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Lawrence C. Andrews[a]* and Herbert J. Bernstein[b]

[a]*Ronin Institute, 9515 NE 137th St, Kirkland, WA, 98034-1820 USA, and* [b]*Ronin Institute, c/o NSLS-II, Brookhaven National Laboratory, Upton, NY, 11973-5000 USA. E-mail: larry6640995@gmail.com*

1. Note

Although this document is written in first person, Herbert Bernstein has edited and agreed to all statements.

2. Response to Reviewer 1

I would like to begin with the response to the issues of the reviewer by quoting the abstract of our article:

> Several attributions exist for the reduced cells of lattices and for the reduction processes to produce them. The actual origins of the terms are compared and a taxonomy created. The terms "Niggli reduction", "Niggli cell", "Delaunay reduction", and "Delaunay cell" most accurately describe the crystallographic usages of reduced cells and reduction methods.

It seems that Reviewer 1 has misunderstood the aim of the article, which is clearly given in the abstract.

Reviewer 1:

> Current manuscript suggests brief description of various approaches to the choice of reduced cells of crystal lattices and associated reduction methods. In fact, the problem of reduction for lattices has much more general mathematical meaning and consists in finding conditions on parameters of an "elementary cell" allowing to find for any lattice the unique (in certain sense) frame characterizing the elementary cell. The application to classification of 3-dimensional crystal lattices is an important physical problem and it is certainly very useful for specialists in crystallography to adapt and to follow terminology and the mathematical results formulated over more than 250-years history of such celebrated mathematical problem. After reading the current.

We completely agree with the reviewer's statements. But this is not the paper, nor the intent, to describe the important topics they allude to. Even our abstract is clear that this is not the intent. If the reviewer would prepare the article they describe, I would eagerly read it.

On the issue of (Delone et al., 1975): unfortunately neither of us knows the Russian language, so we have always consulted the English translation. We agree that that paper is an important one. To point out an important point from that paper, they discuss unit cells as being in a metric space; it is the same space that we have elaborated as S6 in our own work.

The reviewer's comment has led us to include a reference to that very useful article.

3. Response to Reviewer 2

The reviewer writes:

> Ten methods are described in sequence included in alphabetical order of the authors. It is noticeable that the latest date for the references of the methods is 1960 and many are from the 19th century. So I doubt the timeliness and tbe relevance to crystallographers of this contribution.

It is true that the table that lists the types and their taxonomy does only have dates from 1960 and farther into the past. That is because the table lists the types and the dates of their original creations/publication.

The later text cites dates including 2023.

The only other issue this reviewer lists is whether to change the ordering of the descriptions of the types to chronological. While they could be changed, that change would make locating they particular types more difficult for someone who is not familiar with the original dates. I do not see the advantage of such a change.

References

Delone, B. N., Galiulin, R. V. & Shtogrin, M. I. (1975). J. Sov. Math. **4**(1), 79 – 156.
Oishi-Tomiyasu, R. (2012). Acta Cryst. A**68**(5), 525 – 535.