Artificial Intelligence

LOGICAL AGENTS

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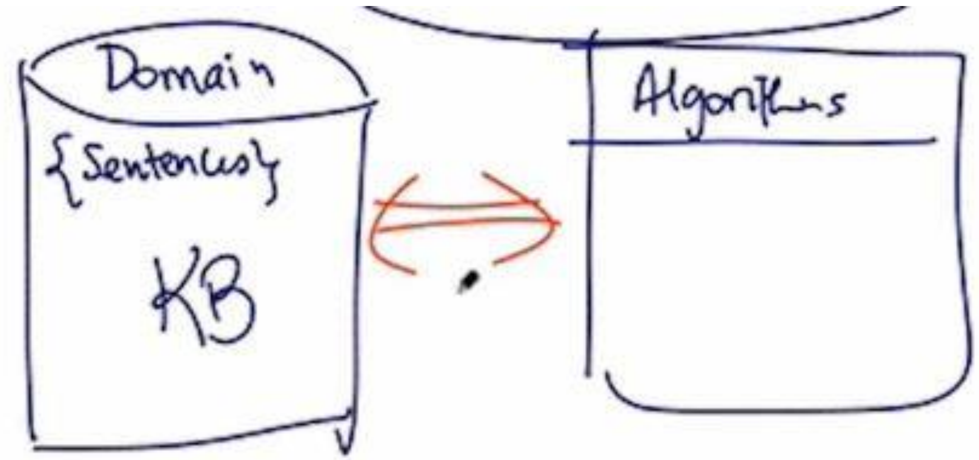
Outline

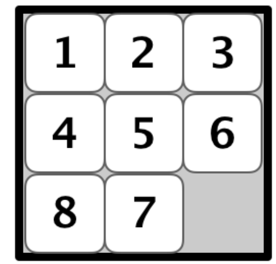
- Knowledge-based agents
- The Wumpus world
- Propositional logic: A very simple logic
- Propositional theorem proving
- Effective propositional model checking



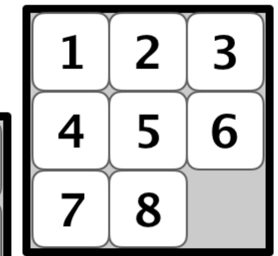
Knowledge-based agents

Problem-solving agents

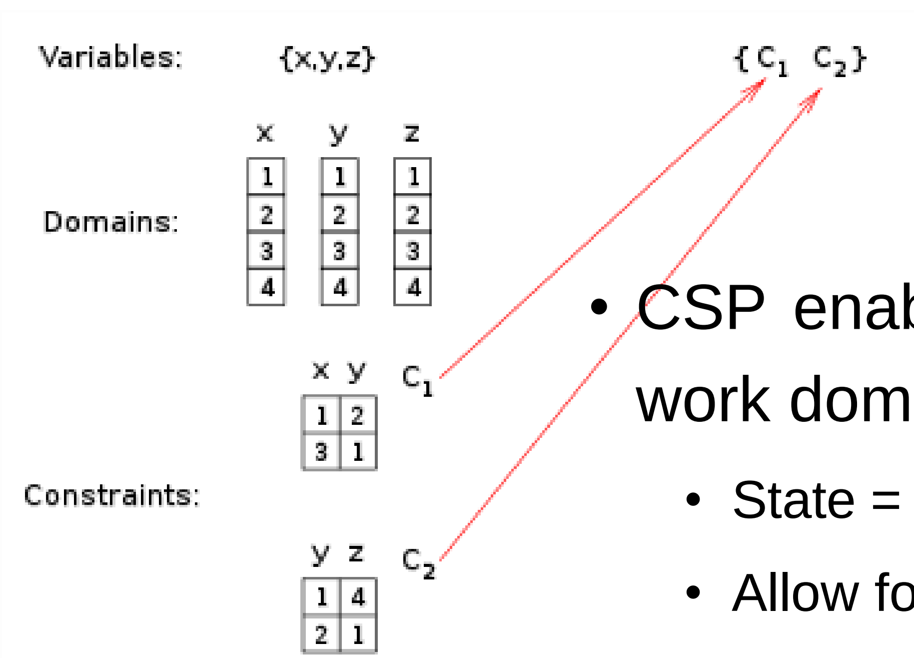
- These agents know things in a very limited, inflexible sense.
  - E.g., an 8-puzzle agent cannot deduce pairs of unsolvable states from their parities.



unsolvable



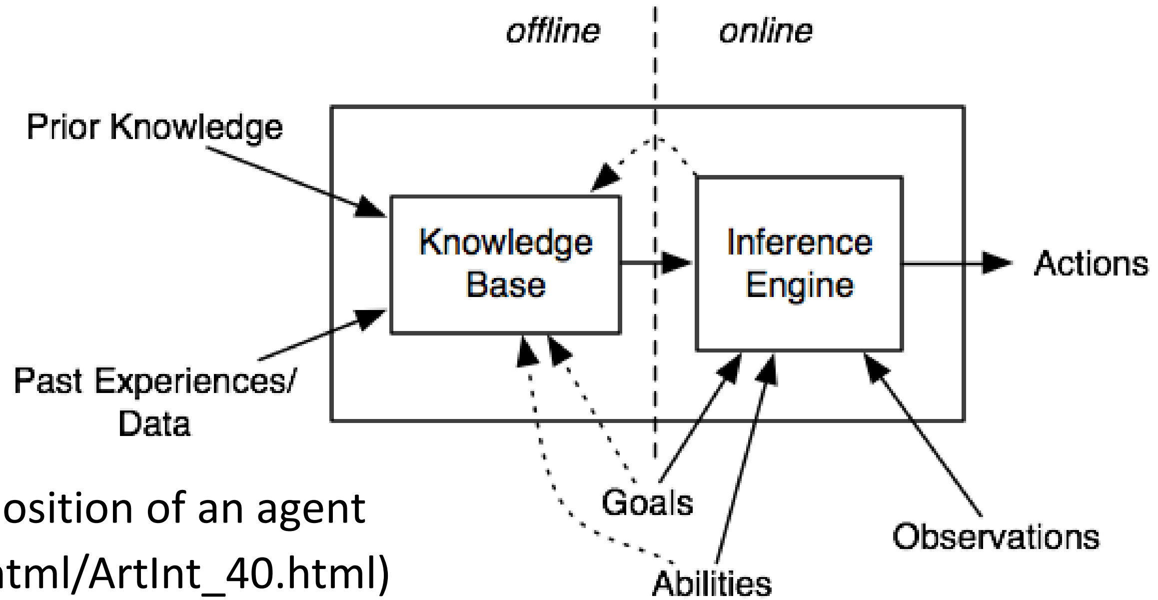
goal



- CSP enables some parts of the agent to work domain-independently
  - State = an assignment of values to variables
  - Allow for more efficient algorithms

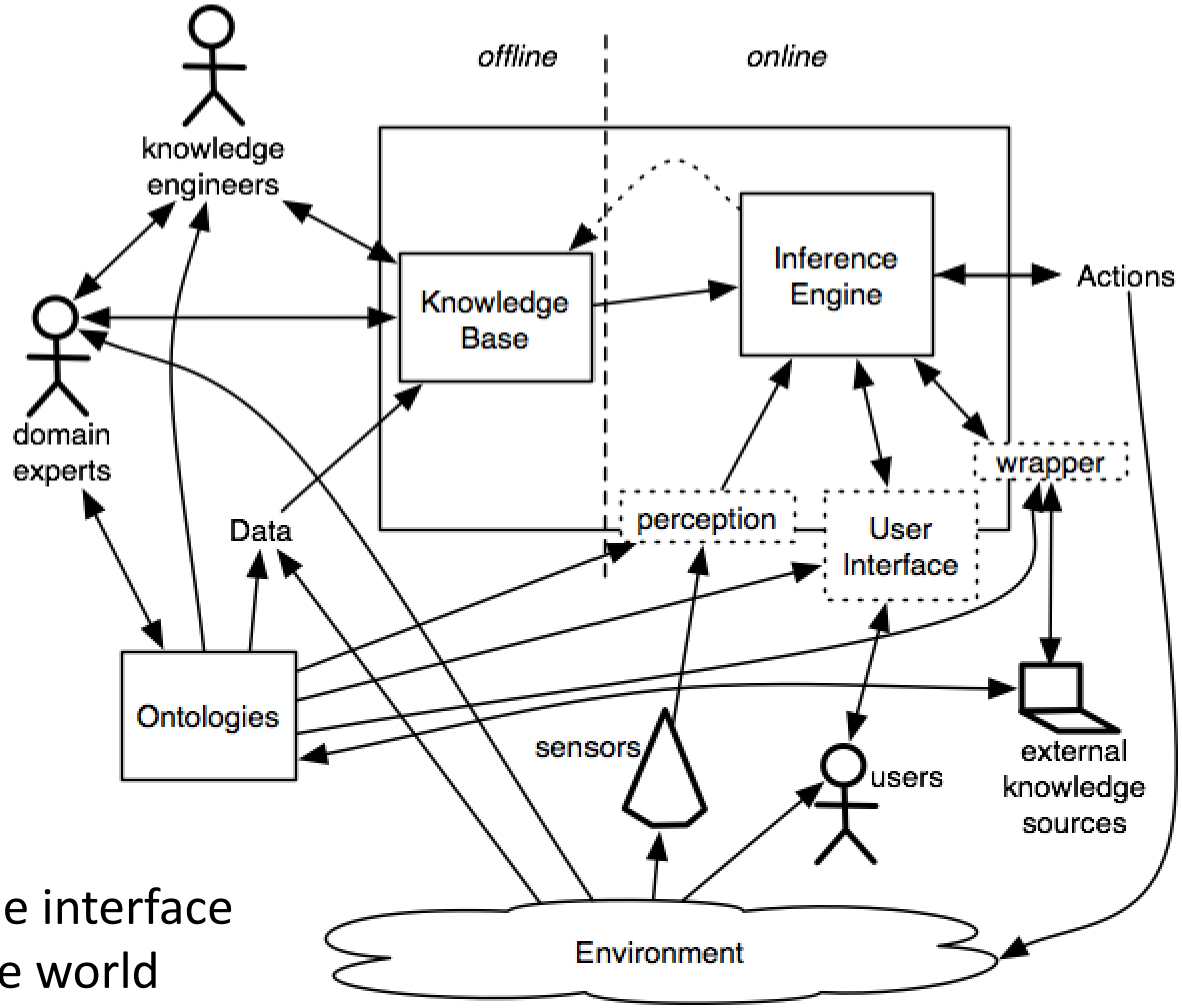
Knowledge-based agents

- Supported by **logic** – a general class of representation
- Combine and recombine information to suit myriad purposes
  - Accept new tasks in the form of explicitly described goals
  - Achieve competence by learning new knowledge of the environment
  - Adapt to changes by updating the relevant knowledge



Offline and online decomposition of an agent
(Credit: https://artint.info/html/ArtInt_40.html)

Knowledge-based agents



A detailed description of the interface between the agents and the world

(Credit: https://artint.info/html/ArtInt_40.html)

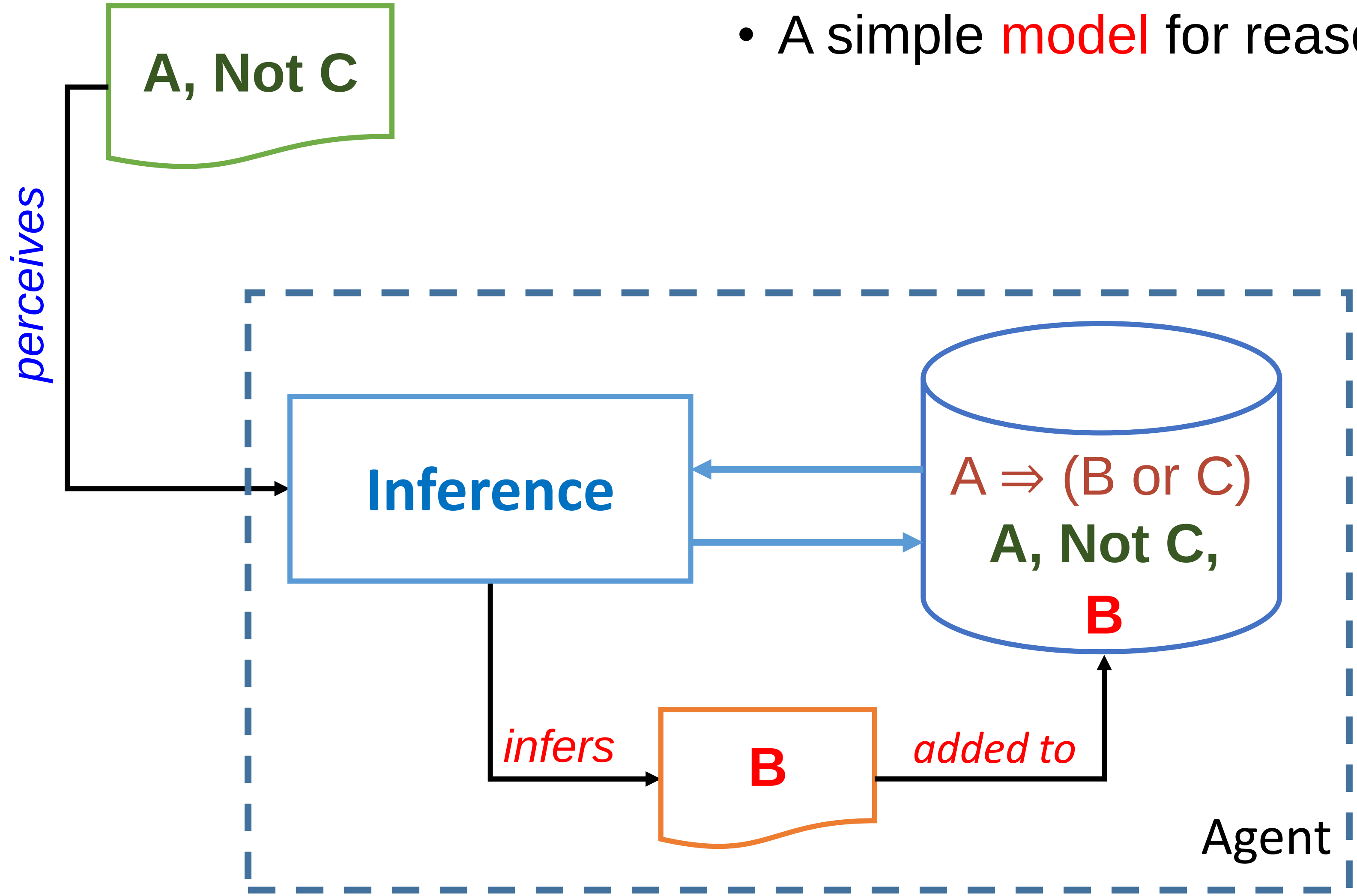
Knowledge-based agents

- Knowledge base (KB): A set of sentences or facts
  - Each sentence represents some assertion about the world.
  - Axiom = sentence that is not derived from other sentences

