# Shanshu Interview Question

**Problem Description.** A transportation network is denoted by $\mathcal{G} = (\mathcal{E}, \mathcal{V})$ with depot set $\mathcal{V} = \{1, 2, ..., n\}$, edge set $\mathcal{E} = \{(i, j)|i, j \in \mathcal{V}\}$, and a distance matrix $\mathcal{D} = \{d_{ij}|i, j \in \mathcal{V}\}$. A logistics company owns a fleet of $m$ homogeneous vehicles. The initial locations of those vehicles are $v_1, ..., v_m \in \mathcal{V}$. There is a pool of customer orders (they are given in advance). Each customer order requires a vehicle picking up goods at one depot (called origin) and delivering to another depot (called destination) within a specific time window. More specifically, customer order $i$, if served, must be picked up between $(\ell_i^p, u_i^p)$ and delivered between $(\ell_i^d, u_i^d)$. Each vehicle can only serve one customer at a time (Full-Truck-Load).

1. Please formulate a mathematical programming model that can find a routing plan for the vehicles (including each vehicle's routing plan and timetable). The objective is to maximize the total mileage of all vehicles that are not running empty (i.e., loaded with goods). Note that it is not necessary to serve all demands (usually the number of vehicle is not enough to serve all demands).

2. The following is an instance of the above problem. The distance matrix and the customer orders are given in Tables 1 and 2. There are $n = 10$ depots and $m = 3$ vehicles in the network with their initial locations at $v_1 = 4, v_2 = 10, v_3 = 2$. In this case, there is only a deadline for the pick-up time for each order (no later than that time) and there is no other time window constraints. Please write a computer program to solve this problem and output the optimal solution and objective value. You may use any programming language/solver you are familiar with (e.g., Python, MATLAB, C++, Java, etc.).

Table 1: Distance matrix (vehicle speed is fixed at 1)

| Depots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 20 | 10 | 3 | 15 | 8 | 17 | 17 | 8 | 10 |
| 2 | 20 | 0 | 24 | 19 | 12 | 12 | 7 | 13 | 18 | 21 |
| 3 | 10 | 24 | 0 | 13 | 14 | 14 | 24 | 15 | 6 | 3 |
| 4 | 3 | 19 | 13 | 0 | 16 | 7 | 15 | 18 | 10 | 13 |
| 5 | 15 | 12 | 14 | 16 | 0 | 9 | 15 | 3 | 9 | 10 |
| 6 | 8 | 12 | 14 | 7 | 9 | 0 | 10 | 12 | 8 | 11 |
| 7 | 17 | 7 | 24 | 15 | 15 | 10 | 0 | 18 | 18 | 21 |
| 8 | 17 | 13 | 15 | 18 | 3 | 12 | 18 | 0 | 10 | 11 |
| 9 | 8 | 18 | 6 | 10 | 9 | 8 | 18 | 10 | 0 | 3 |
| 10 | 10 | 21 | 3 | 13 | 10 | 11 | 21 | 11 | 3 | 0 |

Table 2: Orders

| Orders | Origin depot | Destination depot | Deadline for pick-up |
|---|---|---|---|
| 1 | 9 | 2 | 37 |
| 2 | 4 | 10 | 36 |
| 3 | 2 | 5 | 26 |
| 4 | 5 | 3 | 48 |
| 5 | 7 | 10 | 39 |
| 6 | 9 | 7 | 38 |
| 7 | 7 | 10 | 14 |
| 8 | 7 | 8 | 17 |
| 9 | 5 | 10 | 17 |
| 10 | 4 | 9 | 12 |
| 11 | 1 | 6 | 38 |
| 12 | 4 | 3 | 46 |

Please submit your answer with the formulation (please describe clearly your variables and important formula), and the answer to the above instance. In a separate file, please submit the code you used to solve the problem.