EMBEDDED VISION DESIGN 3

REGRESSION

Prediction from estimating relationships between variables and outcomes

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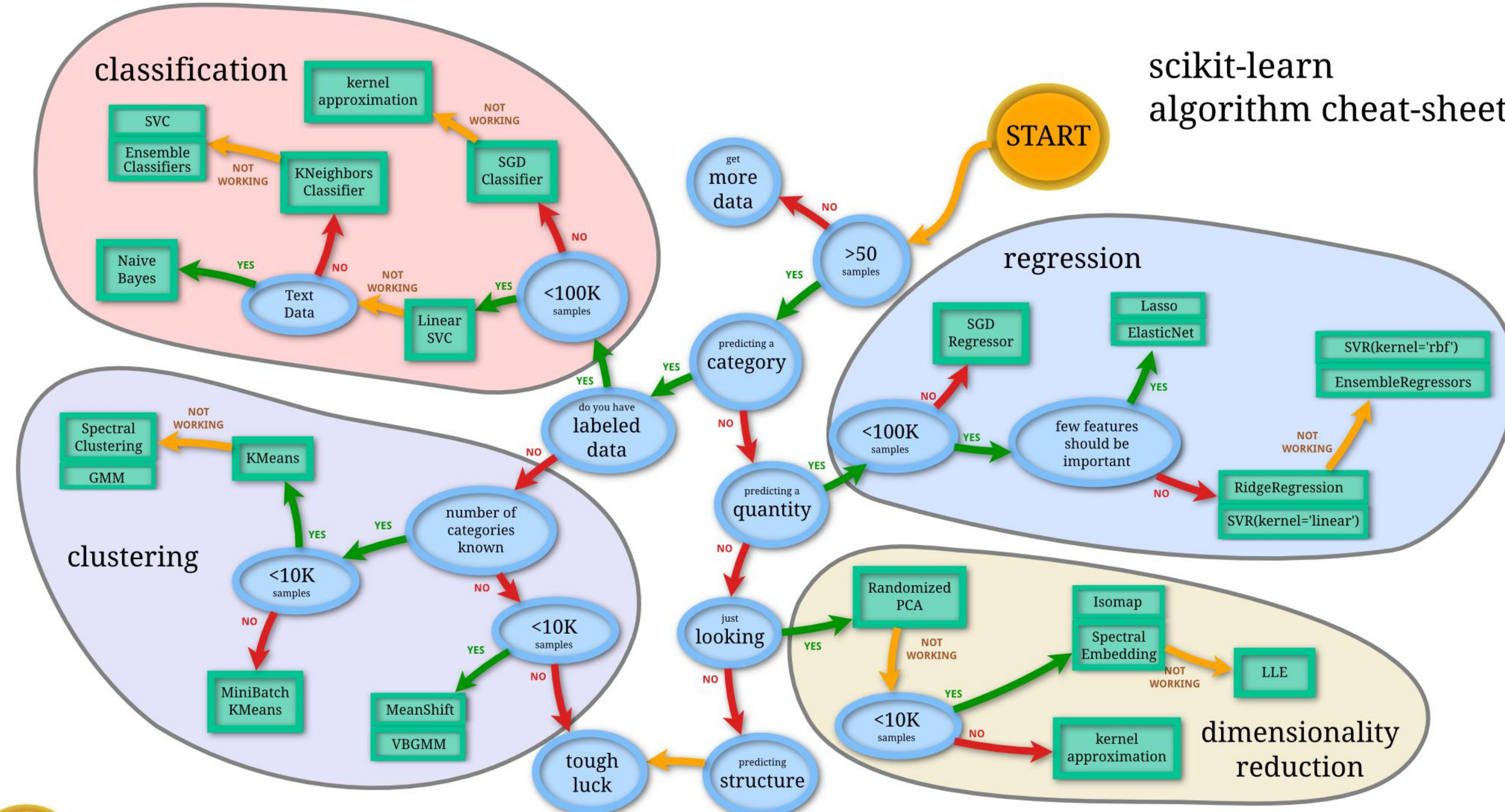
scikit-learn algorithm cheat-sheet



