Create the constrained expression:

Create parameters θ(x) and n(y). Use a univariate constrained expression to force θ(x) to adhere to the following constraints:
$\sqrt{2}\,\text{Sin}(\theta(0)) = (-1)^{\text{UnitStep}(n(0))}$ and $\sqrt{2}\,\text{Cos}(\theta(0)) = (-1)^{\text{UnitStep}(n(1))}$.
This can be rewritten as one constraint:
$\text{Tan}(\theta(0)) = \frac{(-1)\text{^UnitStep }(n(0))}{(-1)^{\text{UnitStep}(n(1))}}$.

In[1]:=
```
θFunc[x_, n_Symbol, θ_Symbol] := θ[x] + ArcTan[(-1)^UnitStep[n[1]], (-1)^UnitStep[n[0]]] - θ[0]
```

Check that the univariate constrained expression we created has the properties we want:

In[2]:=
```
FullSimplify[√2 Cos[θFunc[0, n, θ]] == (-1)^UnitStep[n[1]]]
FullSimplify[√2 Sin[θFunc[0, n, θ]] == (-1)^UnitStep[n[0]]]
FullSimplify[(√2 Cos[θFunc[0, n, θ]])^2 + (√2 Sin[θFunc[0, n, θ]])^2 == 2]
```

Out[2]= True

Out[3]= True

Out[4]= True

Create the rest of the multivariate constrained expression.

In[5]:=
```
s2[x_] := {1, x};
S2 = {s2[0], 2 Integrate[s2[1], {y, 0, 1}]};
α2 = Inverse[S2];
ϕ2[x_] := s2[x].α2;
ρ2[x_, y_, n_Symbol, g_Symbol] :=
  {(-1)^UnitStep[n[y]] - g[0, y], 1 - 2 Integrate[g[1, τ], {τ, 0, 1}]};
u2[x_, y_, n_Symbol, g_Symbol] := g[x, y] + ϕ2[x].ρ2[x, y, n, g];

s1[y_] := {1, y, y^2};
S1 = {s1[0], s1[1], 2 Integrate[s1[y], {y, 0, 1}]};
α1 = Inverse[S1].{{1, 0}, {0, 1}, {0, 0}};
ϕ1[y_] := s1[y].α1;
ρ1[x_, y_, n_Symbol, θ_Symbol, g_Symbol] :=
  {√2 Sin[θFunc[x, n, θ]] - u2[x, 0, n, g], √2 Cos[θFunc[x, n, θ]] - u2[x, 1, n, g]};
u[x_, y_, n_Symbol, θ_Symbol, g_Symbol] := u2[x, y, n, g] + ϕ1[y].ρ1[x, y, n, θ, g];
```

Check the constraints:

In[17]:=
```
FullSimplify[u[0, y, n, θ, g]^2 == 1]
```

Out[17]= True

In[18]:= `FullSimplify[u[x, 0, n, θ, g]^2 + u[x, 1, n, θ, g]^2 == 2]`

Out[18]= True

In[19]:= `FullSimplify[2 Integrate[u[1, y, n, θ, g], {y, 0, 1}] == 1]`

Out[19]=
$$2 \int_0^1 \Big(\sqrt{2}\ y(-2+3y)\,\mathrm{Cos}\big[\mathrm{ArcTan}\big[(-1)^{\mathrm{UnitStep}[n[1]]}, (-1)^{\mathrm{UnitStep}[n[0]]}\big] - \theta[0] + \theta[1]\big] + (2-3y)\,y\,g[1,1] + g[1,y] + 6(-1+y)\,y \int_0^1 g[1,\tau]\,d\tau + (-1+y)\big(g[1,0] - 3y(1+g[1,0]) + \sqrt{2}\,(-1+3y)\,\mathrm{Sin}\big[\mathrm{ArcTan}\big[(-1)^{\mathrm{UnitStep}[n[1]]}, (-1)^{\mathrm{UnitStep}[n[0]]}\big] - \theta[0] + \theta[1]\big]\big)\Big)\,dy == 1$$

One can verify that the above is true the long tedious way, or more simply by proving that the following is true (which it is)

In[20]:= `FullSimplify[2 Integrate[u2[1, y, n, g], {y, 0, 1}] == 1]`

Out[20]=
$$2 \int_0^1 \left(\frac{1}{2} + g[1, y] - \int_0^1 g[1, \tau]\,d\tau\right) dy == 1$$

and observing that $\rho 1$ is not a function of y and $2\int_0^1 \phi[y]\,dy = 0$, so $2\int_0^1 \rho 1 * \phi 1[y]\,dy = 2\,\rho 1 \int_0^1 \phi 1[y]\,dy = 0$

In[21]:= `FullSimplify[2 Integrate[ϕ1[y], {y, 0, 1}]]`

Out[21]= {0, 0}