# Lego Mindstorms NXT for teaching students fundamentals of adaptive control ⋆

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**Abstract:** This article presents content practical course of a control theory used in the Control Systems and Informatics department of the Saint-Petersburg National Research University of Information Technologies Mechanics and Optics. There are many different examples of the realization empirical and mathematical control algorithms. Course begins with the simple line tracer robot and ends with the inverted pendulum on a cart. During the course, students are offered to learn the problem of identification, modelling and programming, based on the Lego NXT.

*Keywords:* LEGO Mindstorms NXT, mobile robots, Ziegler-Nichols ultimate gain method, modal control, Ackermann's formula.

## INTRODUCTION

One of the important problems is the decrease the level of students education. The interest in the natural sciences falls because the students do not know how to apply the theoretical knowledge. "Is it useful for me in my life?" – question that students often ask teachers. But if the students go straight from theory to practice it is very strong motivation to learning fundamental science.

In this year we tried new labs for control theory. But we did it for first year students. It gave big feedback from them. Cause, now they have wide interest in automation control and know, how it can be used in practice. Maybe they didn't understand complex mathematics, but will interested in explanation in future.

## LAB ACTIVITY 1

At first, we want to show to the students simple robotic system, and teach them methods, which does not requires profound knowledges in physics and mathematics. For this task we decide to use Ziegler-Nichols' ultimate gain method (closed-loop method). During this work, students have time to learn NXC program language for NXT brick, and try to use some abilities of Scilab – open source software for numerical computation. Students can use the open source software not only at university, but work at home this own projects. Ziegler-Nichols' method used for tuning PID–controller, one of the most popular algorithm, and good example in control theory.

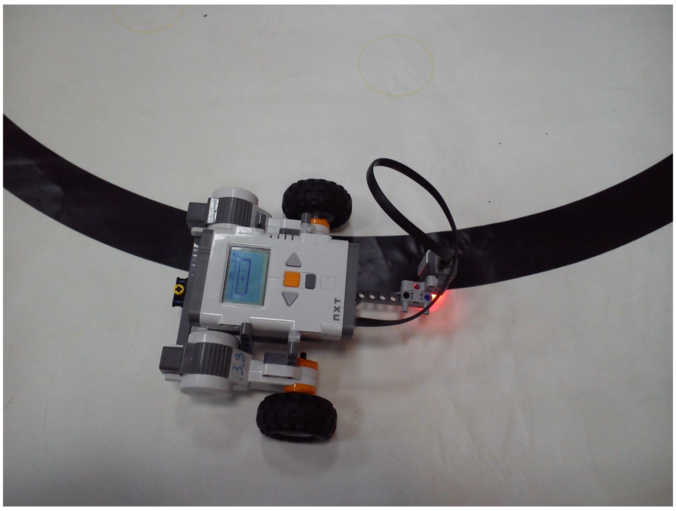



Fig. 1. Line tracer robot.

For this task students construct simple line tracer robot 1 with one light sensor. Our field look like number eight. We begin from the P–control law. After our robot will be able to pass this line, we should increase proportional gain, until the robot starts to make stable oscillations, moving along the line. Maximum of the workable proportional coefficient, it is our ultimate gain. When we got measurements of the light sensor from robot, we should commit period of the signal oscillations.

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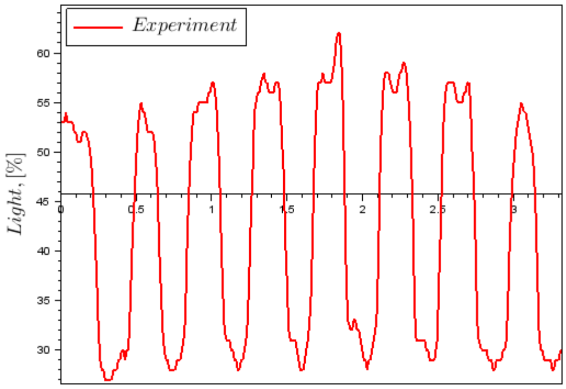


Fig. 2. Measurements of light sensor in time.

