Create the constrained expression:

Create parameters θ(x) and n(y). Use a univariate constrained expression to force θ(x) to adhere to the following constraints:

$\sqrt{2}\ \mathrm{Sin}(\theta(0)) = (-1)^{\mathrm{UnitStep}(n(0))}$ and $\sqrt{2}\ \mathrm{Cos}(\theta(0)) = (-1)^{\mathrm{UnitStep}(n(1))}$.

This can be rewritten as one constraint:

$\mathrm{Tan}(\theta(0)) = \frac{(-1)\text{^UnitStep }(n(0))}{(-1)^{\mathrm{UnitStep}(n(1))}}$.

In[1]:=
```
θFunc[x_, n_Symbol, θ_Symbol] := θ[x] + ArcTan[(-1)^UnitStep[n[1]], (-1)^UnitStep[n[0]]] - θ[0]
```

Check that the univariate constrained expression we created has the properties we want:

In[2]:=
```
FullSimplify[√2 Cos[θFunc[0, n, θ]] == (-1)^UnitStep[n[1]]]
FullSimplify[√2 Sin[θFunc[0, n, θ]] == (-1)^UnitStep[n[0]]]
FullSimplify[(√2 Cos[θFunc[0, n, θ]])^2 + (√2 Sin[θFunc[0, n, θ]])^2 == 2]
```

Out[2]= True

Out[3]= True

Out[4]= True

Create the rest of the multivariate constrained expression.

In[5]:=
```
s1[x_] := {1, x};
S1 = {s1[0], 2 Integrate[s1[1], {y, 0, 1}]};
α1 = Inverse[S1];
ϕ1[x_] := s1[x].α1;
ρ1[x_, y_, n_Symbol, g_Symbol] :=
  {(-1)^UnitStep[n[y]] - g[0, y], 1 - 2 Integrate[g[1, τ], {τ, 0, 1}]};
u1[x_, y_, n_Symbol, g_Symbol] := g[x, y] + ϕ1[x].ρ1[x, y, n, g];

s2[y_] := {1, y, y^2};
S2 = {s2[0], s2[1], 2 Integrate[s2[y], {y, 0, 1}]};
α2 = Inverse[S2].{{1, 0}, {0, 1}, {0, 0}};
ϕ2[y_] := s2[y].α2;
ρ2[x_, y_, n_Symbol, θ_Symbol, g_Symbol] :=
  {√2 Sin[θFunc[x, n, θ]] - u1[x, 0, n, g], √2 Cos[θFunc[x, n, θ]] - u1[x, 1, n, g]};
u[x_, y_, n_Symbol, θ_Symbol, g_Symbol] := u1[x, y, n, g] + ϕ2[y].ρ2[x, y, n, θ, g];
```

Check the constraints:

In[17]:=
```
FullSimplify[u[0, y, n, θ, g]^2 == 1]
```

Out[17]= True

In[18]:= **FullSimplify[u[x, 0, n, θ, g]^2 + u[x, 1, n, θ, g]^2 == 2]**

Out[18]= True

In[19]:= **FullSimplify[2 Integrate[u[1, y, n, θ, g], {y, 0, 1}] == 1]**

Out[19]=
$$2 \int_0^1 \Big(\sqrt{2}\ y\,(-2+3\,y)\,\mathrm{Cos}\Big[\mathrm{ArcTan}\Big[(-1)^{\mathrm{UnitStep}[n[1]]},\ (-1)^{\mathrm{UnitStep}[n[0]]}\Big] - \theta[0] + \theta[1]\Big] + (2-3\,y)\,y\,g[1,\,1] + g[1,\,y] + 6\,(-1+y)\,y \int_0^1 g[1,\,\tau]\,d\tau + (-1+y)\Big(g[1,\,0] - 3\,y\,(1+g[1,\,0]) + \sqrt{2}\ (-1+3\,y)\,\mathrm{Sin}\Big[\mathrm{ArcTan}\Big[(-1)^{\mathrm{UnitStep}[n[1]]},\ (-1)^{\mathrm{UnitStep}[n[0]]}\Big] - \theta[0] + \theta[1]\Big]\Big)\Big)\,dy == 1$$

One can verify that the above is true the long tedious way, or more simply by proving that the following is true (which it is)

In[20]:= **FullSimplify[2 Integrate[u1[1, y, n, g], {y, 0, 1}] == 1]**

Out[20]=
$$2 \int_0^1 \left(\frac{1}{2} + g[1,\,y] - \int_0^1 g[1,\,\tau]\,d\tau \right) dy == 1$$

and observing that $\rho 2$ is not a function of y and $2 \int_0^1 \phi 2[y]$ dy = 0, so $2 \int_0^1 \rho 2 * \phi 2[y]$ dy = $2\,\rho 2 \int_0^1 \phi 2[y]$ dy = 0

In[21]:= **FullSimplify[2 Integrate[ϕ2[y], {y, 0, 1}]]**

Out[21]= {0, 0}