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Short note

While Boolean sets non-gently rip: A theoretical framework on fuzzy sets for mapping landscape patterns

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ABSTRACT

Boolean logic is frequently applied in order to map landscape patterns. Nonetheless this implies to divide the gradual variability of the earth's surface into a finite number of non-overlapping classes, which are considered exhaustive and mutually exclusive. On the contrary, landscapes are expected to be spatially continuous. Fuzzy membership seems to better fit such an issue, by associating for each entity the degree of membership to a class thus maintaining uncertainty information. In this paper, I will disentangle, from a theoretical point of view, the potential of fuzzy set theory for mapping landscape patterns, particularly focusing on those properties of the fuzzy membership concept which are crucial when aiming at extracting the whole information over a landscape.

1. Landscape mapping: the Boolean panacea

The assessment of the ecological complexity of a landscape basically relies on field monitoring (Ferretti and Chiarucci, 2003). Meanwhile, remote sensing offers the capability of obtaining a synoptic information over large areas in order to guide sampling design for improving their efficiency (Rocchini et al., 2005). In this view land use maps are increasingly being used in landscape planning and management (see e.g. Romero-Calcerrada and Perry, 2004; Marignani et al., 2008). The classification of remotely sensed information such as aerial photos or satellite images for deriving land use maps is based on clustering of spatial entities within a spectral space.

The main procedure for classifying images is to create training areas (clusters) by further attributing pixels with unknown classes to such clusters. Boolean membership based on the binary codes 0/1 is often applied by creating thresholds based on a maximum-likelihood criterion. Strictly spoken the lower the distance of unclassified pixels from a cluster A the higher the probability of occurring within A.

In this manner each thematic entity (being a pixel or a polygon) can be represented as a tuple $\{z(x)|s(x)\}$, where $z(x)$ = class for the each entity related to its $s(x)$ spatial component (Goodchild et al., 1999). Nonetheless this implies to divide the gradual variability of the Earth's surface into a finite number of non-overlapping classes. Therefore, classes are considered exhaustive and mutually exclusive (Foody, 1996; Woodcock et al., 1996).

On the contrary, landscapes are expected to be spatially continuous. Thus, no matter how accurately map classes are defined, the uncertainty associated to class mixtures will be never completely eliminated (Costa Fonte and Lodwick, 2004; Ricotta, 2005; Shanmugam et al., 2006).

Noteworthy, one of the most pressing needs in landscape ecology is to severely take into account the uncertainty related to patterns in the landscape (Bolliger, 2005; Green and Sadedin, 2005). Analyzing landscape patterns with a-priori defined thresholds and boundaries may lead to loose the capability of catching their actual complexity by hampering to account for the continuous variability over space (Foody, 1996; Lucieer et al., 2005; Bridges et al., 2007).

The aim of this paper is to disentangle from a theoretical point of view the potential of fuzzy set theory for mapping landscape patterns. In particular I will focus on situations in which fuzzy set theory is almost indispensable in land use mapping. Generally speaking, theory is crucial for further applications; thus I will firstly give some general theoretical foundation of fuzzy sets by further entering the landscape mapping framework in major detail.

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2. Fuzzy set theory for landscape mapping

Fuzzy set theory should aid in maintaining uncertainty information related to each class. The concept of fuzzy sets was first introduced by Zadeh (1965); thus, fuzzy set based approaches have been widely used in ecology dating back to 1980s (see e.g. Feoli and Zuccarello, 1988; Roberts, 1996; Ricotta and Anand, 2006).

The principle behind fuzzy set theory is that the situation of one class being exactly right and all other classes being equally and exactly wrong often does not exist. Conversely, there is a gradual change from membership to non-membership (Gopal and Woodcock, 1994).

A fuzzy set is defined as follows: let U denote a universe of entities u, the fuzzy set F turns out to be:

$$F = \{(u, \mu_f(u)) | u \in U\} \tag{1}$$

where the membership function μ_f associates for each entity $u \in U$ the degree of membership into the set F (see Zimmermann, 2001). The degree of membership ranges in the interval [0,1], i.e. the real range between 0 and 1.

Two major assumptions lead to consider fuzzy sets as a powerful tool for maintaining uncertainty information when aiming at mapping and analysing landscape patterns: (i) membership of ecological entities to classes is not forced to occur within the integer range [0,1] as in Boolean logic, (ii) considering different classes [A, B, ..., N] the sum of membership values $\sum(\mu_A, \mu_B, \ldots, \mu_N)$ does not necessarily equal 1 for each pixel or polygon. Thus, different classes may overlap to different degrees overcoming the traditional restriction on the mutually exclusive nature of map classes (Rocchini and Ricotta, 2007).

