HW2

Consider the following linear stochastic system

$$\begin{aligned} dx(t) &= (Ax(t) + Bu(t) + v(t))\,dt \\ &\quad + C_1 x(t) dw_1 + C_2 x(t) dw_2 \\ y(t) &= Dx(t) + n(t) \end{aligned}$$

where the system matrices are defined as

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\ D = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

In this work, the system state $x(t)$ is defined as $x(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, measurement noise $n(t)$ is given as white noise with zero mean and unit variance, and the external disturbance $v(t)$ is given as $v(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$.

Please design a mixed H_2/H_∞ observer-based controller to achieve suboptimal H_2 control performance and H_∞ robust control performance.

In your report, a well designed controller gain and observer gain should be attached. Moreover, please show the state trajectories and estimated state trajectories in your report.

Hint1 : (3.116)-(3.129) in textbook

Hint2 : Check controllability and observability of the system.

Hint3 : $\bar{Q} = \begin{bmatrix} I & 0 \\ 0 & 4I \end{bmatrix}$, $R = I$ in (3.120), where I is the 2×2 identity matrix.

Hint4 : Use Runge-Kutta 4th method to generate your state trajectories.