In fig. 2 for our line tracer ultimate gain $K_u = 7$ and the period of oscillation $T_u = 0.43\ sec$. After determination of the ultimate gain, can be calculated gains for PID-controller. Fig. 3 show to us processes in the system with different control laws.

## LAB ACTIVITY 2

In the second lab activity we start a series of works that will help us to describe the pendulum system in the state space. For beginning students should get constants which associate the values of control voltage and output torque. The first necessary value is mechanical time constant. We can determine it with assignment constant control signal, and write the value of turn angle of the motor. We can calculate value of mechanical constant using formula 1 dependence angle with time and least squares method.

$$\phi = \omega_{nls} t - T_m \omega_{nls} + T_m \omega_{nls} \exp(-t/T_m), \qquad (1)$$

where $t$ – time, $\phi$ – turn angle, $\omega_{nls}$ – no load speed, $T_m$ – mechanical time constant. Using this equation students should process data from motor in Scilab application. The least squares method gives no load speed and mechanical time constant for NXT motor (fig. 4). In our case we got $\omega_{nls} = 14.2 \frac{rad}{sec}$ and $T_m = 0.079\ sec$. Control voltage value is $U = 7V$. If we consider that in the our motor no viscous friction and other disturbances, we can use formulas for ideal engine. It means that back EMF $U_emf$ value equals to the control voltage, and in the rotating motor current is $0\ A$ (the real value $I = 0,05\ A$).

$$K_e = \frac{U_{emf}}{\omega_{nls}}. \qquad (2)$$

For our motor $K_e = 0.49\ V * sec$. The next step in this lab work, checking parameters in the open-loop model. Students should synthesize control law $U = 7 * sin(\pi * t)$, where $U$ – the control signal.

As you can see, these curves too close to each other. It means that the parameters of motor identified correctly. But the initial point of oscillation is moving in the negative part of graph, this is because the maximum voltage of PWM in different direction has difference $0.2V$. This value disperses on the control switch.