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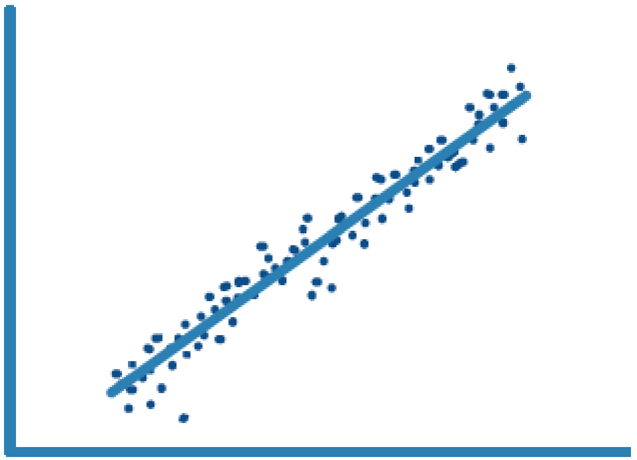
CONTENTS

- Linear regression
- Polynomial regression
- Regularized linear models
- Logistic regression
- Classification and regression

LINEAR REGRESSION

- Easy to interpret
- Fast to fit
- Benchmark



Source: Mathworks, Applying Supervised Learning

Equation 4-1. Linear Regression model prediction

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

In this equation:

- $\hat{y}$ is the predicted value.
- n is the number of features.
- x_i is the i^{th} feature value.
- θ_j is the j^{th} model parameter (including the bias term θ_0 and the feature weights $\theta_1, \theta_2, \cdots, \theta_n$).

Source: Géron, ISBN: 9781492032632

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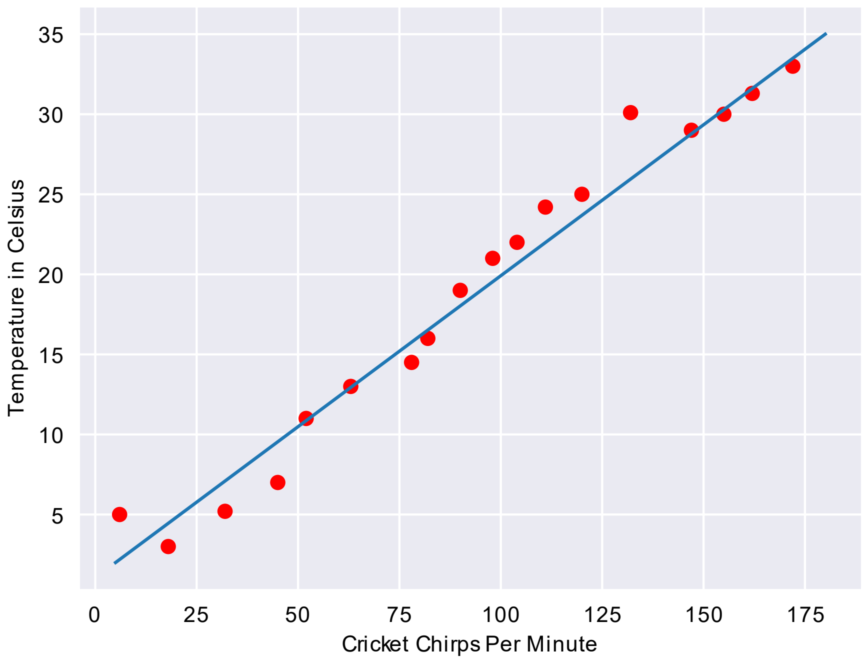
SIMPLE EXAMPLE

- Cricket chips vs temperature



y is the temperature in Celsius—the value we're trying to predict.

m is the slope of the line.

x is the number of chirps per minute—the value of our input feature.

b is the y-intercept.