- Inference: Derive (infer) new sentences from old ones
  - Add new sentences to the knowledge base and query what is known

Model for reasoning: An example

- A simple model for reasoning



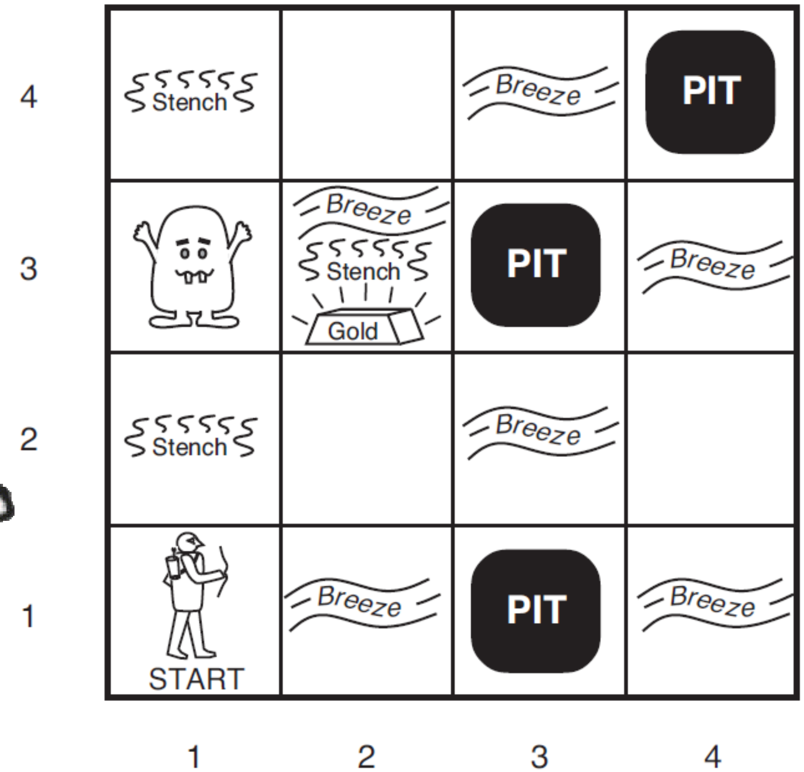
A generic knowledge-based agent

function KB-AGENT(*percept*) **returns** an *action*
persistent: *KB*, a knowledge base
t, a counter, initially 0, indicating time
TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))
action ← ASK(*KB*, MAKE-ACTION-QUERY(*t*))
TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))
t ← *t* + 1
return *action*

Inference mechanisms are hidden inside TELL and ASK

A generic knowledge-based agent

- Declarative approach
    - Empty KB $\rightarrow$ TELL the agent the facts, one by one until it knows how to operate in its environment
- Procedural approach
    - Encode desired behaviors directly as program code
- Combined approach $\rightarrow$ Partially autonomous
- Learning approach (Chapter 18) $\rightarrow$ Fully autonomous
    - Provide a knowledge-based agent with mechanisms that allow it to learn for itself



The Wumpus world

PEAS Description

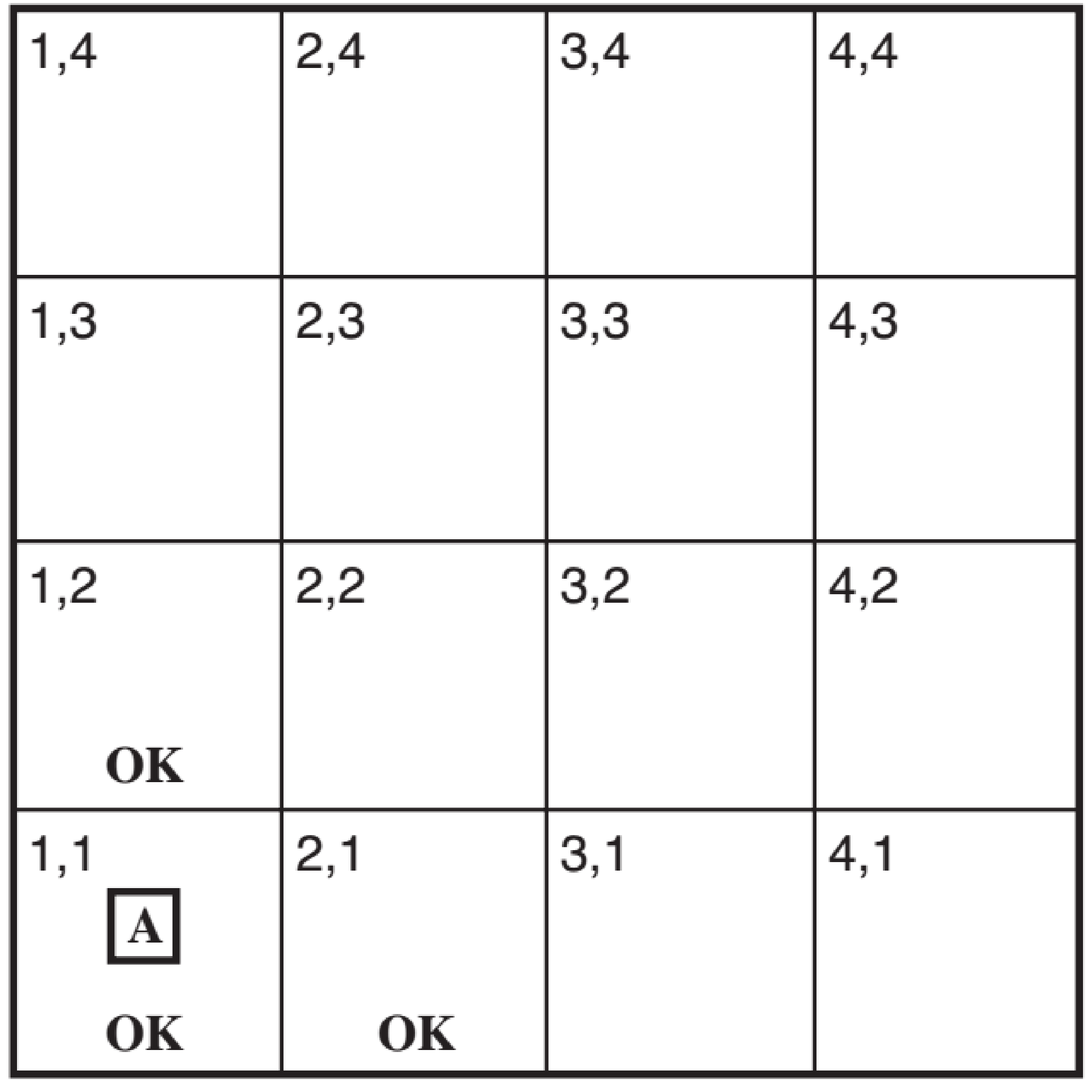
- Environment
  - 4×4 grid of rooms, agent starts in the square [1,1], facing to the right
  - The locations of Gold and Wumpus are random
  - Each square can be a pit, with probability 0.2
- Performance measure
  - +1000 for climbing out of the cave with gold, -1000 for death
  - -1 per step, -10 for using the arrow
  - The game ends when agent dies or climbs out of the cave
- Actuators: $Forward$, $TurnLeft/TurnRight$ by 90º, $Grab, Shoot, Climb$
- Sensors: $Stench, Breeze, Glitter, Bump, Scream$
- Percept: $[Stench, Breeze, None, None, None]$

Characterize the Wumpus world

- Fully Observable: No – only local perception
- Deterministic: Yes – outcomes exactly specified
- Episodic: No – sequential at the level of actions
- Static: Yes – Wumpus and Pits do not move
- Discrete: Yes
- Single-agent: Yes – Wumpus is essentially a natural feature

Exploring a Wumpus world

$\boxed{\mathbf{A}}$ = *Agent*
B = *Breeze*
G = *Glitter, Gold*
OK = *Safe square*
P = *Pit*
S = *Stench*
V = *Visited*
W = *Wumpus*



Exploring a Wumpus world

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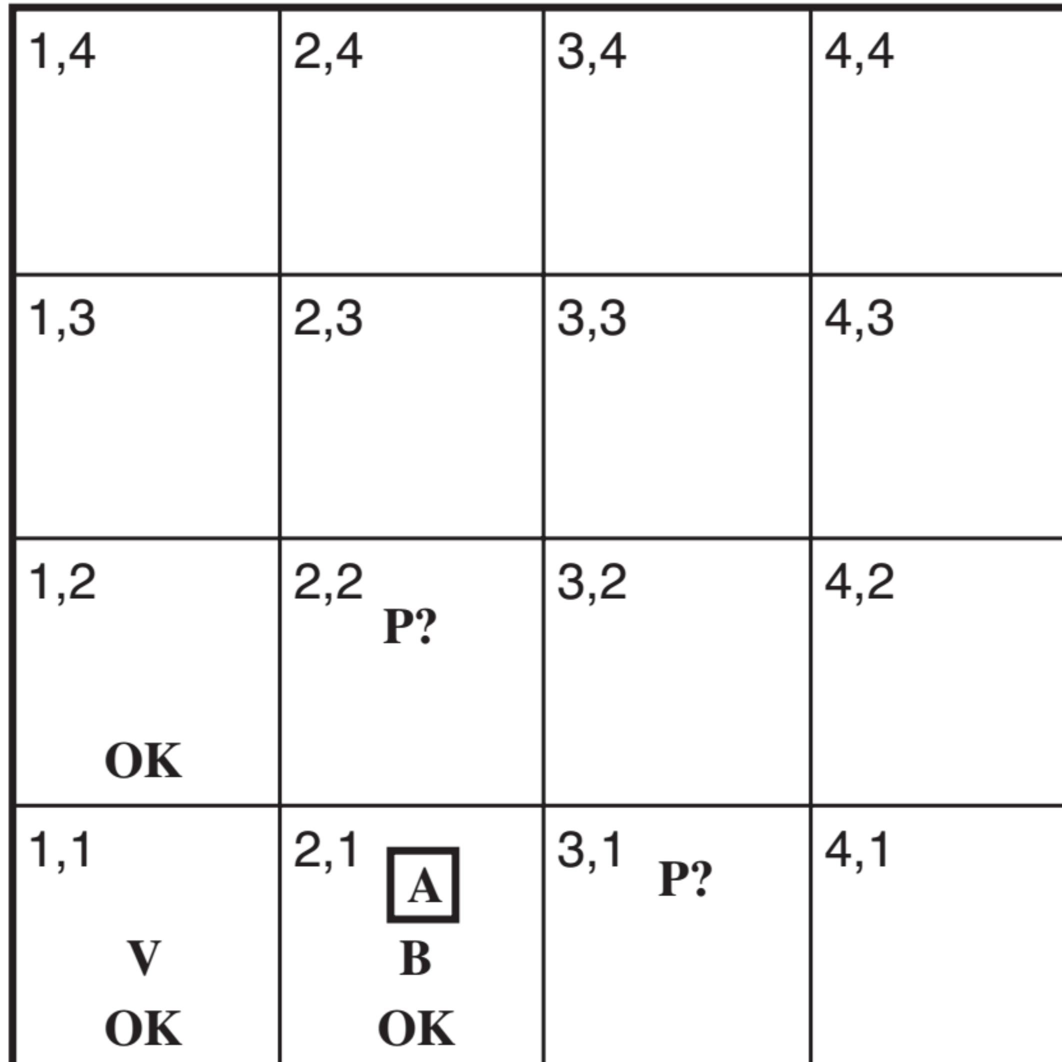

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W = *Wumpus*

1,4	2,4	3,4	4,4
1,3 **W!**	2,3	3,3	4,3
1,2 $\boxed{\text{A}}$ **S** **OK**	2,2 **OK**	3,2	4,2
1,1 **V** **OK**	2,1 **B** **V** **OK**	3,1 **P!**	4,1