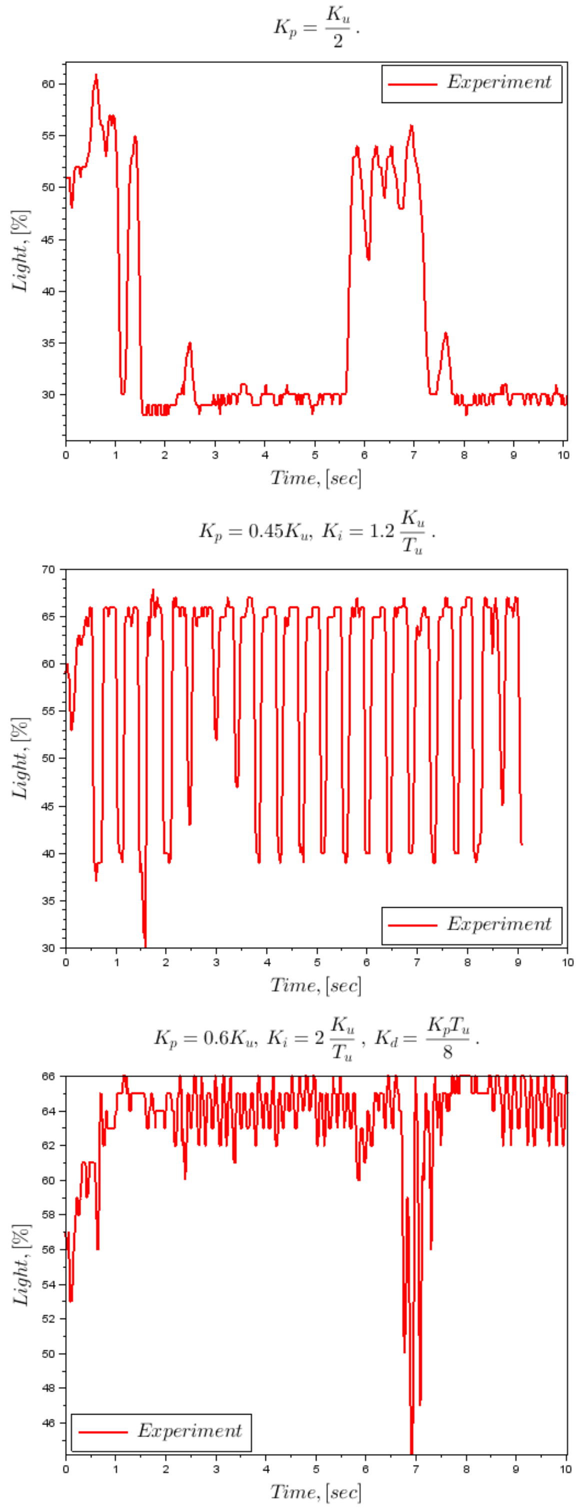


Fig. 3. P,PI,PID control laws and formulas for gains calculations.

## LAB ACTIVITY 3

In this lab activity students should check obtained result in the closed-loop system. For this task we should synthesize control law for steering the angle of the motor by the

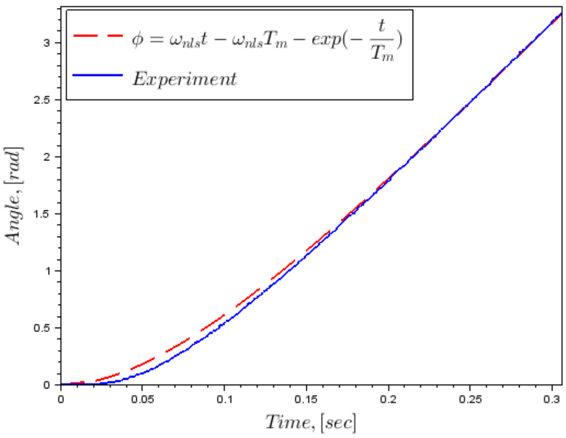


Fig. 4. Identification of model parameters.

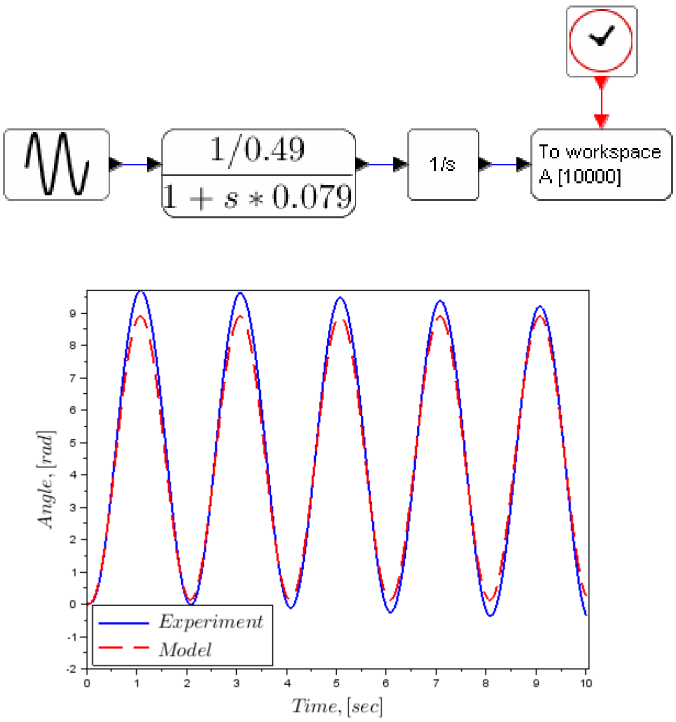


Fig. 5. Checking model parameters.

proportional controller. This work includes programming and modelling closed-loop system, cause students should be able to associate mathematical formulas and text of the program.