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LINEAR REGRESSION MODEL FITTING

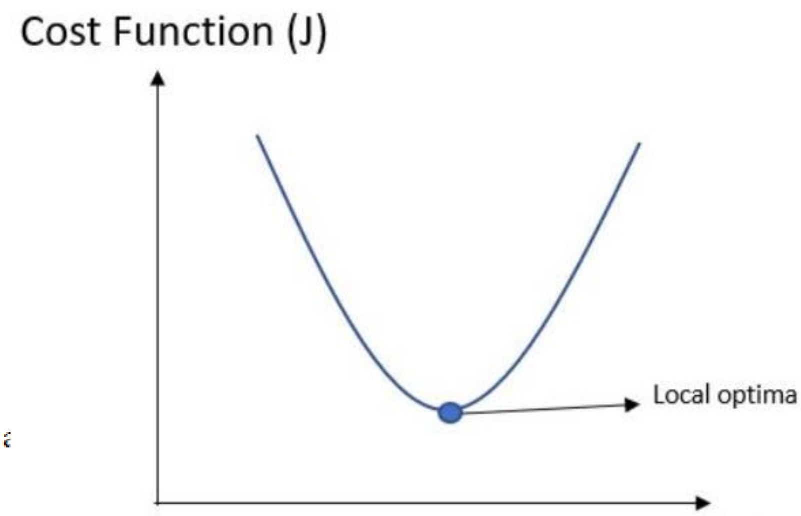
- Performance measure

Equation 4-3. MSE cost function for a Linear Regression model

$$\mathrm{MSE}(\mathbf{X}, h_{\boldsymbol{\theta}}) = \frac{1}{m}\sum_{i=1}^{m}\left(\boldsymbol{\theta}^{\mathsf{T}}\mathbf{x}^{(i)} - y^{(i)}\right)^2$$

- Aka cost function (J)

- Note the vectorized form
  - $\boldsymbol{\theta}$ is the model's *parameter vector*, containing the bias term θ_0 weights θ_1 to θ_n.
  - $\mathbf{x}$ is the instance's *feature vector*, containing x_0 to x_n, with x_0 always equal to 1.
  - $\boldsymbol{\theta} \cdot \mathbf{x}$ is the dot product of the vectors $\boldsymbol{\theta}$ and $\mathbf{x}$, which is of course equal to $\theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots + \theta_n x_n$.
  - $h_{\boldsymbol{\theta}}$ is the hypothesis function, using the model parameters $\boldsymbol{\theta}$.



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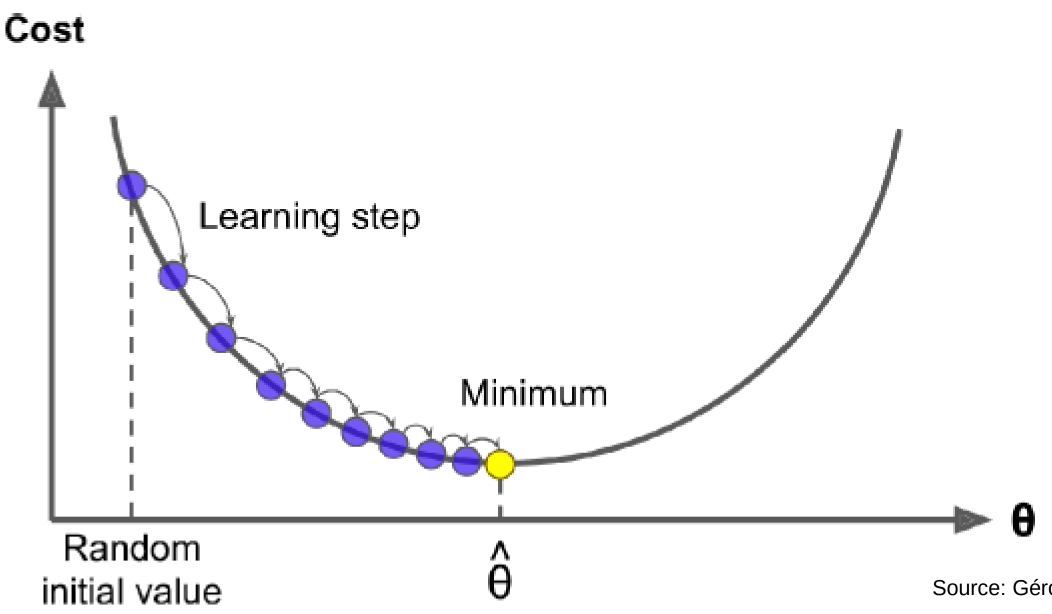
GRADIENT DESCENT