Exploring a Wumpus world

- $\boxed{A}$ = *Agent*
- **B** = *Breeze*
- **G** = *Glitter, Gold*
- **OK** = *Safe square*
- **P** = *Pit*
- **S** = *Stench*
- **V** = *Visited*
- **W** = *Wumpus*

1,4	2,4 **P?**	3,4	4,4
1,3 **W!**	2,3 $\boxed{A}$ **S G B**	3,3 **P?**	4,3
1,2 **S V OK**	2,2 **V OK**	3,2	4,2
1,1 **V OK**	2,1 **B V OK**	3,1 **P!**	4,1

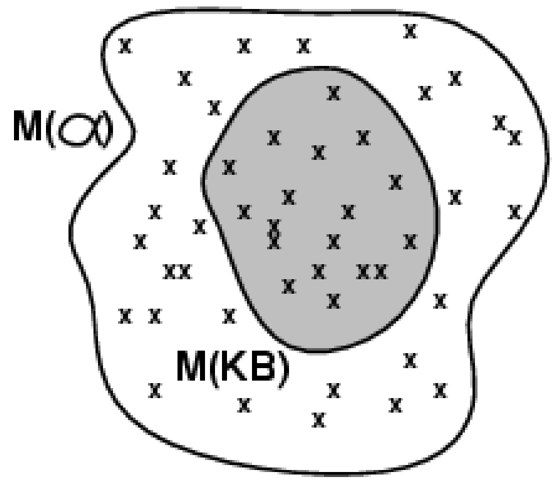
Propositional logic

Logics in general

- Models (or possible worlds) are mathematical abstractions that fix the truth or falsehood of every relevant sentence.
  - E.g., all possible assignments of real numbers to x and y
- m satisfies (or is a model of) α if α is true in model m
- $M(\alpha) =$ the set of all models of α

Entailment in logic

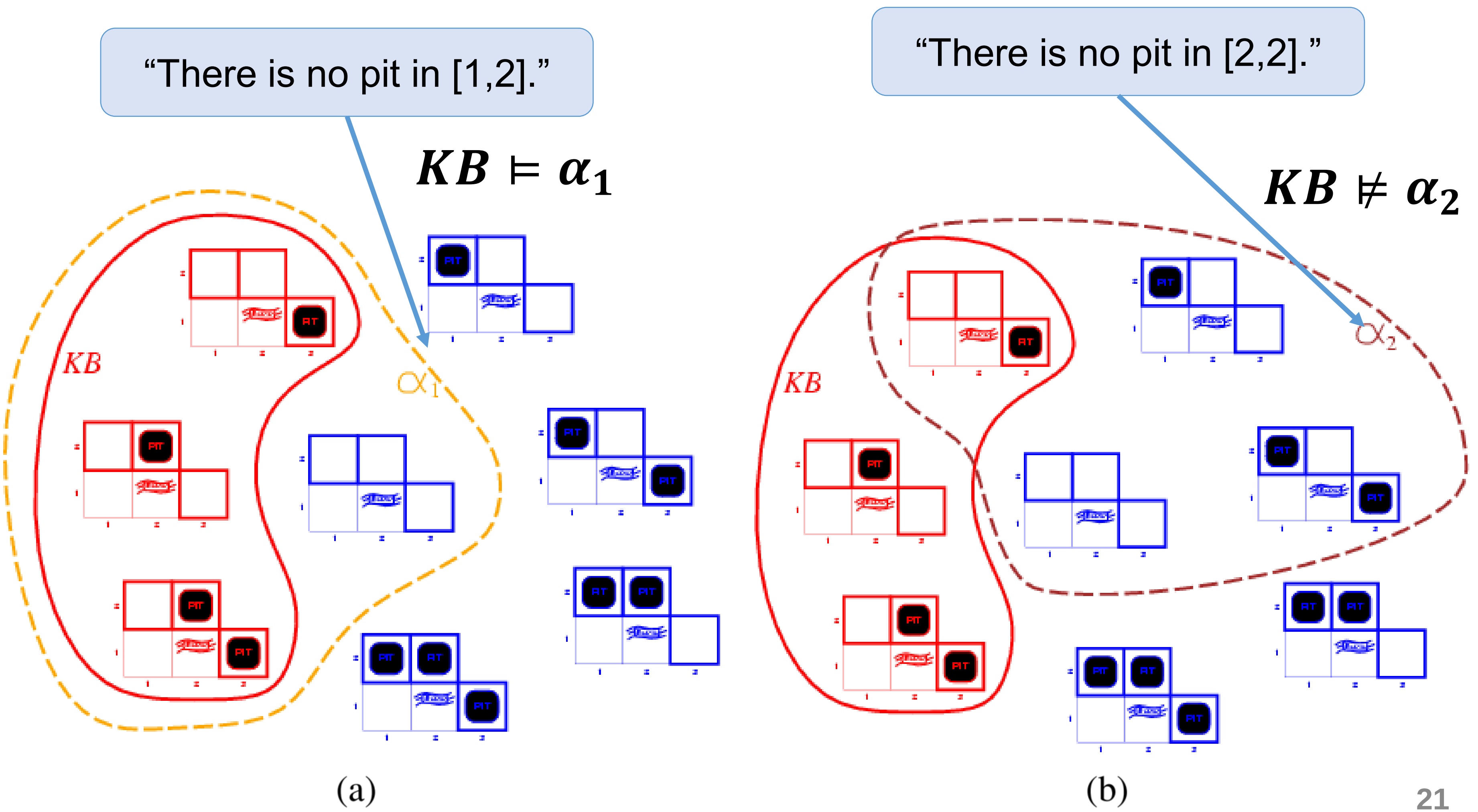
- A sentence follows logically from another sentence: $\boldsymbol{\alpha} \vDash \boldsymbol{\beta}$
- $\boldsymbol{\alpha} \vDash \boldsymbol{\beta}$ if and only if, in every model in which $\boldsymbol{\alpha}$ is true, $\boldsymbol{\beta}$ is also true, i.e., $M(\alpha) \subseteq M(\beta)$



- For example,
  - $x = 0$ entails $xy = 0$
  - The KB containing “Apple is red” and “Tomato is red” entails “Either the apple or the tomato is red”
- Entailment is a relationship between sentences (i.e., syntax) that is based on semantics.

Entailment in logic: Wumpus world

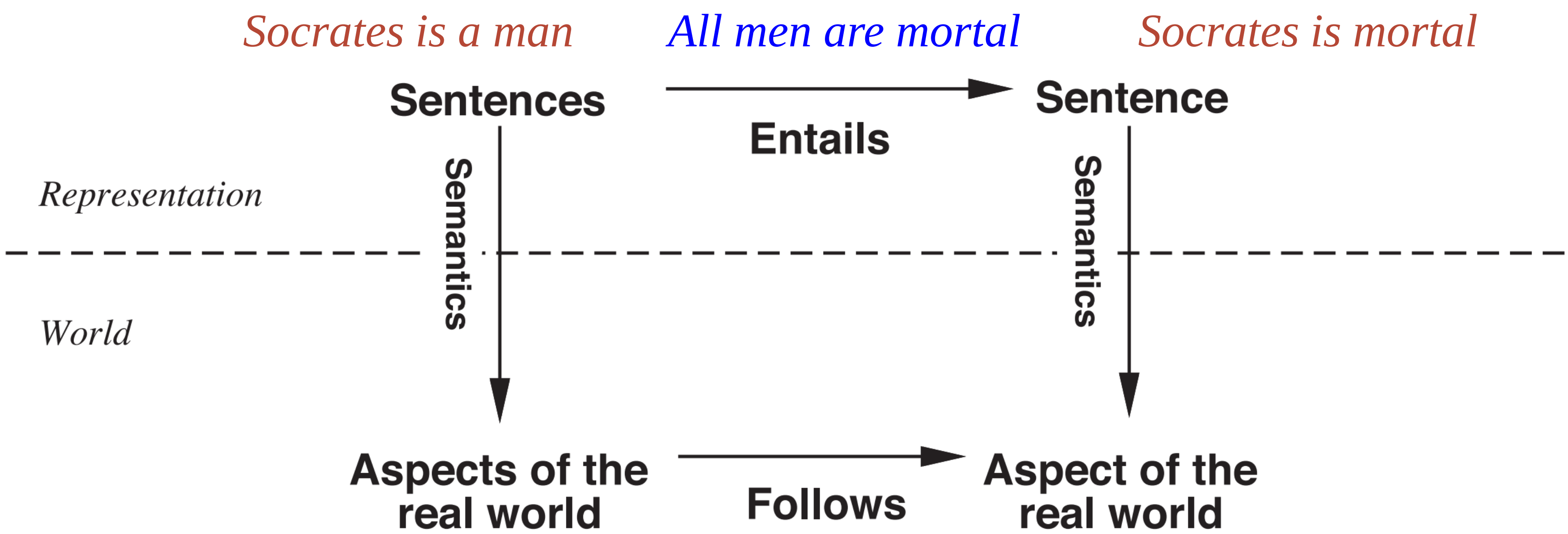
- Consider two possible conclusions α_1 and α_2

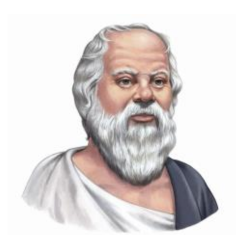


Logical inference

- $KB \vDash_i \alpha$ means α can be derived from KB by procedure i
- Soundness: i is sound if whenever $KB \vDash_i \alpha$, it is also true that $KB \vDash \alpha$
- Completeness: i is complete if whenever $KB \vDash \alpha$, it is also true that $KB \vDash_i \alpha$
- That is, the procedure will answer any question whose answer follows from what is known by the *KB*.

World and representation

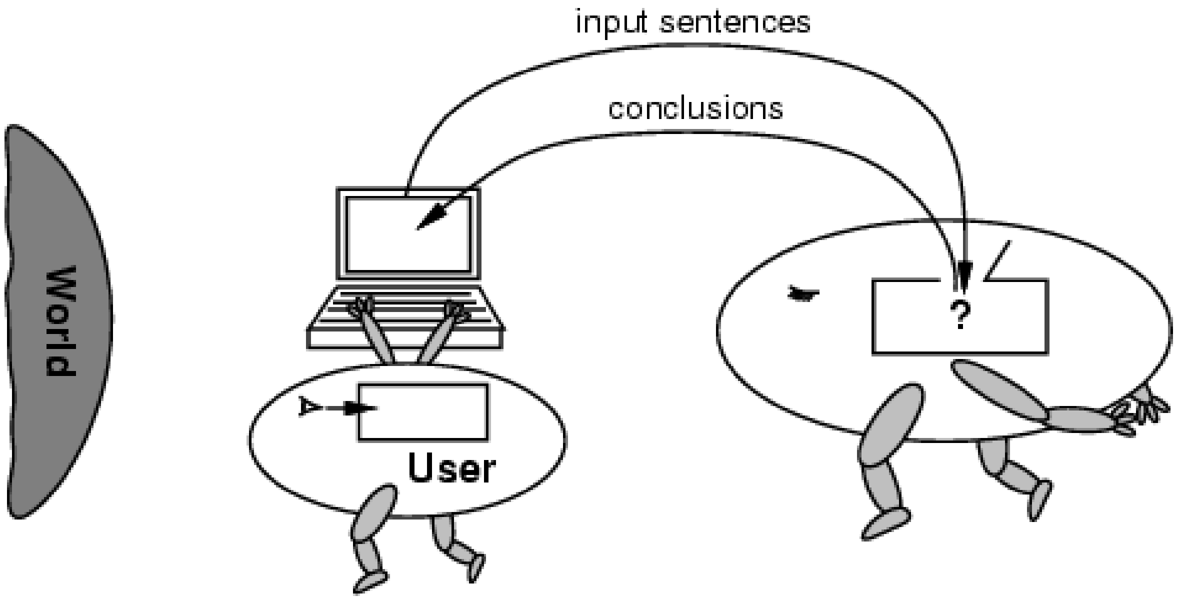




[470 – 399 BC]

No independent access to the world

- The reasoning agent gets its knowledge about the facts of the world as a sequence of logical sentences
- Conclusions must be drawn only from those → without agent's independent access to the world
- Thus, it is very important that the agent's reasoning is sound!



Propositional logic: Syntax

- Constants: **TRUE** or **FALSE**
- Symbols stand for propositions (sentences): $P, Q, P_1, W_{1,3}, \ldots$
- Logical connectives

NOT	$\neg$	Negation
AND	$\wedge$	Conjunction
OR	$\vee$	Disjunction
IMPLIES	$\Rightarrow$	Implication (if..then)
IFF	$\Leftrightarrow$	Equivalence, biconditional

- Literal: atomic sentence (P) or negated atomic sentence ($\neg$P)

Propositional logic: Syntax

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \Rightarrow Sentence \\
& \mid & Sentence \Leftrightarrow Sentence \\
\\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Propositional logic: Semantics

- Each model specifies true/false for each proposition symbol.
  - E.g., $m_1 = \{P_{1,2} = false, P_{2,2} = false, P_{3,1} = true\}$, 8 possible models

- Rules for evaluating truth with respect to a model m

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

- Simple recursive process evaluates an arbitrary sentence.
  - E.g.,$\neg P_{1,2} \wedge \left(P_{2,2} \vee P_{3,1}\right) = true \wedge (true \vee false) = true \wedge true = true$

A simple knowledge base

- Symbols for each position $[i, j]$
  - $P_{i,j}$: there is a pit in $[i, j]$
  - $W_{i,j}$: there is a Wumpus in $[i, j]$
  - $B_{i,j}$: there is a breeze in $[i, j]$
  - $S_{i,j}$: there is a stench in $[i, j]$

- Sentences in Wumpus world's KB

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 $\boxed{A}$ B OK	3,1 P?	4,1

A simple inference procedure

- **Given:** a set of sentences, $\boldsymbol{KB}$, and sentence $\boldsymbol{\alpha}$
- **Goal:** answer $\boldsymbol{KB} \vDash \boldsymbol{\alpha}?$ = "Does $\boldsymbol{KB}$ semantically entail $\boldsymbol{\alpha}$?"
  - In all interpretations in which KB's sentences are true, is α also true?
  - E.g., in the Wumpus world, $KB \vDash P_{1,2}?$ = "Is there a pit in [1,2]?"