Our control algorithm compares the current angle from a given, in our case it is $\pi$, and based on the error generates a control signal to the motor.

$$U = Kp * (\pi - \phi). \tag{3}$$

Now we have result for closed-loop system, and experimental curve have error not more than 2%. All our calculations made correctly, and we can proceed to the calculation of torque constant.

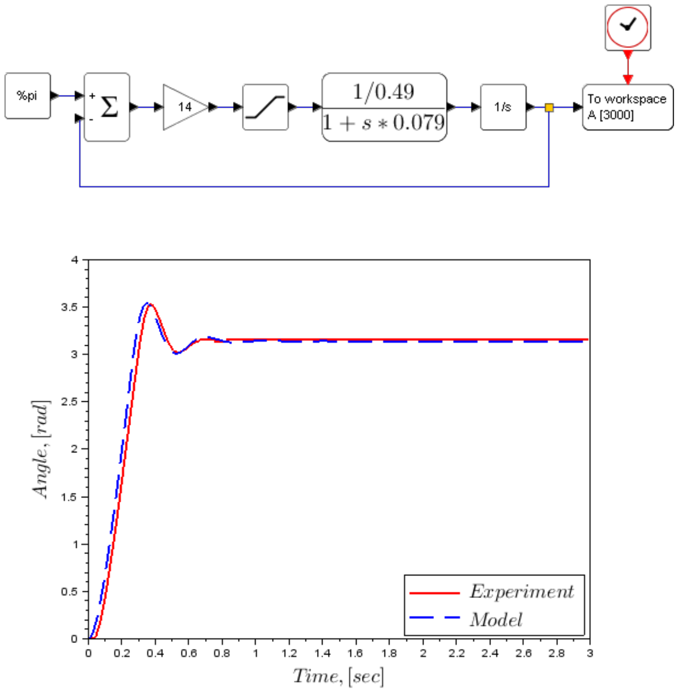


Fig. 6. Model of the P-controller and graph of real motor.

LAB ACTIVITY 4

We need to make mathematical description of our robotic system, for this we should control output torque of the motor. And we need for this task torque constant $K_t$. Consider the equation of the motor without the viscous friction:

$$\begin{cases} \dot{\omega} = \frac{K_t}{J} I, \\ \dot{I} = \frac{1}{L} U - \frac{K_e}{L} \omega - \frac{R}{L} I. \end{cases} \tag{4}$$

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where $J = 10^{-6}\ kg * m^2$ – moment of inertia of the rotor, $R = 6\ Omh$ – electric resistance, $L = 0.0047\ H$ – inductance of armature, $\omega$ – rotor speed, $I$ – the rotor current. Now, following formula 5, calculate the value of the torque constant.

$$K_t = \frac{M_{lrt} i^2}{I_{lrc}}, M_{lrt} = \frac{\omega_{nls} J}{T_m}, \tag{5}$$

where $M_{lrt}$ – the locked rotor torque, $I_{lrt}$ – the locked rotor current, $i = 48$ – the NXT motor gear ratio. Substitute values of the parameters of our motor. Torque constant and back EMF constant values should be close, for NXT these constants are $K_t = 0.42\ \frac{N m * m}{Ah}$ and $K_e = 0.49\ V * sec$. For input/output model we also need electrical time constant $T_e = \frac{L}{R} = 0.008\,sek$. Lego NXT motor steered by PWM. But in our equation used continuous control signal. Now students should check, that using PWM and DC control gives same result. The graph 7 shows simulation results for NXT motor with reduction gear.

It should be noted, that measurements of the voltmeter show us correct constant value of the voltage. For demonstration of the PWM signal, should be used oscilloscope.