- Tweak parameters iteratively in order to minimize a cost function



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GRADIENT DESCENT

Equation 4-5. Partial derivatives of the cost function

$$\frac{\partial}{\partial \theta_j} \text{MSE}(\boldsymbol{\theta}) = \frac{2}{m} \sum_{i=1}^{m} \left(\boldsymbol{\theta}^\intercal \mathbf{x}^{(i)} - y^{(i)} \right) x_j^{(i)}$$

Equation 4-7. Gradient Descent step

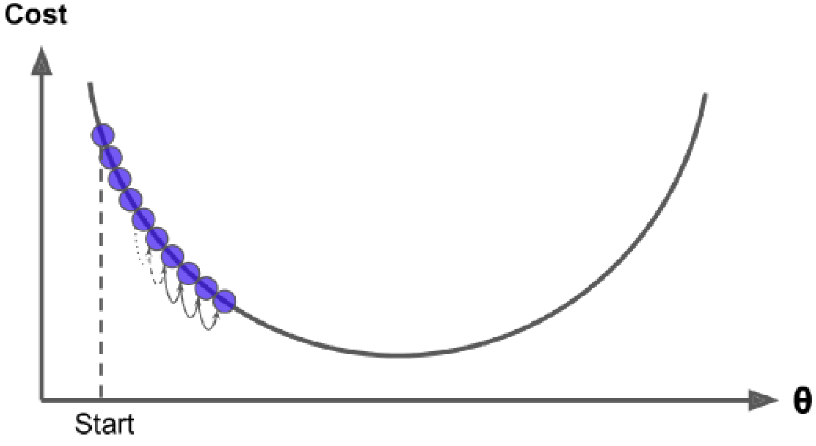
$$\boldsymbol{\theta}^{(\text{next step})} = \boldsymbol{\theta} - \boxed{\eta}\boxed{\nabla_{\boldsymbol{\theta}} \text{MSE}(\boldsymbol{\theta})}$$

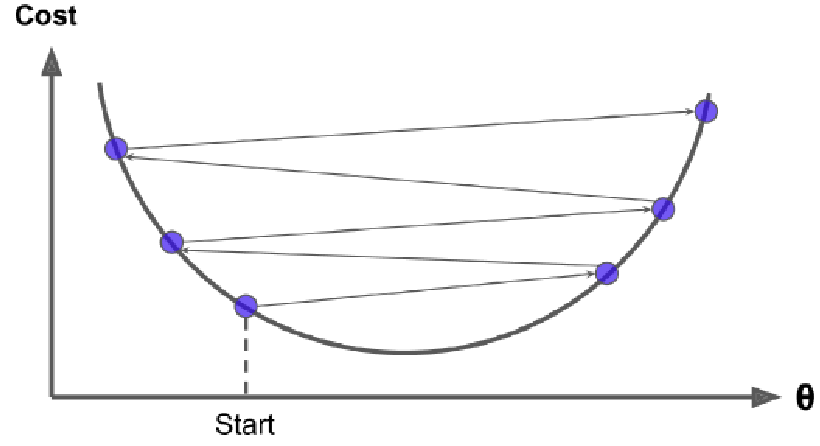
Learning rate

$$\nabla_{\boldsymbol{\theta}} \text{MSE}(\boldsymbol{\theta}) = \begin{pmatrix} \frac{\partial}{\partial \theta_0} \text{MSE}(\boldsymbol{\theta}) \\ \frac{\partial}{\partial \theta_1} \text{MSE}(\boldsymbol{\theta}) \\ \vdots \\ \frac{\partial}{\partial \theta_n} \text{MSE}(\boldsymbol{\theta}) \end{pmatrix}$$