Model-checking approach (Inference by enumeration)

Inference rules

Conversion to the inverse SAT problem (Resolution refutation)

Model-checking approach

- Check if α is true in every model in which KB is true.
    - E.g., the Wumpus's KB has 7 symbols → 2^7 = 128 models
- Draw a truth table for checking

No pit in [1,2]

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference by (depth-first) enumeration

function TT-ENTAILS?(*KB*,α) **returns** *true* or *false*
inputs: *KB*, the knowledge base, a sentence in propositional logic
α, the query, a sentence in propositional logic
symbols ← a list of the proposition symbols in *KB* and α
return TT-CHECK-ALL(*KB*,α,*symbols*,{ })

---

function TT-CHECK-ALL(*KB*,α,*symbols*,*model*) **returns** *true* or *false*
if EMPTY?(*symbols*) **then**
if PL-TRUE?(*KB*,*model*) **then return** PL-TRUE?(α,*model*)
else return *true* // *when KB is false, always return true*
else do
P ← FIRST(*symbols*)
rest ← REST(*symbols*)
return (TT-CHECK-ALL(*KB*,α,*rest*,*model* ∪ {*P* = *true*})
and TT-CHECK-ALL(*KB*,α,*rest*,*model* ∪ {*P* = *false*}))

sound and complete

Time complexity $O(2^n)$, space complexity $O(n)$

Quiz 01: Model-checking approach

- Given a KB containing the following rules and facts

  R_1: IF hot AND smoky THEN fire

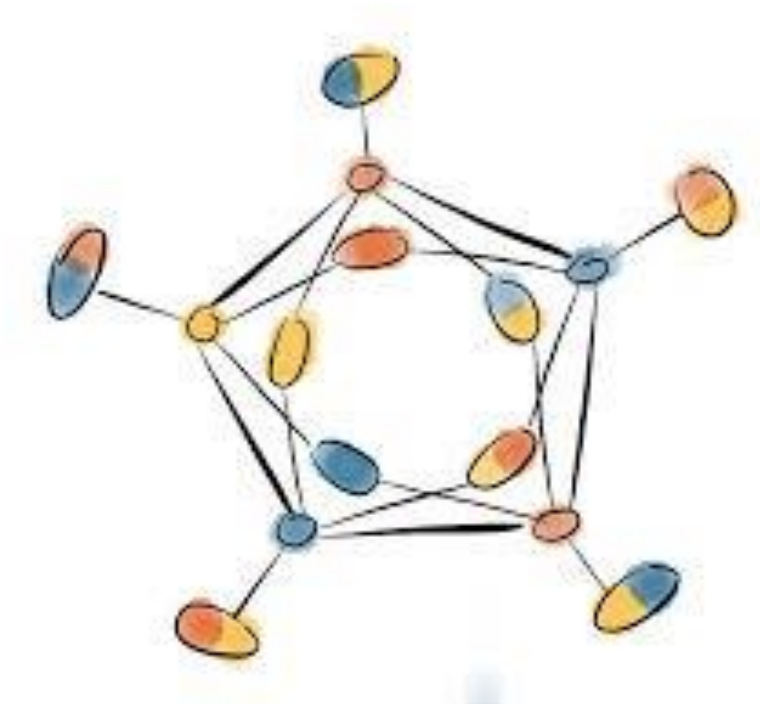
  R_2: IF alarm_beeps THEN smoky

  R_3: IF fire THEN sprinklers_on

  F_1: alarm_beeps

  F_2: hot

- Represent the KB in propositional logic with given symbols
  - H = hot, S = smoky, F = fire, A = alarms_beeps, R = sprinklers_on
- Answer the question “Sprinklers_on?” by using the model-checking approach.

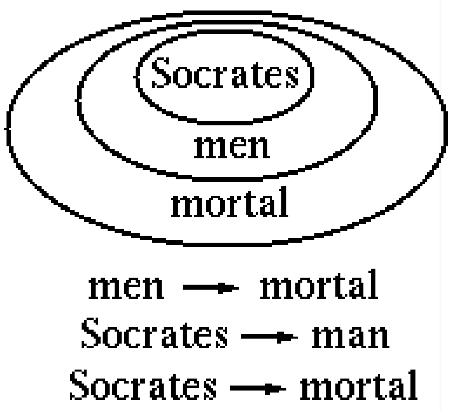


Propositional theorem proving

- *Proof by Resolution*
- *Forward and Backward Chaining*

Inference rules approach

- Theorem proving: Apply rules of inference directly to the sentences in KB to construct a proof of the desired sentence **without** consulting models
- More efficient than model checking when the number of models is large, yet the length of the proof is short



Logical equivalence

- Two sentences, α and β, are logically equivalent if they are true in the same set of models.

$$\boldsymbol{\alpha \equiv \beta \;\; iff \;\; \alpha \vDash \beta \;\; and \;\; \beta \vDash \alpha}$$

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Validity

- A sentence is valid if it is true in all models.
  - E.g., $P \vee \neg P$, $P \Rightarrow \neg P$, $(\mathrm{P} \wedge (P \Rightarrow Q)) \Rightarrow Q$
- Valid sentences are also known as tautologies.
- Validity is connected to inference via the Deduction Theorem

$$\alpha \vDash \beta \;\; iff \;\; \alpha \Rightarrow \beta \; is \; valid$$

Satisfiability

- A sentence is satisfiable if it is true in some model.
  - E.g., $P \vee Q$, P
- A sentence is unsatisfiable if it is true in no models.
  - E.g., $P \wedge \neg P$
- Satisfiability is connected to inference via the following

$$\alpha \vDash \beta \;\; iff \;\; \alpha \wedge \neg\beta \; is \; unsatisfiable$$

$\rightarrow$ Refutation or proof by contradiction

- The **SAT problem** determines the satisfiability of sentences in propositional logic (NP-complete)
  - E.g., in CSPs, the constraints are satisfiable by some assignment.

Quiz 02: Validity and Satisfiability

- Check the validity and satisfiability of the below sentences using the truth table

  1. $A \lor B \Rightarrow A \land C$
  2. $A \land B \Rightarrow A \lor C$
  3. $(A \lor B) \land (\neg B \lor C) \Rightarrow A \lor C$
  4. $(A \lor \neg B) \Rightarrow A \land B$

Inference and Proofs

- **Proof**: A chain of conclusions leads to the desired goal
- Example sound rules of inference

$$\frac{\alpha \Rightarrow \beta \qquad \alpha}{\therefore\ \beta}$$

Modus Ponens

$$\frac{\alpha \Rightarrow \beta \qquad \neg\beta}{\therefore\ \neg\alpha}$$

Modus Tollens

$$\frac{\alpha \qquad \beta}{\therefore\ \alpha \wedge \beta}$$

AND-Introduction

$$\frac{\alpha \wedge \beta}{\therefore\ \alpha}$$

AND-Elimination

Inference rules: An example

KB
$P \wedge Q$
$P \Rightarrow R$
$Q \wedge R \Rightarrow S$

$\boldsymbol{S}$?

No.	Sentences	Explanation
1	$P \wedge Q$	From KB
2	$P \Rightarrow R$	From KB
3	$Q \wedge R \Rightarrow S$	From KB
4	P	1 And-Elim
5	R	4,2 Modus Ponens
6	Q	1 And-Elim
7	$Q \wedge R$	5,6 And-Intro
8	$\boldsymbol{S}$	3,7 Modus Ponens

Inference rules in Wumpus world

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof: $\neg P_{1,2}$

- Bi-conditional elimination to R_2 : R_6 : $\left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)$
- And-Elimination to R_6 : R_7 : $(P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}$
- Logical equivalence for contrapositives: R_8 : $\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})$
- Modus Ponens with R_8 and the percept R_4 : R_9: $\neg(P_{1,2} \vee P_{2,1})$
- De Morgan’s rule: R_{10} : $\neg P_{1,2} \wedge \neg P_{2,1}$

Proving by search

- Search algorithms can be applied to find a sequence of steps that constitutes a proof.
  - INITIAL STATE: the initial knowledge base
  - ACTIONS: apply all inference rules to all the sentences that match the top half of the inference rule
  - RESULT: add the sentence in the bottom half of the inference rule
  - GOAL: a state that contains the sentence need to be proved
- The proof can ignore irrelevant propositions, no matter how many of them there are → more efficient
  - E.g., in the Wumpus world, $B_{2,1}$, $P_{1,1}$, $P_{2,2}$ and $P_{3,1}$ are not mentioned.

Monotonicity

- The set of entailed sentences only increases as information is added to the knowledge base.

$$if\ \ KB \vDash \alpha \ \ then\ \ KB \wedge \beta \vDash \alpha$$

- Additional conclusions can be drawn without invalidating any conclusion α already inferred.

Proof by Resolution

- Proof by Inference Rules: sound but not complete
  - If the rules are inadequate, then the goal is not reachable.
- **Resolution:** sound and complete, a single inference rule
  - A **complete** inference algorithm when coupled with any complete search algorithm
  - Unit resolution inference rule

$$\frac{l_1 \vee \cdots \vee l_k \qquad m}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k}$$

where l_i and m are **complementary literals**

  - Full resolution rule

$$\frac{l_1 \vee \cdots \vee l_k \qquad m_1 \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where l_i and m_j are complementary literals

Inference rules in Wumpus world

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

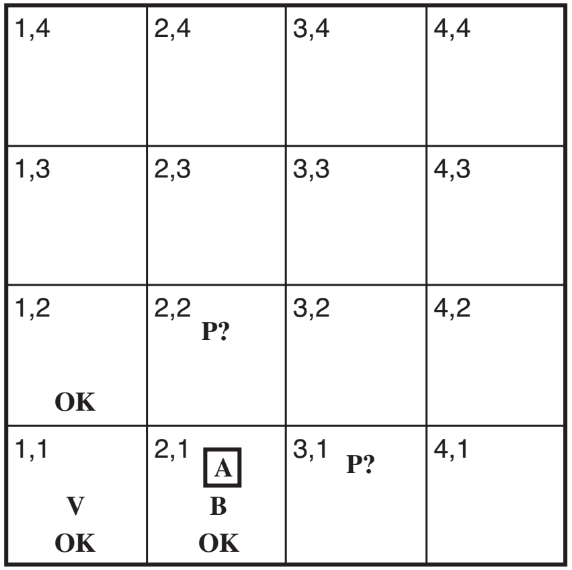
R_6: $\left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)$

R_7: $\neg P_{1,2} \wedge \neg P_{2,1} \Rightarrow B_{1,1}$

R_8: $\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})$

R_9 :$\neg(P_{1,2} \vee P_{2,1})$

R_{10} : $\neg P_{1,2} \wedge \neg P_{2,1}$



Inference rules in Wumpus world

R_1: $\neg P_{1,1}$

…

R_{11}: $\neg B_{1,2}$

R_{12}: $B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})$

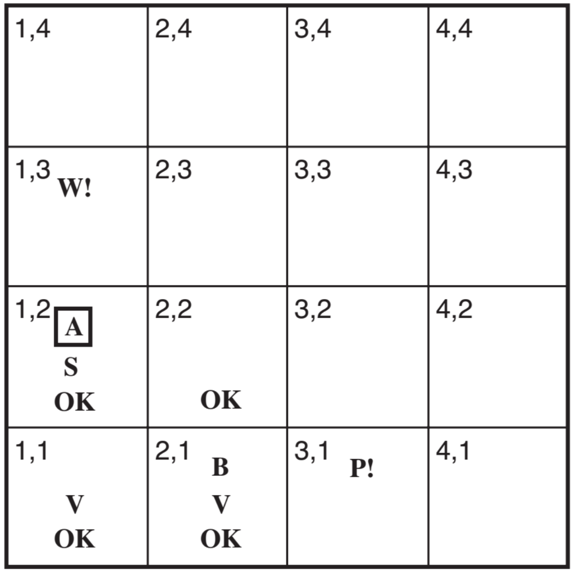
R_{13}: $\neg P_{2,2}$

R_{14}: $\neg P_{1,3}$

R_{15}: $P_{1,1} \vee P_{2,2} \vee P_{3,1}$

R_{16}: $P_{1,1} \vee P_{3,1}$

R_{17}: $P_{3,1}$



$\neg P_{2,2}$ resolves with $P_{2,2}$

$\neg P_{1,1}$ resolves with $P_{1,1}$

Proof by Resolution

- Factoring: the resulting clause should contain only one copy of each literal.
  - E.g., resolving $(A \vee B)$ with $(A \vee \neg B)$ obtains $(A \vee A)$ $\rightarrow$ reduced to A

- For *any pair of sentences*, α and β, *in propositional logic*, a resolution-based theorem prover *can decide whether* $\alpha \vDash \beta$.