Strictly spoken, one pixel or polygon may show a high membership to broadleaf forests (e.g. μ_{bf} = 0.8) and to grassland (e.g. μ_g = 0.7) as well. Noteworthy pixels are expected to include several classes. In fact, the spectral signatures for these mixed pixels (Small, 2004) are due to a combination of classes (Gibson and Power, 2000). This may hardly be solved by a simple dominance criterion. Moreover, property (ii) aids avoiding difficulties in building a-priori exhaustive hierarchical classification schemes. A worked example of fuzzy set theory applied to landscape mapping is provided in Box 1.

Box 1. A theoretical example of fuzzy set theory applied to landscape mapping

Imagine that a set of pixels has been assigned to class A, e.g. broadleaf forests, by Boolean logic. When fuzzy set theory has been applied, the percentage of pixels showing different broadleaf forest fuzzy membership (μ_{bf}) values may be extracted. Fig. 1 shows two different situations (lines), considering the same number of pixels belonging to the crisp class ''broadleaf forest''.

The solid line represents a situation where a low amount of landscape variability exists. In fact, all pixels show high values of fuzzy membership μ_{bf}, exceeding 0.7 with a peak corresponding to $\mu_{bf} \sim 0.85$. The dashed line shows a very different situation. Pixels generally show high values of fuzzy membership, with a peak at $\mu_{bf} \sim 0.7$. However a queue composed by pixels with medium to very low values persists. The left part of such a queue may represent pixels erroneously classified as broadleaf forests while medium values may derive from a heterogeneous landscape mosaic. In this case mixed classes are expected. Several examples of such a situation with complex and highly interspersed landscapes exist considering different habitats and biogeographical areas: from evergreen vegetation of the Mediterranean basin (Carranza et al., 2001; Calvão and Palmeirim, 2004; Acosta et al., 2005), to grasslands of Central Europe mountain chains (Koellner et al., 2004; Dufour et al., 2006), to tropical forests (Thessler et al., 2005; Chust et al., 2006), etc.

In order to solve this issue one should seriously take into account fuzzy set theory. In fact, facing class mixing by only relying on Boolean logic is a hard task to deal with. For instance, one could just apply Boolean logic creating mixed classes as the mathematical intersection of two or more classes. However this may lead to a huge propagation of the number of classes.

Another way to face the problem by Boolean logic is to base the classification on inner dominance of classes within each spatial entity, i.e. a pixel or a polygon. In such a case, while the number of classes does not propagate their uncertainty is inevitably lost.

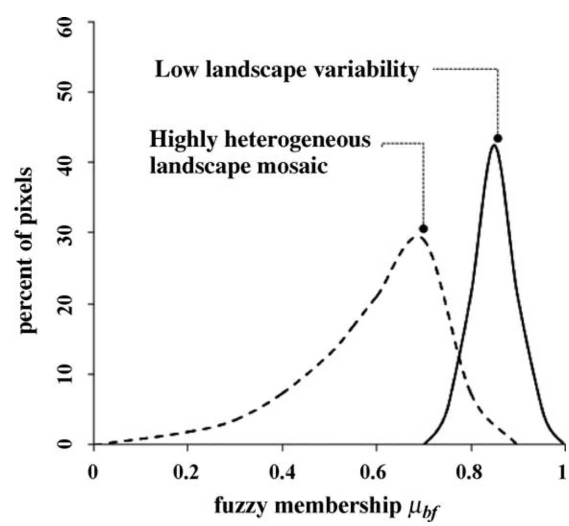

Fig. 1. Theoretical frequency distribution with respect to fuzzy membership μ_{bf} of pixels assigned to the same crisp class, e.g. broadleaf forest. Solid line: low landscape variability, all pixels show high values of fuzzy membership μ_{bf}. Dashed line: highly heterogeneous landscape mosaic, the queue towards medium and low μ_{bf} values indicates high class mixing.

3. Issues related to uncertainty in landscape mapping and their fuzzy-based solution

There are several issues related to uncertainty in the classification framework which are deeply intermingled with one another, such as: (i) mixed pixels, (ii) mapping error related to thematic attribution or to boundary uncertainty, and (iii) the representation of uncertainty.

3.1. Mixed pixels

As previously pointed out, it is well known that pixels composing input images are not explicit spatial units but empirical models of the reality. That is, they are expected to be mixed in their very nature (Fisher, 1997; Small, 2004). From an ecological point of view, the chief source of variation within a remotely sensed image is caused by differences among different land use classes (Woodcock and Strahler, 1987; Ricotta et al., 1999).