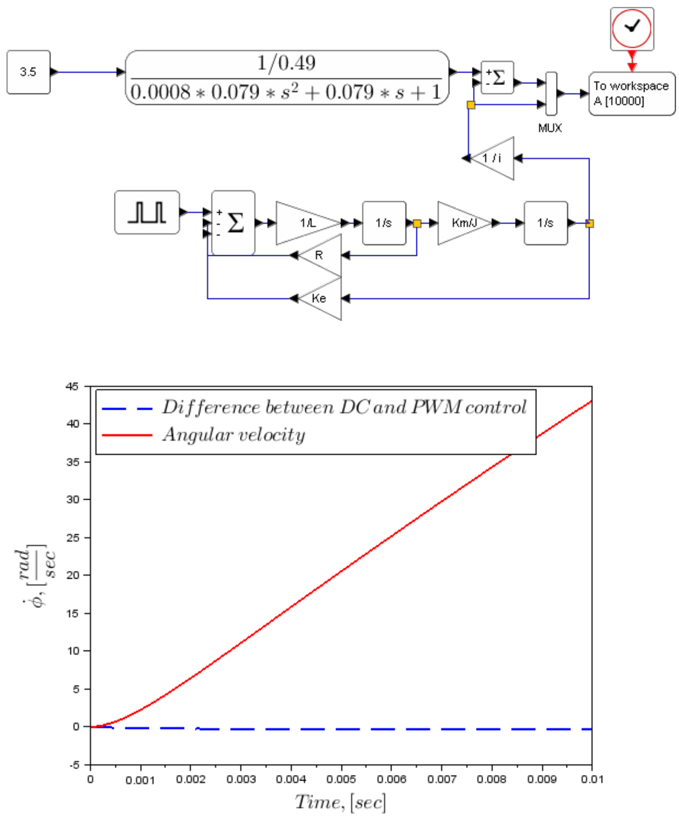


Fig. 7. Comparison of DC and PWM control.

After this students sure, that for our calculation can be used continuous control.

## LAB ACTIVITY 5

Now we have all data for making equations of our robotic system. For demonstration the main results of this activities, we chose well-known problem inverted pendulum on a cart. We take the HiTechnic Angle Sensor for measuring of the angle of pendulum.

From the EulerLagrange equation we get mathematical formulas describing inverted pendulum on a cart. For calculations of the formulas students use Maxima, computer algebra system.

$$\begin{cases} m_p lr\ddot{\theta} + (J_p + m_p l^2)\ddot{\psi} - m_p gl\psi = 0, \\ k_1 r^2\ddot{\theta} + m_p lr\ddot{\psi} + 2\dfrac{K_t K_e}{R}\dot{\theta} = 2\dfrac{K_t}{R}U, \end{cases} \tag{6}$$

where $k_1 = (\frac{4J_w}{r^2} + m_c + 4m_w + m_p)$, $m_p$ – mass of the pendulum, $l$ – distance to the center of mass of the pendulum, $r$ – radius of the wheel, $\theta$ – turning angle of the wheel, $J_p$ – inertia moment of the pendulum, $\psi$ – turning angle of the pendulum, $J_w$ – inertia moment of the wheel, $m_c$ – mass of the cart, $m_w$ – mass of the wheel.

Rewrite this system in matrix form

$$E\ddot{X} + F\dot{X} + GX = HU. \tag{7}$$

For our pendulum $X^T = [\theta, \psi]$. Divide the equation by $E$:

