cap.exp[2,1:4] = 0.7673244
```

$$C_{pk} = \frac{Z_{EQ}}{3}$$

Assess Capability to Conform to Nominal – Cpm

For unilateral specifications, normal

$$C_{pm} = \frac{2|\bar{X} - SL|}{\sqrt{s^2 + (\bar{X} - T)^2}}$$

Assess Capability to Conform to Nominal – Cpm

For unilateral specifications, non-normal

$$C_{pm} = \frac{2\left|\tilde{X} - SL\right|}{6\sqrt{\left(\frac{NT}{6}\right)^2 + \left(\tilde{X} - T\right)^2}}$$

Assess Capability to Conform to Nominal – Cpm

Using Exponential Distribution

$$C_{pm} = \frac{2|13.245 - 48|}{6\sqrt{\left(\frac{60}{6}\right)^2 + (13.245 - 36)^2}}$$

$$C_{pm} = 0.4661$$

Process Performance Measures

If the process is ***not*** in control, use the estimate of the standard deviation based on the overall raw data (ignoring underlying distribution)

```
# Calculate the natural tolerance
s<-sd(RFP$Time) = 8.927889
nt_s<-6*s = 53.56733
```

Performance Measures – Pp

```
# Pp only
cap.exp[4,1:4] = 1.200629
```

$$P_p = \frac{2\left|\overline{X} - SL\right|}{6\hat{\sigma}}$$

Performance Measures – Ppk

```
# Ppk only
cap.exp[5,1:4] = 1.200629
```

$$P_{pk}U = \frac{USL - \mu}{3\sigma} = \frac{2\left|\bar{X} - SL\right|}{6\hat{\sigma}}$$

Performance Measures – Ppm

```
# Ppm only
cap.exp[6,1:4] = 0.4862165
```

$$P_{pm} = \frac{2|\bar{X} - SL|}{6\sqrt{\hat{\sigma}^2 + (\mu - \text{Nominal})^2}}$$

Capability & Performance Summary

statistic	eq	n	value
Cp	=		1.0718
Cpk	=		0.7673
Cpm	=		0.4764
Pp	=		1.2006
Ppk	=		1.2006
Ppm	=		0.4862
Obs. n / PPM (lower)	=	NA	NA
Obs. n / PPM (upper)	=	1	13333.0000
Obs. n / PPM (total)	=	1	13333.0000
Pot. PPM (lower)	=		NA
Pot. PPM (upper)	=		10668.0000
Pot. PPM (total)	=		10668.0000

Sources

The material used in the PowerPoint presentations associated with this course was drawn from a number of sources. Specifically, much of the content included was adopted or adapted from the following previously-published material:

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982
- Luftig, J. Advanced Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1984.
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- Luftig, J. Guidelines for Reporting the Capability of Critical Product Characteristics. Anheuser-Busch Companies, St. Louis, MO. 1994
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- Luftig, J. and Petrovich, M. Quality with Confidence in Manufacturing. SPSS, Inc. Chicago, IL 1997
- Littlejohn, R., Ouellette, S., & Petrovich, M. Black Belt Business Improvement Specialist Training, Luftig & Warren International, 2000
- Ouellette, S. Six Sigma Champion Training, ROI Alliance, LLC & Luftig & Warren, International, Southfield, MI 2005