LEARNING RATE

- Stepsize
- Convergence
- Stopping criterion





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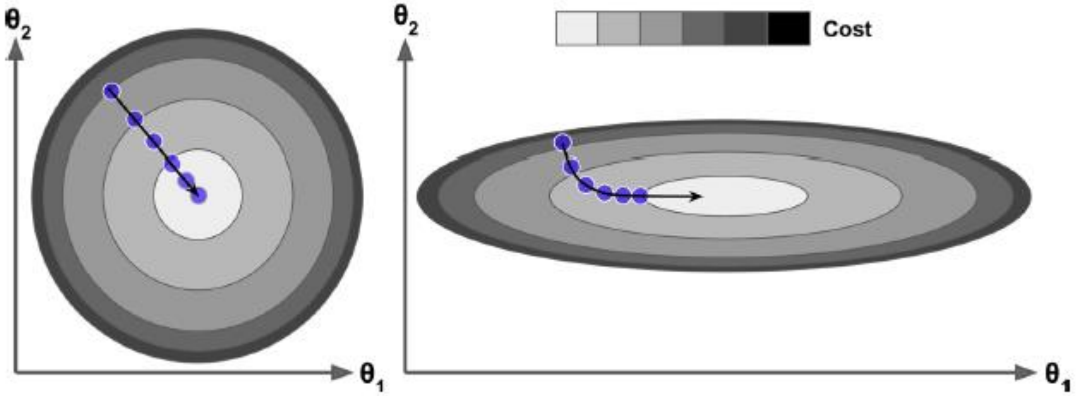
PITFALLS

- Unbalanced features



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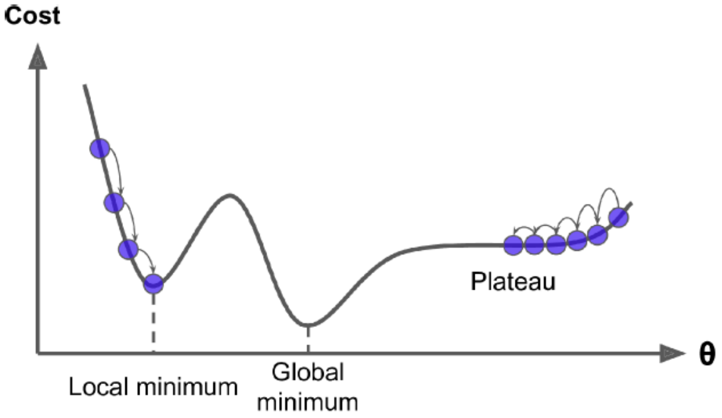
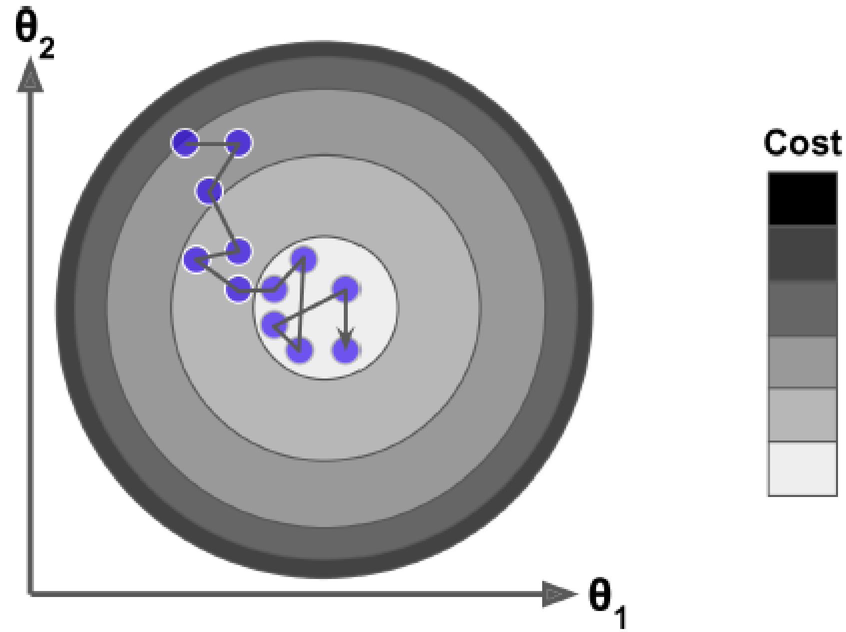
- Local minima