Obviously, the higher the pixel dimension the higher the sub-pixel heterogeneity being lost, each pixel corresponding to a mixture of field objects' signatures. In this view, hyperspatial data sources – such as QuickBird or Ikonos (2.88 and 4 m of spatial resolution in the multispectral channels, respectively) – permit the identification of transition zones and heterogeneous habitats, avoiding the problem of a lack of detectability of sub-pixel heterogeneity. This type of data is very likely to discern small scale heterogeneity.

Nonetheless, it is worth stressing that high spatial resolution may in some cases confound the issue. Nagendra and Rocchini (2008) demonstrated that pixels being smaller than the objects (land use classes) which should be represented will increase local variability within each object (land use class) by adding noise instead of improving information (see e.g. Fig. 2 and Woodcock and Strahler, 1987; Ricotta et al., 1999; Song and Woodcock, 2002 for major information).

Mixed pixels represent a problem for land use statistical classification because (i) they violate the major assumption of most algorithms (like the widely used maximum-likelihood) of spectral homogeneity within each class and (ii) there are major physical inconsistencies between the thematic classes sought and the discrimination power of moderate spatial resolution sensors (Small, 2005). It is worth remembering that the signal of any given pixel actually arises as a result of the contributions from objects lying therein (Fisher and Pathirana, 1990; Cracknell, 1998).

While Boolean methods will be prevented from producing accurate results, modelling a combination of surface reflectances within each pixel would lead to more reliable output maps (Small, 2005). In this view, the mostly used technique for solving the problem is the spectral unmixing (see Richards and Jia, 2005). Spectral unmixing seeks to derive fractions of endmembers (classes) from the mixed pixel. Mathematically speaking, let $R(\lambda)$ be a continuous reflectance profile as a function of the wavelength λ. Its additive partitioning may lead to estimating the corresponding endmember fractions based on:

$$R(\lambda) = \sum f_c E_c \tag{2}$$

where f_c = endmember fraction for the class c having a spectrum E_c.

In this case for each pixel the membership to the various land use classes sum up to 1, while in fuzzy set theory classes do not necessarily be exhaustive (see property (ii) in Section 2). Meanwhile, spectral unmixing clearly represents an extension of fuzzy set theory, in the sense that it is a technique based on the fuzzy nature of complex entities. I remind to Ju et al. (2005), Small (2005), Feingersh et al. (2007) and Nichol and Wong (2007) for further mathematical details and empirical examples.

3.2. Mapping error

Mapping error may derive from both thematic and geometric errors. In most cases, thematic error is measured by posterior probability estimators of map accuracy (sensu Stehman and Czaplewski, 1998). Strictly spoken, once a sampling design has been chosen, a sample of points representative of the whole population (i.e. the map) is visited in the field. Another method relies on resampling the training sets used for classifying the image by applying a bootstrap cross-validation procedure. Nonetheless, as demonstrated by Steele (2005), posterior probability estimators were demonstrated to yield map accuracy estimators with substantially less bias and smaller mean square error than resampling-based cross-validation.

Thus, let P be a vector of plots $[p_1, p_2, p_3, \ldots, p_n]$ which have been checked in the field. Error matrices of plots versus classes $[P \times C]$ are then produced and accuracy measures are derived (see Congalton, 1991, on the user versus producer accuracy). However, experts going in the field are obliged to specify one single class per plot by next making a binary comparison (right or wrong) with the map labels (the matrix $[P \times C]$). Nonetheless, Bacaro et al. (2009), dealing with botanists involved in the field, recently demonstrated that different operators might "jeopardize the whole set of results". In other words, the fact that an observer is constrained to choose only one class per plot implies an additional bias to the accuracy assessment procedure which is not accounted for. For this reason, Woodcock and Gopal (2000) proposed a soft accuracy assessment of map classification based on a linguistic value like 'absolutely right', 'good answer', 'reasonable', etc. As stressed by the authors, one limitation of the used approach is that actually it does not make use of fuzzy sets whose membership values usually vary continuously from 0 to 1. Meanwhile this represents an example which may allow to estimate ratings of class membership based on field survey directly accounting for vegetation heterogeneity. I remind to Laba et al. (2002) for a similar example.

Besides thematic attribution, mapping error may derive from spatial boundaries among classes. For instance, the idea that a given landscape is usually composed of distinguishable units or patches is one of the basic assumptions of landscape ecology.

