Assignment 02

Summary on Gödel's incompleteness theorems

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Gödel's incompleteness theorems , published by Kurt Gödel in 1931, are two theorms of mathematical logic that deals with the limitations of provability in the formal axiom theory. The first incompleteness theorem says that for any set of axioms that can be considered a theorm , there will always be true facts about natural numbers that cannot be proved by these axioms. The second incompleteness theorm says that these axioms or systems cannot prove its own consistency. These statements are applicable within the formal systems which can be defined as abstract framework that follows a set of rules to derive theorems from axioms. The properties of a formal system includes completeness, consistency, and the existence of an effective axiomatization. The incompleteness theorm demonstrates that systems with a significant amount of arithmetic cannot have all three features. These theorms are formally stated as:

1.First Incompleteness Theorem: "Any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F."

2.Second Incompleteness Theorem: "Assume F is a consistent formalized system which contains elementary arithmetic. Then F $\nvdash$ Cons(F).

In the second theorm it is assumed that this canonical consistency statement Cons(F) will not be provable in F and the term "formalized system" also includes an assumption that F is effectively axiomatized.

The incompleteness theorm is applicable within these systems. The incompleteness theorems demonstrate that systems with a significant amount of arithmetic cannot have all three features.

The second theorem is stronger than the first incompleteness theorem because the first incompleteness theorem does not directly express the consistency of the system. The proof of the second incompleteness theorem is obtained by formalizing the proof of the first incompleteness theorem within the system F itself.