Conjunctive Normal Form (CNF)

- Resolution applies only to clauses, i.e., disjunctions of literals

  $\rightarrow$ Convert all sentences in KB into clauses (CNF form)

- For example, convert $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ into CNF

  $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

  $\rightarrow$ A conjunction of 3 clauses

Conversion to CNF

1. Eliminate ⇔: $\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$
2. Eliminate ⇒: $\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$
3. The operator ¬ appears only in literals: "move ¬ inwards"

   $\neg\neg\alpha \equiv \alpha$ (double-negation elimination)

   $\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$ (De Morgan)

   $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$ (De Morgan)
4. Apply the distributivity law to distribute ∨ over ∧

$$(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$$

Quiz 03: Conversion to CNF

- Convert the following sentences into CNF

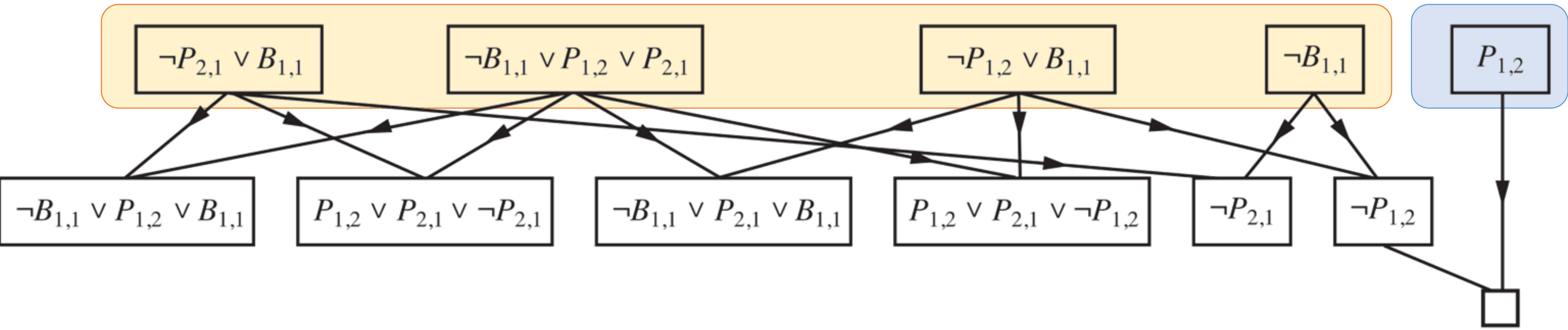
1. $(A \wedge B) \Rightarrow (C \Rightarrow D)$
2. $P \vee Q \Leftrightarrow R \wedge \neg Q \Rightarrow P$

The resolution algorithm

- Proof by contradiction (resolution refutation): To show that $KB \vDash \alpha$, prove $KB \wedge \neg\alpha$ is unsatisfiable

function PL-RESOLUTION(KB,α) **returns** *true* or *false*
    inputs: KB, the knowledge base, a sentence in propositional logic
                α, the query, a sentence in propositional logic
    clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
    new ← { }
    loop do
        for each pair of clauses C_i, C_j **in** *clauses* **do**
            resolvents ← PL-RESOLVE(C_i, C_j)
            if *resolvents* contains the empty clause **then return** *true*
            new ← *new* ∪ *resolvents*
        if *new* ⊆ *clauses* **then return** *false*
        clauses ← *clauses* ∪ *new*

The resolution algorithm



- Many resolution steps are pointless.
- Clauses with two complementary literals can be discarded.
  - E.g., $B_{1,1} \vee \neg B_{1,1} \vee P_{2,1} \equiv True \vee P_{2,1} \equiv True$

Quiz 04: The resolution algorithm

- Given the following hypotheses
  - If it rains, Joe brings his umbrella.
  - If Joe brings his umbrella, Joe does not get wet.
  - If it does not rain, Joe does not get wet.
- Prove that Joe does not get wet.

Quiz 04: The resolution algorithm

- The KB contains facts and hypotheses

KB
$R \Rightarrow U$
$U \Rightarrow \neg W$
$\neg R \Rightarrow \neg W$

- Check if the sentence

$\neg W$ is entailed by KB?

Horn clauses and Definite clauses

- Definite clause: a disjunction of literals of which exactly one is positive.
  - E.g., $\neg P \vee \neg Q \vee R$ is a definite clause, whereas $\neg P \vee Q \vee R$ is not.
- Horn clause: a disjunction of literals of which at most one is positive.
  - All definite clauses are Horn clauses
- Goal clause: clauses with no positive literals
- Horn clauses are closed under resolution
  - Resolving two Horn clauses will get back a Horn clause.

Backus normal form (BNF)

$$
\begin{array}{rcl}
CNFSentence & \rightarrow & Clause_1 \wedge \cdots \wedge Clause_n \\
Clause & \rightarrow & Literal_1 \vee \cdots \vee Literal_m \\
Literal & \rightarrow & Symbol \mid \neg Symbol \\
Symbol & \rightarrow & P \mid Q \mid R \mid \ldots \\
HornClauseForm & \rightarrow & DefiniteClauseForm \mid GoalClauseForm \\
DefiniteClauseForm & \rightarrow & (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow Symbol \\
GoalClauseForm & \rightarrow & (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow False
\end{array}
$$

KB of definite clauses

- KB containing only definite clauses are interesting.
- Every definite clause can be written as an implication.
  - Premise (**body**) is a conjunction of positive literals and Conclusion (**head**) is a single positive literal (**fact**) $\rightarrow$ easier to understand
  - E.g., $\neg P \vee \neg Q \vee R \equiv (P \wedge Q) \Rightarrow R$
- Inference can be done with forward-chaining and backward-chaining algorithms
  - This type of inference is the basis for **logic programming**.
- Deciding entailment can be done in linear time.

KB: Horn clauses vs. CNF clauses

Disjuctions of literals
$(l_1 \lor l_2 \lor \cdots \lor l_m)$

CNF clauses

Clause 1 $\land$ Clause 2 $\land \ldots \land$ Clause *n*

Disjunctions of literals of which **at most one is positive**
$(\neg l_1 \lor \neg l_2 \lor \cdots \lor l_m)$

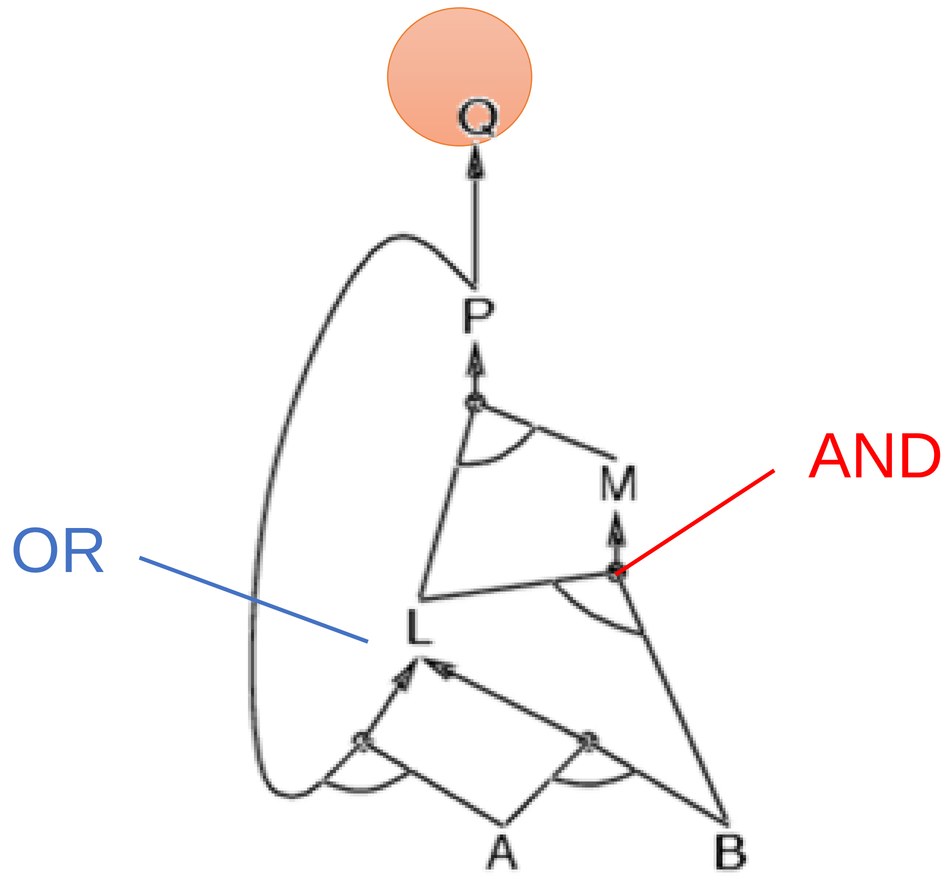
Horn clauses

Restricted form

Forward chaining

- **Key idea:** Fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until the query is found.

$$
\begin{aligned}
&P \Rightarrow Q \\
&L \wedge M \Rightarrow P \\
&B \wedge L \Rightarrow M \\
&A \wedge P \Rightarrow L \\
&A \wedge B \Rightarrow L \\
&A \\
&B
\end{aligned}
$$



The forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional definite clauses
          q, the query, a proposition symbol
  count ← a table, where count[c] is the number of symbols in c's premise
  inferred ← a table, where inferred[s] is initially false for all symbols
  agenda ← a queue of symbols, initially symbols known to be true in KB
  while agenda is not empty do
    p ← POP(agenda)
    if p = q then return true
    if inferred[p] = false then
      inferred[p] ← true
      for each clause c in KB where p is in c.PREMISE do
        decrement count[c]
        if count[c] = 0 then add c.CONCLUSION to agenda
return false
```

Sound and complete

Forward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

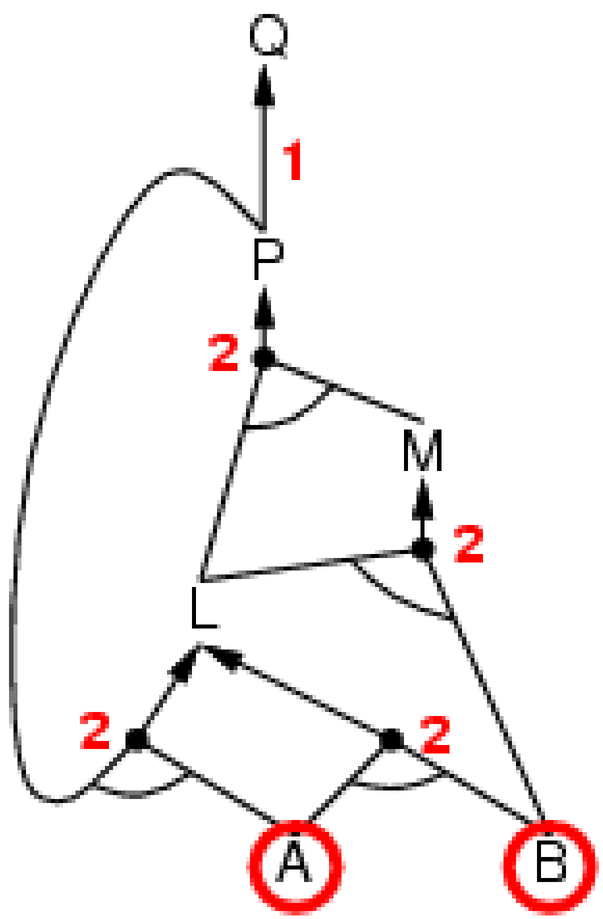
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Forward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

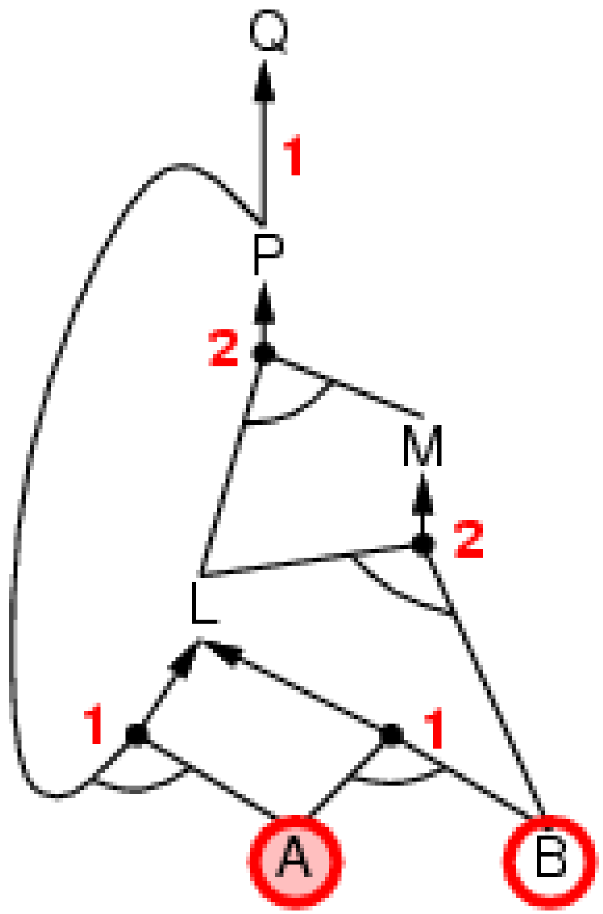
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Forward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

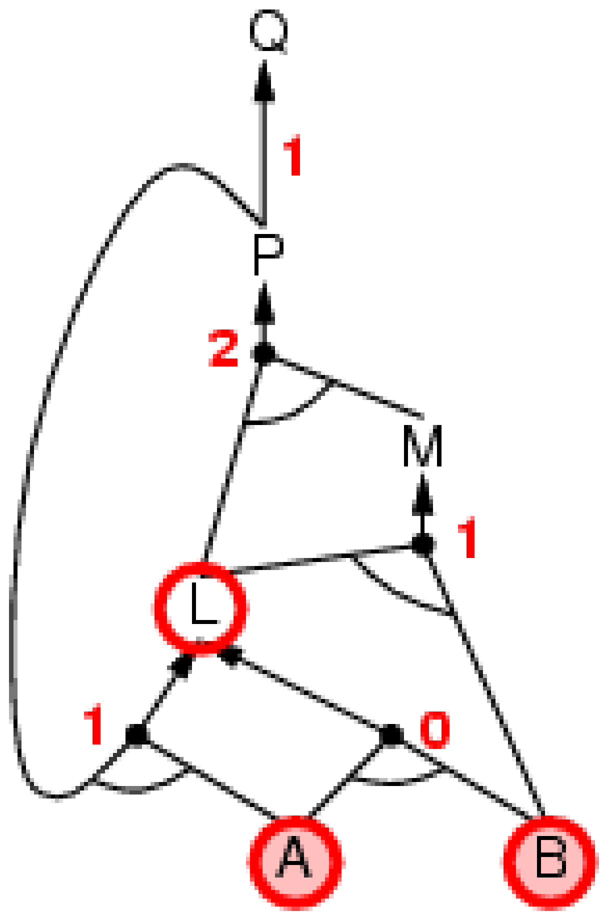
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

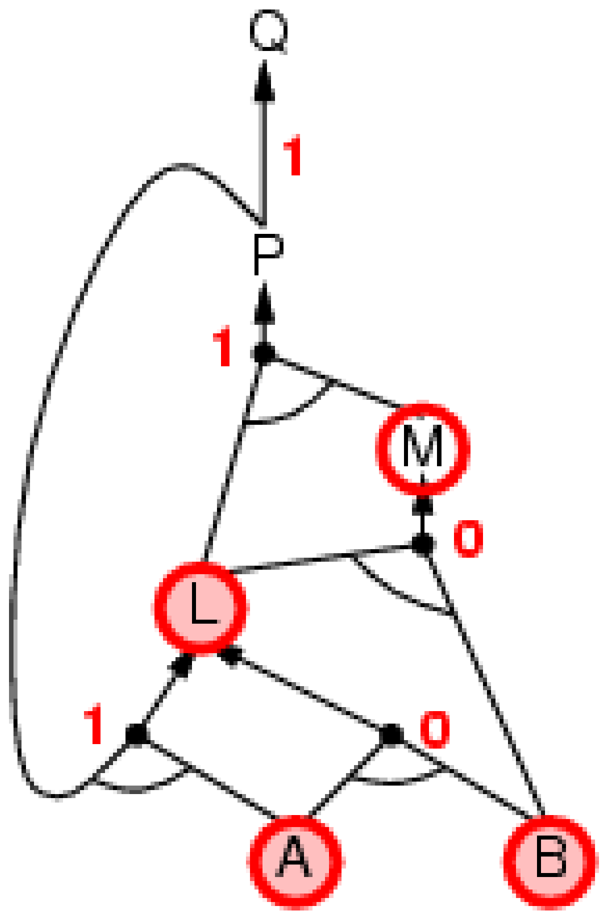
A

B



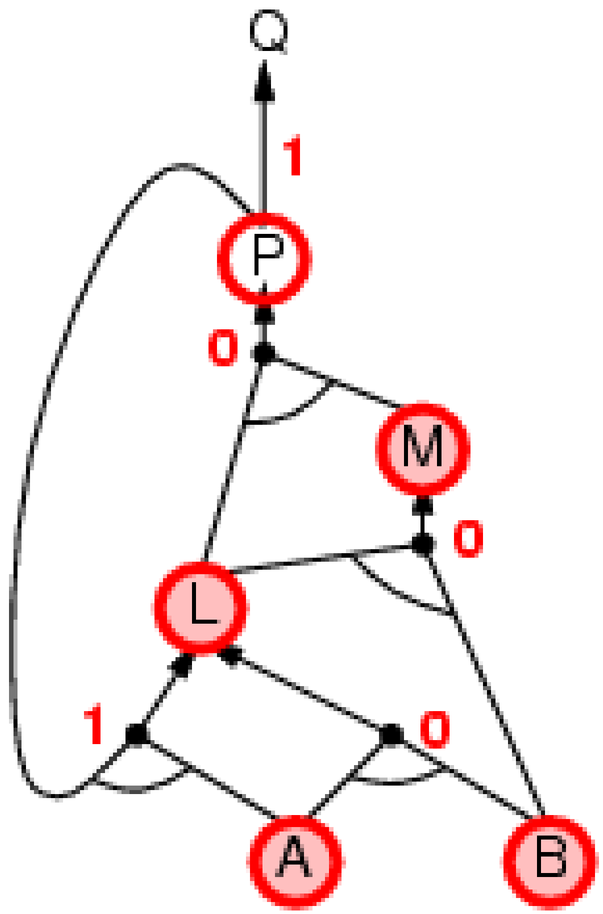
Forward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



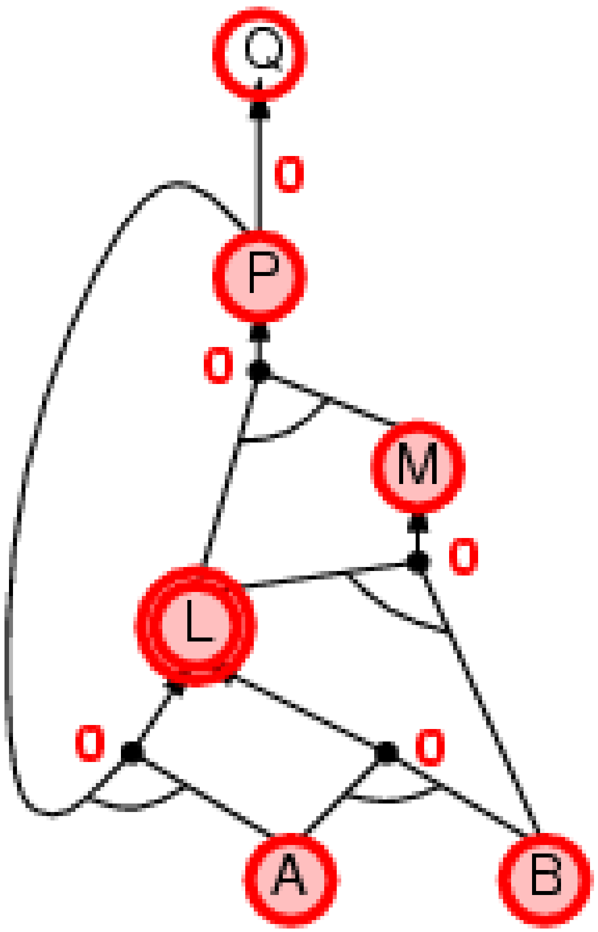
Forward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



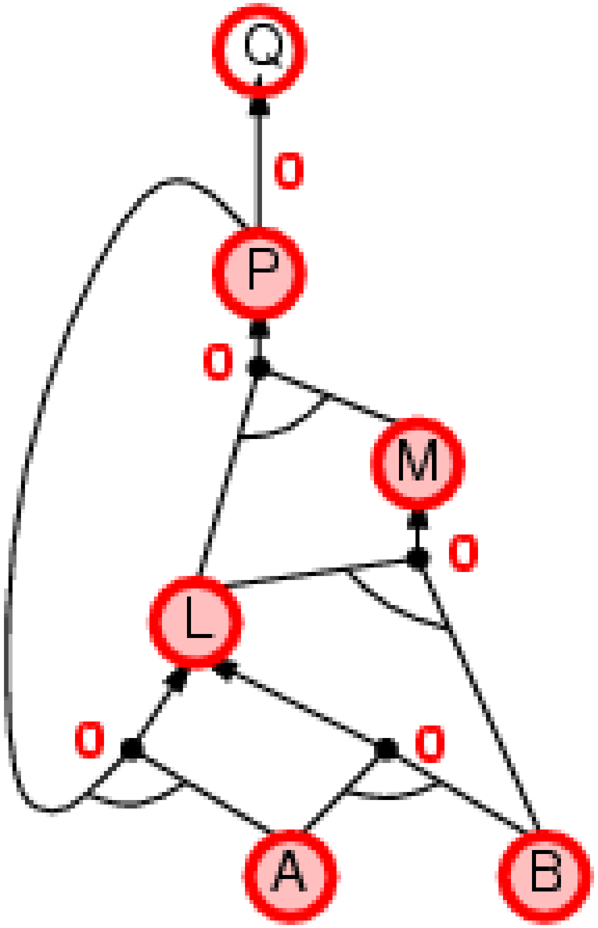
Forward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



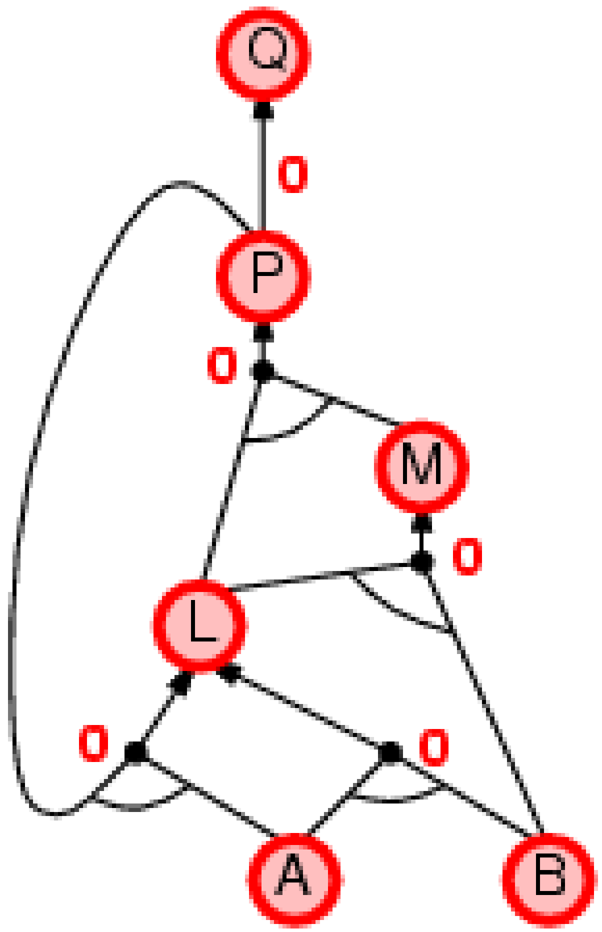
Forward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining: Another example

KB
$A \wedge B \Rightarrow C$
$C \wedge D \Rightarrow E$
$C \wedge F \Rightarrow G$
A
B
D

$\boldsymbol{E}$?

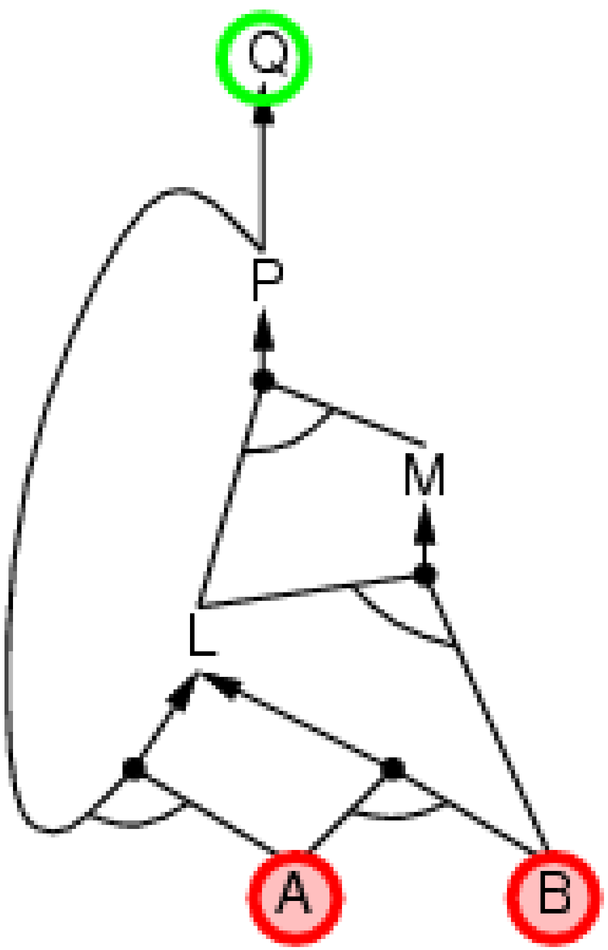
No.	Sentences	Explanation
1	$A \wedge B \Rightarrow C$	From KB
2	$C \wedge D \Rightarrow E$	From KB
3	$C \wedge F \Rightarrow G$	From KB
4	A	From KB
5	B	From KB
6	D	From KB
7	C	1, 4 and 5
8	$\boldsymbol{E}$	2, 6, and 7

Backward chaining

- **Key idea:** Work backwards from the query q
    - Check if q is known already, or
    - Recursively prove by BC all premises of some rule concluding q
- Avoid loops: A new subgoal is already on the goal stack?
- Avoid repeated work: A new subgoal has already been proved true, or has already failed?

Backward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

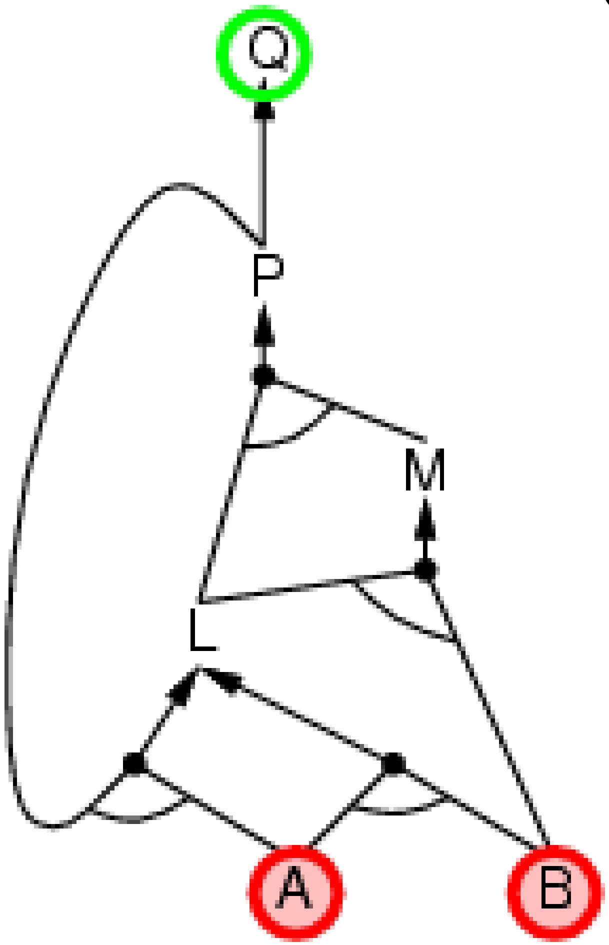
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



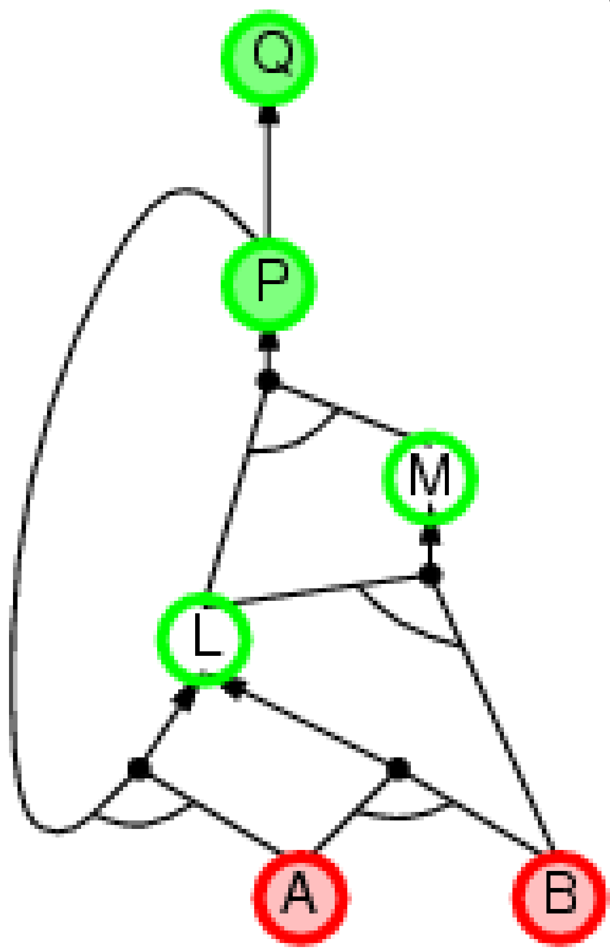
Q?

P?

P ⇒ Q

Backward chaining: An example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Q?
P?
L?

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

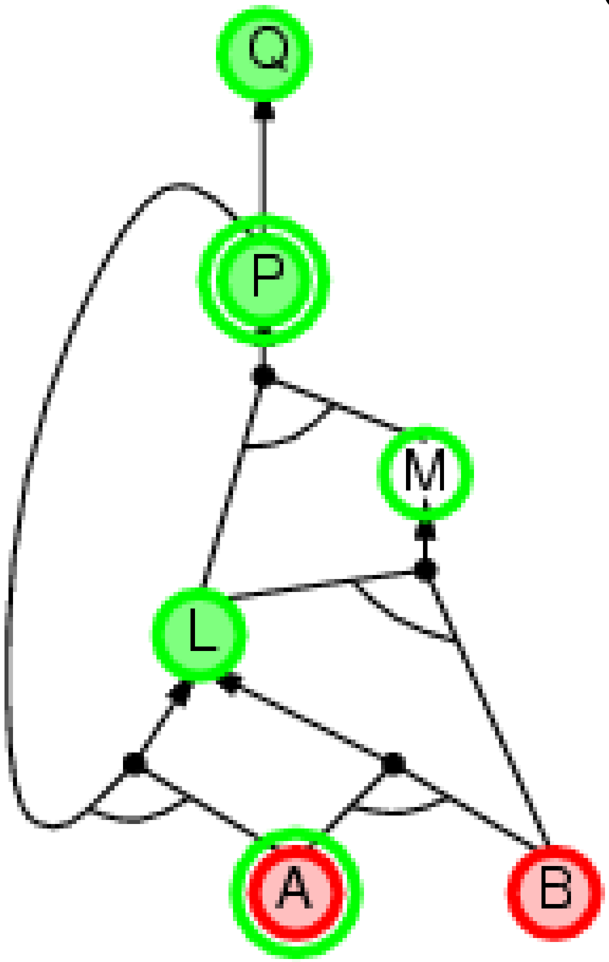
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q?
P?
L?
A?

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

$A \wedge B \Rightarrow L$

✓

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

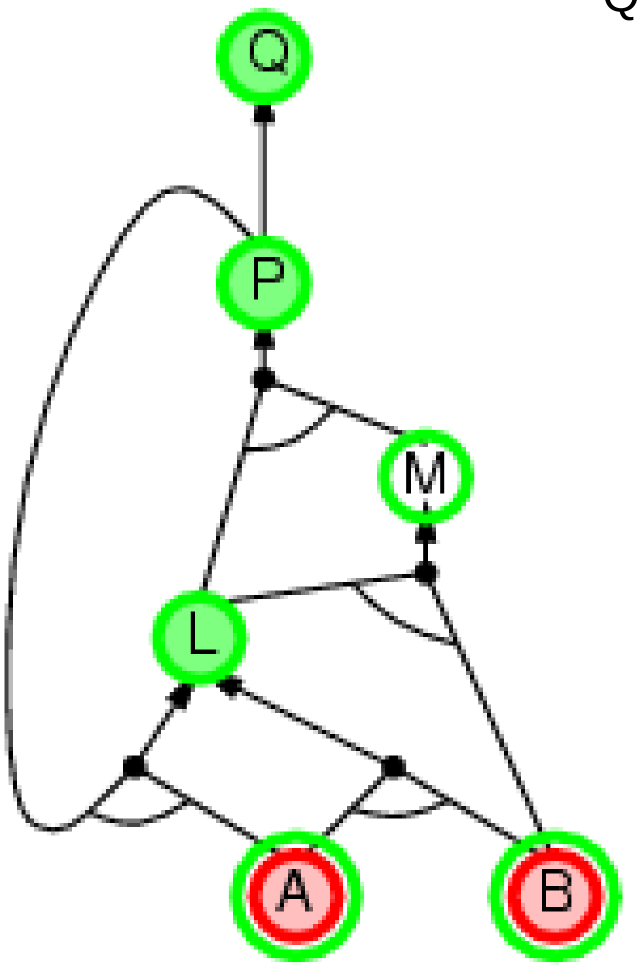
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q?

P?

L?

A?

B?

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

$A \wedge B \Rightarrow L$

✓

✓

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

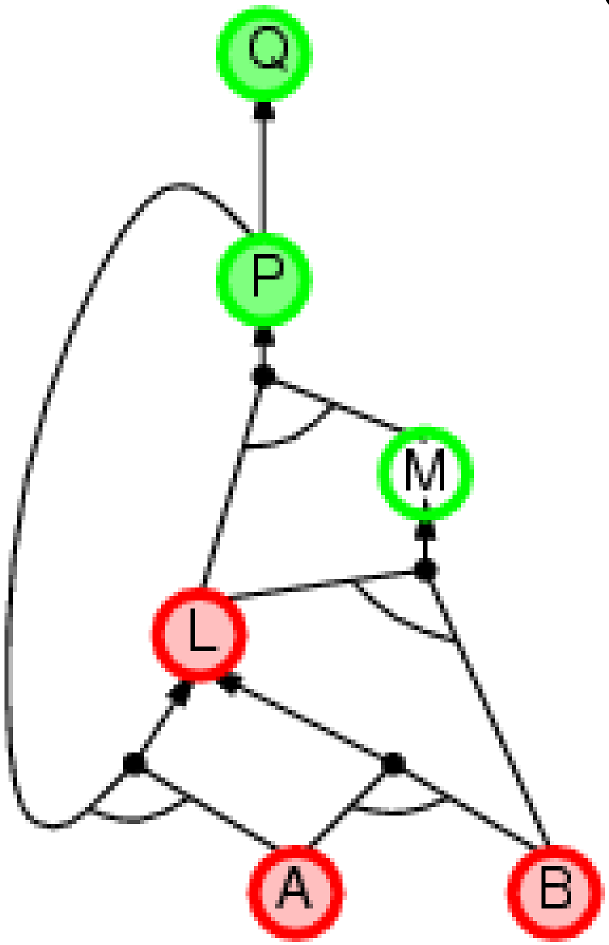
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q? $\quad$ P ⇒ Q

  P? $\quad$ L ∧ M ⇒ P

    L? ✓

      A? ✓

      B? ✓

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

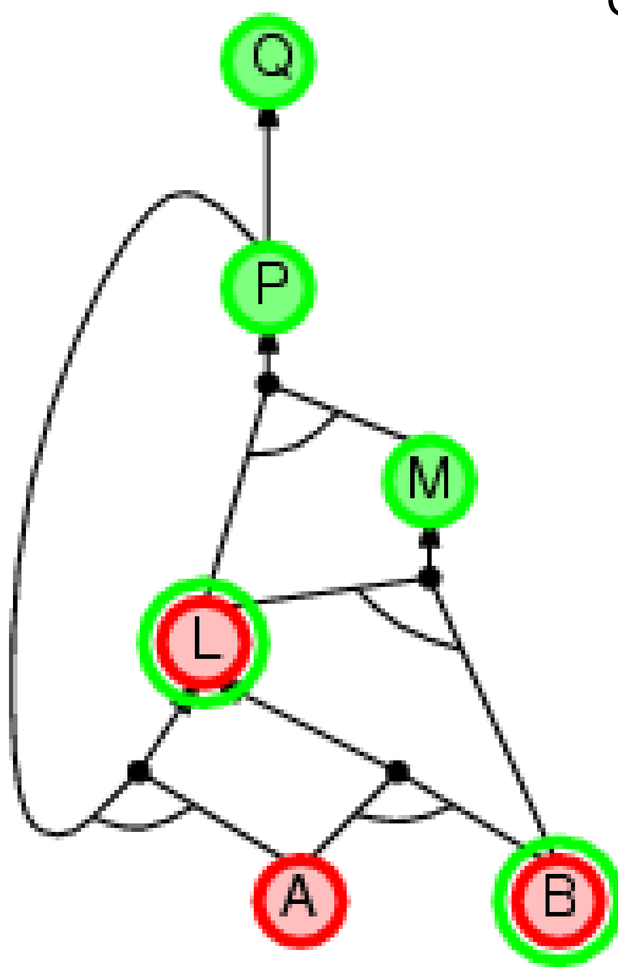
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q?	$P \Rightarrow Q$
P?	$L \wedge M \Rightarrow P$
L? ✓	
A?	✓
B?	✓
M?	$L \wedge B \Rightarrow M$
L?	
B?	

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

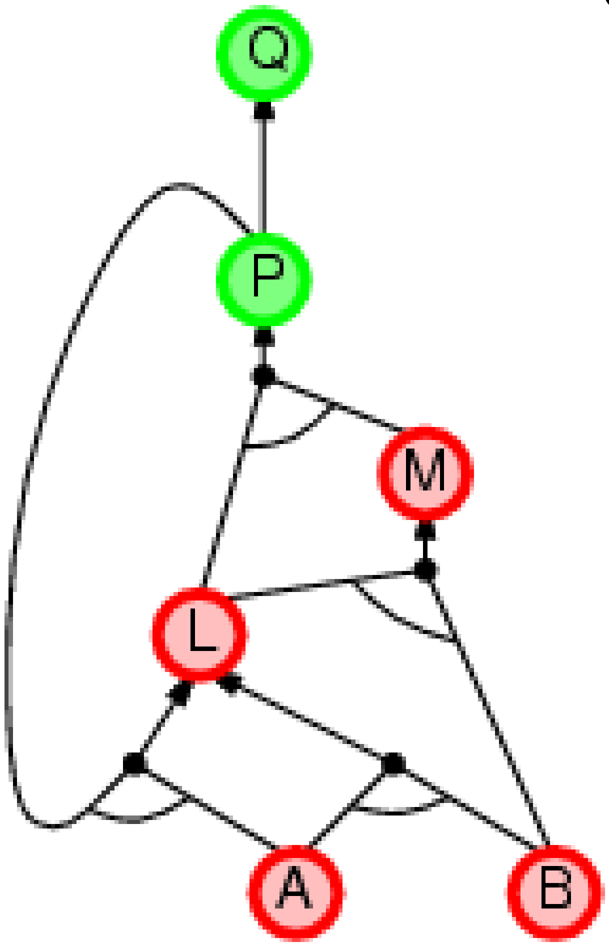
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q?
P?
L? ✓
A? ✓
B? ✓
M? ✓
L? ✓
B? ✓

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

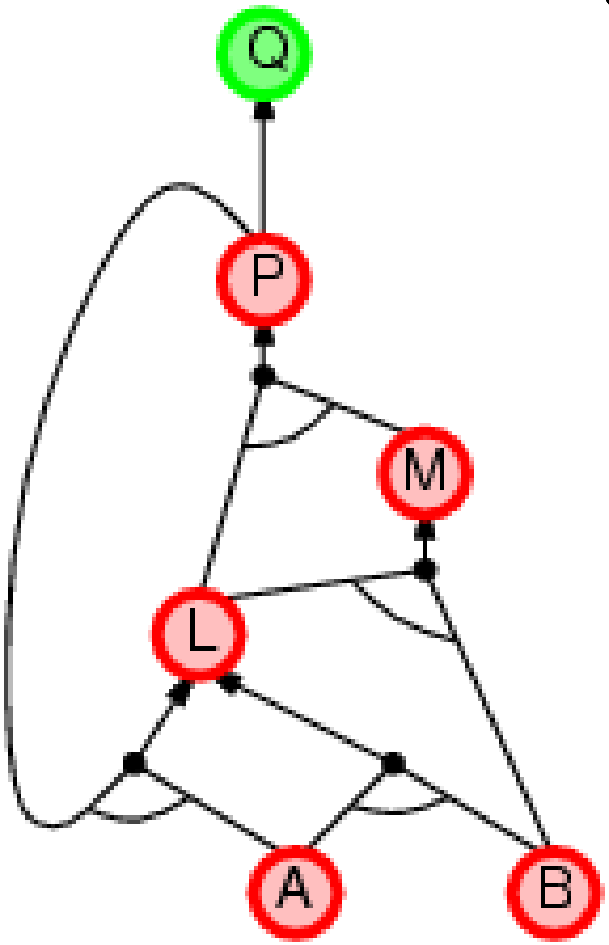
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q? ✓
P? ✓
L? ✓
A? ✓
B? ✓
M? ✓
L? ✓
B? ✓

Backward chaining: An example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

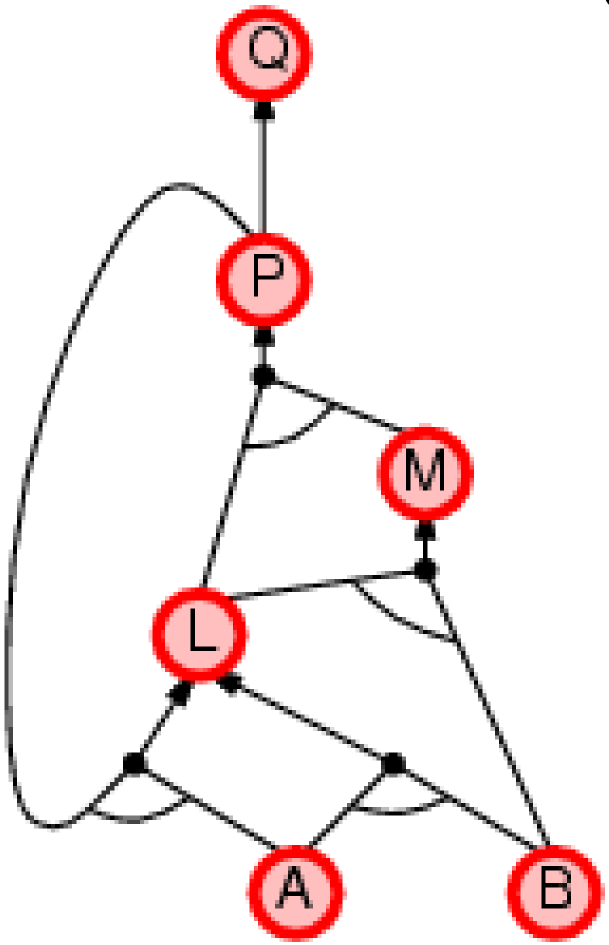
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Q? ✓
  P? ✓
    L? ✓