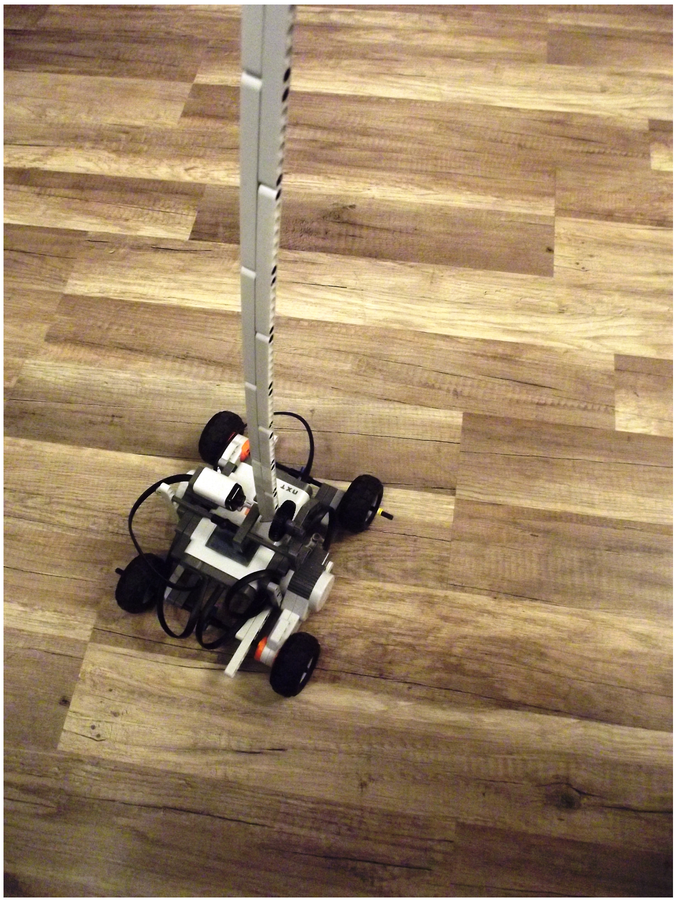

Fig. 8. Inverted pendulum on a cart based on the Lego NXT.

$$\ddot{X} = -E^{-1}F\dot{X} - E^{-1}GX + E^{-1}HU. \tag{8}$$

We need to present this equation in state space form for calculation of the feedback gains.Rewrite this equation in extended form $\dot{X} = AX + BU$, where $X^T = [\psi, \dot{\theta}, \dot{\psi}]$:

$$A = \begin{vmatrix} 0 & 0 & 1 \\ E^{-1}G[1,2] & E^{-1}F[1,1] & E^{-1}F[1,2] \\ E^{-1}G[2,2] & E^{-1}F[2,1] & E^{-1}F[2,2] \end{vmatrix} \tag{9}$$

$$B = \begin{vmatrix} 0 \\ E^{-1}H[1,2] \\ E^{-1}H[1,2] \end{vmatrix}. \tag{10}$$

Obtain an etalon model from the binom of Newton $a^3 + 3a^2 + 3a$. $a = \frac{\tau_0}{\tau}$, where $\tau_0$ an etalon transient time, $\tau$ a desired transient time. For our inverted pendulum $\tau = 0.31\ sec$. Also we need to calculate a roots of the etalon model $\alpha_n$.

Now we can use Ackermann's formula for calculating the state variable feedback matrix.

$$K = [0\,0\,\ldots\,0\,1]{P_c}^{-1}q(A), \tag{11}$$

$$q(A) = A^n + \alpha_{n-1}A^{n-1} + \ldots + \alpha_1 A + \alpha_0 I, \tag{12}$$

where $P_c$ – controllability matrix. We get the raw state variable feedback matrix, cause we don't impose limits on the value of the control signal. We should simulate this system and chose the parameters, which provide our limits. Simulation results presented by the figures 9,10.