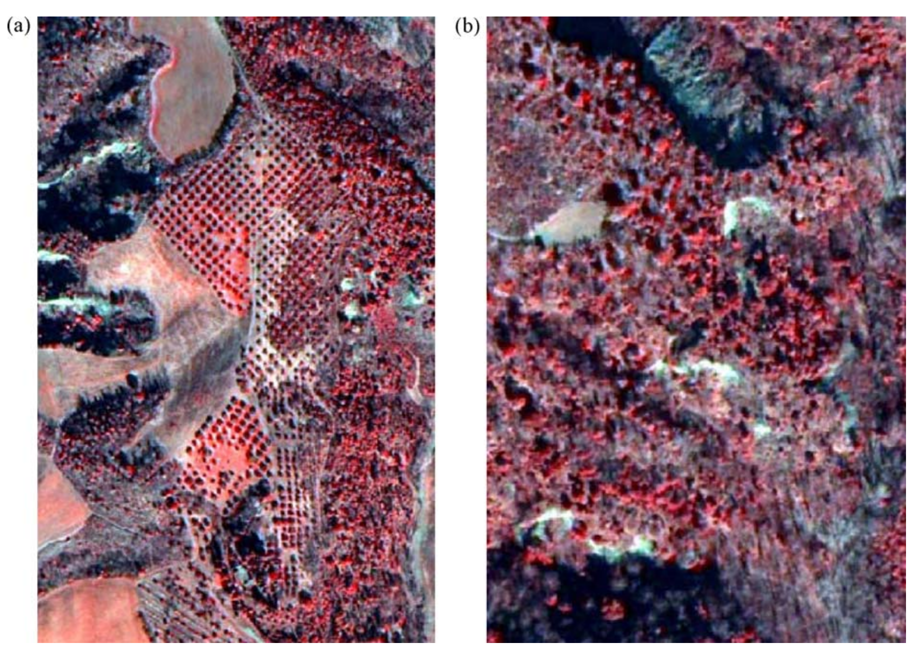

Fig. 2. High variability within human made and seminatural classes like (a) crops and (b) seminatural woodlands. In some cases, high spatial resolution data (here a QuickBird pan-sharpened image in false colour, pixel size = 0.66 m) will lead to an increase of local variability within each object (land use class) by adding noise instead of improving information. While for crops boundary detection is still possible (a), it would be difficult to delineate seminatural woodlands (b) when very high spatial resolution data are used.

However, frequently this is not the case. In order to solve problems related to boundary uncertainty non-site specific algorithms for testing accuracy were proposed (see e.g. Foody, 1999). The general aim is to assess the area covered by each class without considering the spatial location of the classifier. Despite the basic robustness of the method, Gómez et al. (2008) proposed a possible drawback of this procedure, stating that it does not take into account the spatial localisation of the classifier, thus overwhelming a-priori the problem without solving it..

A serious problem is that while man-made boundaries are frequently sharp, landscape ecotones are difficult to be detected or modelled. In these cases, those techniques frequently used for the detection of boundaries (e.g. kernels) should fail since no peaks in pixel reflectance variability are expected in case of gradual change (Fortin and Dale, 2005). Therefore, boundaries of discrete objects may be uncertain as a result of uncertainties in locating gradual transitions (Unwin, 1996). As noted by Fisher et al. (2006), areas mapped as e.g. 'forests' may also expand and contract seasonally due to the growth of more lush vegetation in the edges (ecotones) of the forest patches. The same authors stressed that working with fuzzy sets means that it is possible to map ecotones and their changes over time since it is implicit in the method (fuzzy set theory) that no sharp boundaries actually exist apart for a low number of cases like water versus land or, as previously stated, man-made boundaries (e.g. urban areas). Following this view, Berry (1996) proposed a fascinating method for mapping boundary uncertainty by gridding a landscape and depicting the uncertainty of boundaries based on the proximity of each pixel from the digitised edges.

Of course, so long as there is the possibility of identifying homogeneous patches (crisp landscapes), then fuzzy-based methods still allow the quantification of the residual uncertainty deriving from intra-patch boundaries, depending on the scale at which the analysis is performed. The previously cited Fig. 2 depicts this kind of situation using high spatial resolution data, where boundaries between crops (orchards) and neighbour patches are certainly crisp while within-patch uncertainty is very high. In this case high spatial resolution data will increase the local variability, leading to the detection of several objects within each crop, e.g. single trees, bare soil, shadows, etc. From a machine learning point of view, distinguishing orchards from non-orchards is a hard task. Hence, at the intra-patch scale, crops may show a high uncertainty which could be solved only by fuzzy set theory.