      A? ✓
      B? ✓
    M? ✓
      L? ✓
      B? ✓

Backward chaining: Another example

KB
$A \land B \Rightarrow C$
$C \land D \Rightarrow E$
$C \land F \Rightarrow G$
A
B
D

$\boldsymbol{E}?$

- E? $\quad C \land D \Rightarrow E$
  - C? $\quad A \land B \Rightarrow C$
    - A?
    - B?
  - D?
- A, B and D are given $\rightarrow$ All needed rules are satisfied $\rightarrow$ The goal is proven.

Forward vs. Backward chaining

- Forward chaining: data-driven, automatic, unconscious processing
  - E.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- Backward chaining: goal-driven, good for problem-solving
  - E.g., Where are my keys? How do I get into a PhD program?
  - Complexity can be much less than linear in size of KB

Quiz 05: Forward vs. Backward chaining

- Given a KB containing the following rules and facts

  R_1: IF hot AND smoky THEN fire

  R_2: IF alarm_beeps THEN smoky

  R_3: IF fire THEN sprinklers_on

  F_1: alarm_beeps

  F_2: hot

- Represent the KB in propositional logic with given symbols
  - H = hot, S = smoky, F = fire, A = alarms_beeps, R = sprinklers_on
- Answer the question “Sprinklers_on?” by using the forward chaining and backward chaining approaches

Effective model checking

- *A complete backtracking algorithm*
- *Local search algorithms*

Efficient propositional inference

- The SAT problem (checking satisfiability)
  - Testing entailment, $\alpha \vDash \beta$? = testing **un**satisfiability of $\alpha \wedge \neg\beta$

- Two families of efficient algorithms for general propositional inference based on model checking
  1. Complete backtracking search algorithms
     - **DPLL** algorithm (*Davis, Putnam, Logemann, Loveland*)
  2. Incomplete local search algorithms (hill-climbing)
     - **`WalkSAT`** algorithm

The DPLL algorithm

- Often called the Davis-Putnam algorithm (1960)
- Determine whether an input propositional logic sentence (in CNF) is satisfiable.
  - A recursive, depth-first enumeration of possible models.
- Improvements over truth table enumeration
  1. Early termination
  2. Pure symbol heuristic
  3. Unit clause heuristic

Improvements in DPLL

- **Early termination:** A clause is true if any literal is true, and a sentence is false if any clause is false.
  - Avoid examination of entire subtrees in the search space
  - E.g., $(A \vee B) \wedge (A \vee C)$ is true if A is true, regardless B and C
- **Pure symbol heuristic:** A pure symbol always appears with the same "sign" in all clauses.
  - E.g., $(A \vee \neg B), (\neg B \vee \neg C), (A \vee C)$, A and B are pure, C is impure.
  - Make a pure symbol true $\rightarrow$ Doing so never make a clause false
- **Unit clause heuristic:** there is only one literal in the clause and thus this literal must be true
  - **Unit propagation:** if the model contains $B = true$ then $(\neg B \vee \neg C)$ simplifies to a unit clause $\neg C \rightarrow C$ must be false (so that $\neg C$ is true) $\rightarrow A$ must be true (so that $A \vee C$ is true)

The DPLL procedure

function DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*
  inputs: *s*, a sentence in propositional logic
  clauses ← the set of clauses in the CNF representation of *s*
  symbols ← a list of the proposition symbols in *s*
  return DPLL(*clauses*, *symbols*,{ })

function DPLL(*clauses, symbols, model)* **returns** *true* or *false*
  if every clause in *clauses* is *true* in model **then return** *true*
  if some clause in *clauses* is *false* in model **then return** *false*

1. Early Termination

  P, value ← 2 FIND-PURE-SYMBOL(*symbols, clauses, model*)
  if *P* is non-null **then return** DPLL(*clauses, symbols – P, model* ∪ {*P=value*})
  P, value ← 3 FIND-UNIT-CLAUSE(clauses, model)
  if *P* is non-null **then return** DPLL(*clauses, symbols – P, model* ∪ {*P=value*})
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses, rest, model* ∪ {*P=true*}) **or**
    DPLL(*clauses, rest, model* ∪ {*P=false*}))

The Davis-Putnam procedure

function DP(Δ)

for φ **in** *vocabulary* (Δ) **do**

var $\Delta' \leftarrow \{\,\}$;

for Φ_1 **in** Δ **for** Φ_2 **in** Δ such that $\varphi \in \Phi_1$ $\neg\varphi \in \Phi_2$ **do**