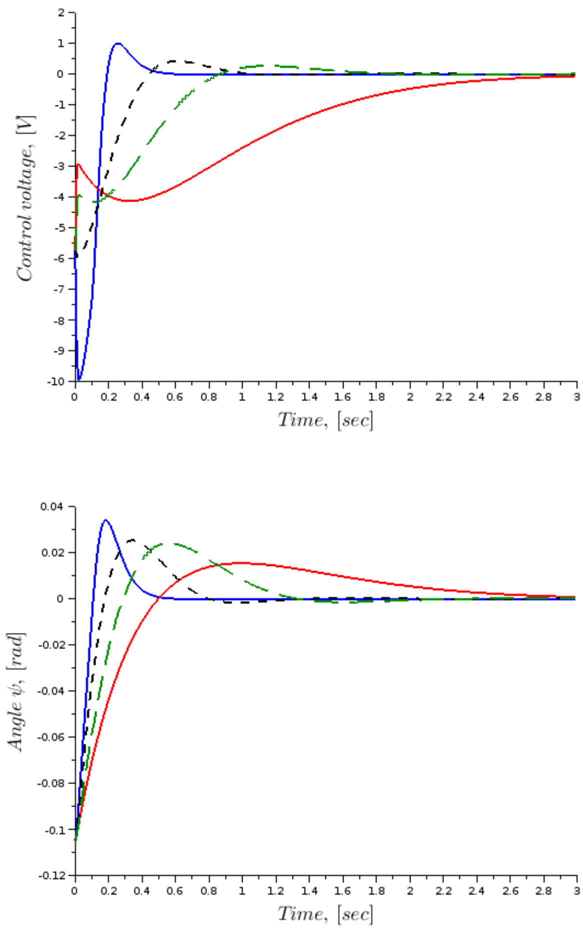


Fig. 9. Transients in the system at various of the feedback gains.

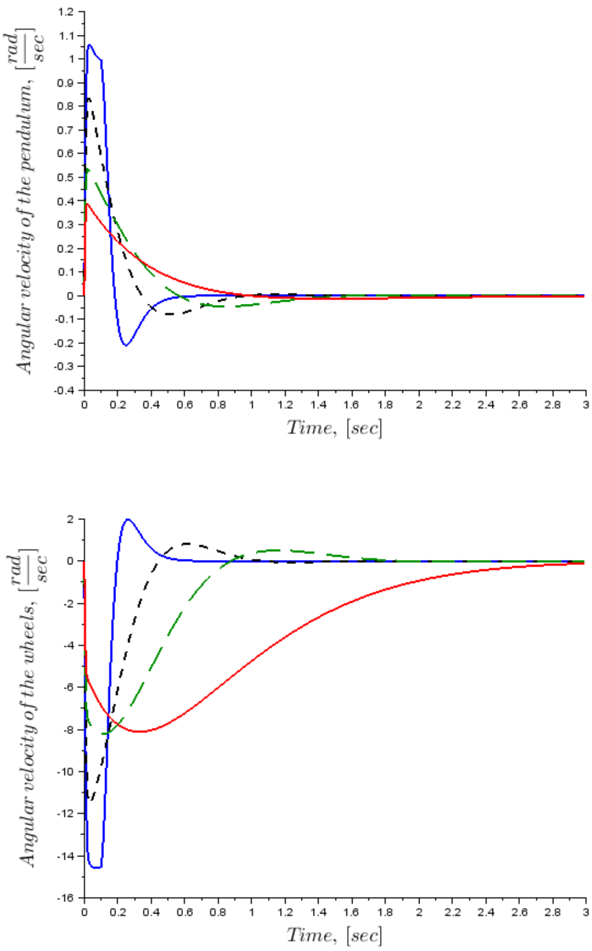


Fig. 10. Angular velocity of the pendulum and wheels.

## CONCLUSION

The article is a compilation of various methods of control theory, implemented on the basis of Lego NXT. Control of the pendulum system is popular task of the control theory. The article shows how the toy can unite many areas of scientific activity. Based on the Lego NXT constructed many different pendulum systems SegWay, Furuta pendulum, Kapitza pendulum, inverted pendulum on a cart. All projects of the Control Systems and Informatics department of the Saint-Petersburg National Research University of Information Technologies Mechanics and Optics are available on our youtube channel `http://www.youtube.com/itmo4robots`.

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