Answer for Shanshu Interview Question

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1 Notations

- $i \in K$: Order set
- $S = \{v_m | m \in M\}$: Start node set
- $m \in M$: Vehicle set
- c_j: time for order j, $c_j = d(j_1, j_2)$; j_1, j_2: origin and destination of order j.
- $t_{ij} = d(i_2, j_1)$.
- 0: Virtual end node, $t_{i0} = 0, \forall i \in K$; $c_0 = 0$
- $U = K \cup S \cup 0$

1.1 Variables

- $x_{ijm} = 1$ if order j is served after i by m, 0 otherwise.
- T_{im}: the time at which m begins to serve order i

2 Formulation

$$
\begin{aligned}
\max \quad & \sum_{m \in M} \sum_{i \in U} \sum_{j \in U} c_j x_{ijm} && && (1)\\
s.t. \quad & \sum_{m \in M} \sum_{i \in K \cup S} x_{ijm} \leqslant 1 && \forall j \in K \cup S && (2)\\
& \sum_{j \in K} x_{v_m jm} = 1 && \forall m \in M && (3)\\
& \sum_{j \in U} x_{jim} - \sum_{j \in U} x_{ijm} = 0 && \forall i \in K, m \in M && (4)\\
& \sum_{i \in K} x_{i0m} = 1 && \forall m \in M && (5)\\
& T_{jm} \geqslant (T_{im} + c_i + t_{ij}) x_{ijm} && \forall i \in U, j \in U, m \in M && (6)\\
& l_i^p \leqslant T_{im} \leqslant u_i^p && \forall i \in U, m \in M && (7)\\
& l_i^d \leqslant T_{im} + c_i \leqslant u_i^d && \forall i \in U, m \in M && (8)\\
& x_{ijm} \in \{0, 1\} && \forall i \in U, j \in U, m \in M && (9)\\
& T_{im} \geqslant 0 && \forall i \in U, m \in M && (10)
\end{aligned}
$$

Consider $s \in S$, 0 are virtual orders. constraints (2) ensure each order is only served once. constraints (3), (4) and (5) define a feasible path in $\mathcal{G}$ for each vehicle. constraints (6) ensure consistency of time. constraints (7) and (8) are time windows.

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2.1 Linearization

Let $Q_{ijm} = T_{im}x_{ijm}$. Constraints in (6) can be linearized using *big*-$\mathbb{M}$ method:

$$
\begin{aligned}
&T_{jm} \geqslant (c_i + t_{ij})x_{ijm} + Q_{ijm} && \forall i \in U, j \in U, m \in M && (11)\\
&Q_{ijm} \leqslant \mathbb{M}x_{ijm} && \forall i \in U, j \in U, m \in M && (12)\\
&Q_{ijm} \leqslant T_{im} && \forall i \in U, m \in M && (13)\\
&Q_{ijm} \geqslant T_{im} - (1 - x_{ijm})\mathbb{M} && \forall i \in U, j \in U, m \in M && (14)\\
&Q_{ijm} \geqslant 0 && \forall i \in U, j \in U, m \in M && (15)
\end{aligned}
$$

3 Solution

The model is solved using Python 3 and Gurobi 8.0. The optimal solution found:

- Objective Value: 130.0
- Routing Plan (Node(Time)):
- Vehicle 1: 4(0.0)–10(13.0)–9(16.0)–7(34.0)–10(55.0)
- Vehicle 2: 10(0.0)–9(3.0)–2(21.0)–5(33.0)–3(47.0)
- Vehicle 3: 2(0.0)–7(7.0)–10(28.0)–4(41.0)–3(54.0)