3.3. *The representation of uncertainty*

For successful and efficient spatial reasoning, a rigorous treatment of uncertainty is necessary since entities derived from images are inherently uncertain (Meidow et al., 2009). Fuzzy set theory is a powerful theoretical framework for accounting for uncertainty problems. However, it is not actually free from discrepancies and drawbacks. As an example, Fisher et al. (2007), dealing with the individuation of elevation peaks, demonstrated that "if a fuzzy set membership is defined, then the parameters of the function are expressed as specific real number values, with a numerical precision that is contrary to the idea that the set is uncertain (one of the main problems people have with fuzzy sets)".

In the reality, the membership function is approximated by expert knowledge with no precise numerical values. As an example, I reported in this paper a case where no membership 'numbers' have been used but discursive descriptions were done instead (see Section 3.2 and Woodcock and Gopal, 2000). When using real numbers (membership function) based on e.g. the spectral distance from a class, other parameters – e.g. the class definition – which are not numeric in their very nature may not be accounted for. In other words, any statement about the existence of a vague phenomenon or its properties must be vague (Sorensen, 1985).

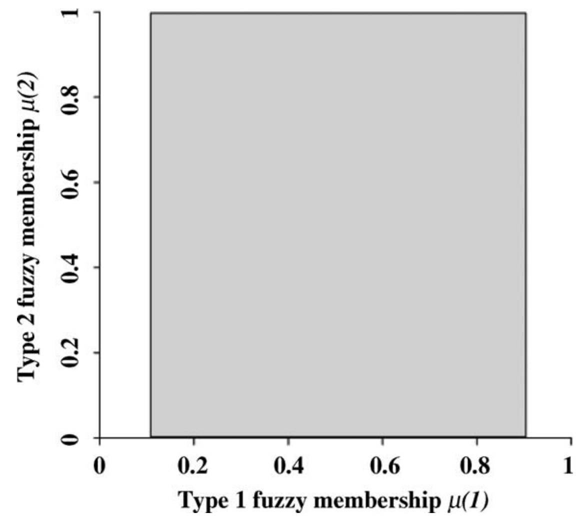

Fig. 3. Higher order vagueness summarized by the type 2 fuzzy membership $\mu(2)$. Considering an object and its fuzzy membership to a class (type 1 fuzzy membership $\mu(1)$, x axis, with e.g. $0.1 \le \mu(1) \le 0.9$), there is an associated fuzzy membership value (type 2 fuzzy membership $\mu(2)$, y axis, second order vagueness). This concept may be extended to nth-order vagueness. I refer to the main text for major explanations.

As stressed by Zadeh (1965) the equation relating each object to a class by a membership is deterministic. This leads to the paradox of describing uncertainty with values which are a-priori suspected to be certain! Meanwhile, this may be overwhelmed by assuming that there is, for every level of the fuzzy membership function, a fuzzy set membership. Extending on his previous theory, Zadeh (1975) summarized this concept as a type 2 fuzzy set, or second order vagueness (Fig. 3) I refer to Williamson (1999), Varzi (2003) and Fisher et al. (2007) for the description of nth-order vagueness, also referred to as higher order vagueness.

4. Concluding remarks

In this paper I mainly dealt with the theoretical amount of information lost by crisp classification based on Boolean logic, discussing problems deriving from a crisp view of ecological entities which are being mapped. Categorical data derive from classification. Hence, (i) they should be provided with a vector of probabilities or memberships (one for each pixel or polygon) to the recognised classes instead of a Boolean (certain) membership, and (ii) because of the need of generalisation, there will be inclusions of land within mapping units that do not show the mapped property (see Unwin, 1996).

I am not claiming at dismissing crisp classification, even if some papers have stressed the higher map accuracy reached by fuzzy classification with respect to a crisp one (see e.g. Shanmugam et al., 2006).

Meanwhile, habitats are expected to gently and continuously vary within a landscape rather than abruptly change. Thus, it is crucial that geographical maps and databases, which are rapidly created after the spread of GIS, account for uncertainty problems (Fisher, 2000; Baja et al., 2002). This is particularly true overall considering that fuzzy sets are not only qualitative but even quantitative ways to characterise uncertainty (Li and Rykiel, 1996).

Classification is a subjective matter in its very nature (Palmer et al., 2002; Foody, 2008). Therefore creating abrupt thresholds among landscape objects (if they actually exist) should only create misleading information for reserve and landscape managers and administrators.

Therefore, by this paper I claim at combining Boolean logic and fuzzy set theory that should help solving mixing problems by providing an approach to straightforwardly extract the whole information over a landscape (Arrell et al., 2007). Quoting Li and Rykiel (1996) "wherever we have been forced by the dictates of binary logic to draw artificially sharp boundaries in ecology, we can now draw more realistic distinctions in terms of fuzzy sets".

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