var $\Phi' \leftarrow \Phi_1 - \{\varphi\} \cup \Phi_2 - \{\neg\varphi\}$;

if not *tautology*(Φ') **then** $\Delta' \leftarrow \Delta' \cup (\Phi')$;

$\Delta \leftarrow \Delta - \{\Phi \in \Delta \mid \varphi \in \Phi \text{ or } \neg\varphi \in \Phi\} \cup \Delta'$;

return {**if** $\{\,\} \in \Delta$ **then** *unsatisfiable* **else** *satisfiable*};

function *tautology*(Φ)

$\varphi \in \Phi$ and $\neg\varphi \in \Phi$

DPLL procedure vs. DP procedure

- DP can cause a quadratic expansion every time it is applied.
  - This can easily exhaust space on large problems.
- DPLL attacks the problem by sequentially solving smaller problems.
  - Basic idea: Choose a literal. Assume true, simplify clause set, and try to show satisfiable. Repeat for the negation of the literal.
  - Good because we do not cross multiply the clause set

DPLL procedure vs. DP procedure

Problem	Tautology	DP	DPLL
Prime	30.00	0.00	0.00
Prime4	0.02	0.06	0.04
Prime9	18.94	2.98	0.51
Prime10	11.40	3.03	0.96
Prime11	28.11	2.98	0.51
Prime16	> 1 hour	*	9.15
Prime17	> 1 hour	*	3.87
Mkadder32	>> 1 hour	6.50	7.34
Mkadder42	>> 1 hour	22.95	46.86
Mkadder52	>> 1 hour	44.83	170.98
Mkadder53	>> 1 hour	38.27	250.16
Mkadder63	>> 1 hour	*	1186.4
Mkadder73	>> 1 hour	*	3759.9

Reference: http://logic.stanford.edu/classes/cs157/2011/lectures/lecture04.pdf

The `WalkSAT` algorithm

- Incomplete, local search algorithm
- Evaluation function: **min-conflict** heuristic, to minimize the number of unsatisfied clauses
- Balance between greediness and randomness

function WALKSAT($clauses$, p, max_flips) **returns** a satisfying model or *failure*
  inputs: $clauses$, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move, typically around 0.5
    max_flips, number of flips allowed before giving up

  $model \leftarrow$ a random assignment of *true*/*false* to the symbols in $clauses$
  for $i = 1$ **to** max_flips **do**
    if $model$ satisfies $clauses$ **then return** $model$
    $clause \leftarrow$ a randomly selected clause from $clauses$ that is false in $model$
    with probability p flip the value in $model$ of a randomly selected symbol from $clause$
    else flip whichever symbol in $clause$ maximizes the number of satisfied clauses
  return *failure*

The `WalkSAT` algorithm

- The algorithm returns a model $\rightarrow$ satisfiable
- The algorithm returns false $\rightarrow$ unsatisfiable **OR** more time is needed for searching

- `WalkSAT` cannot always detect unsatisfiability
  - It is most useful when a solution is expected to exist.
- For example,
  - An agent cannot *reliably* use WALKSAT to prove that a square is safe in the Wumpus world.
  - Instead, it can say, “I thought about it for an hour and couldn’t come up with a possible world in which the square *isn’t* safe.”

Inference-based agents in the Wumpus world

- A Wumpus-world agent using propositional logic will have a KB of **64** distinct proposition symbols, **155** sentences.

$$\neg P_{1,1}$$

$$\neg W_{1,1}$$

$$B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$$

$$S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$$

$$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$$

$$\neg W_{1,1} \vee \neg W_{1,2}$$

$$\neg W_{1,1} \vee \neg W_{1,3}$$

…

Propositional logic: Limitations

- The propositional logic encounters expressiveness limitation.
- KB contains "physics" sentences for every single square
  - Too many propositions to handle
    - E.g., the statement “Do not go forward if the Wumpus is in front of you” requires 16 squares $\times$ 4 orientations = 64 propositional rules.
  - The changes of KB over time is difficult to represent
    - E.g., for every time t and location $[x, y]$

$$L_{x,y} \wedge FacingRight_t \wedge Forward_t \Rightarrow L_{x+1,y}$$

    - This means we have a separate KB for every time point.
- It will take thousands of rules to build an agent.

Quiz 06: DPLL and DP

- Given a KB as shown aside

KB
$A \Rightarrow B \vee C$
$A \Rightarrow D$
$C \wedge D \Rightarrow \neg F$
$B \Rightarrow F$
A

- Using either DPLL or DP to check whether KB entails each of the following sentences
  - $\boldsymbol{C}$
  - $\boldsymbol{B \Rightarrow \neg C}$

THE END