STOCHASTIC GRADIENT DESCENT

- Pick a random sample and compute gradient

- Start with flexible learning rate, and reducing it as the number of examples grows -> learning schedule



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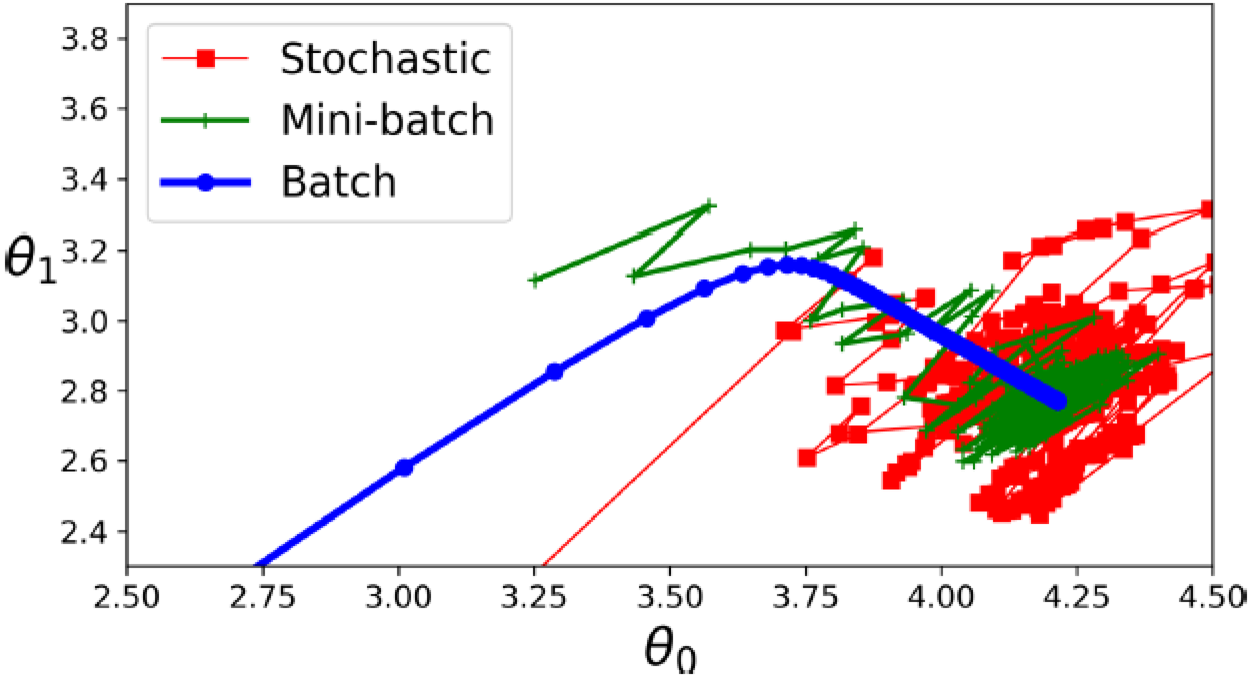
MINI-BATCH SGD

- Dealing with large datasets



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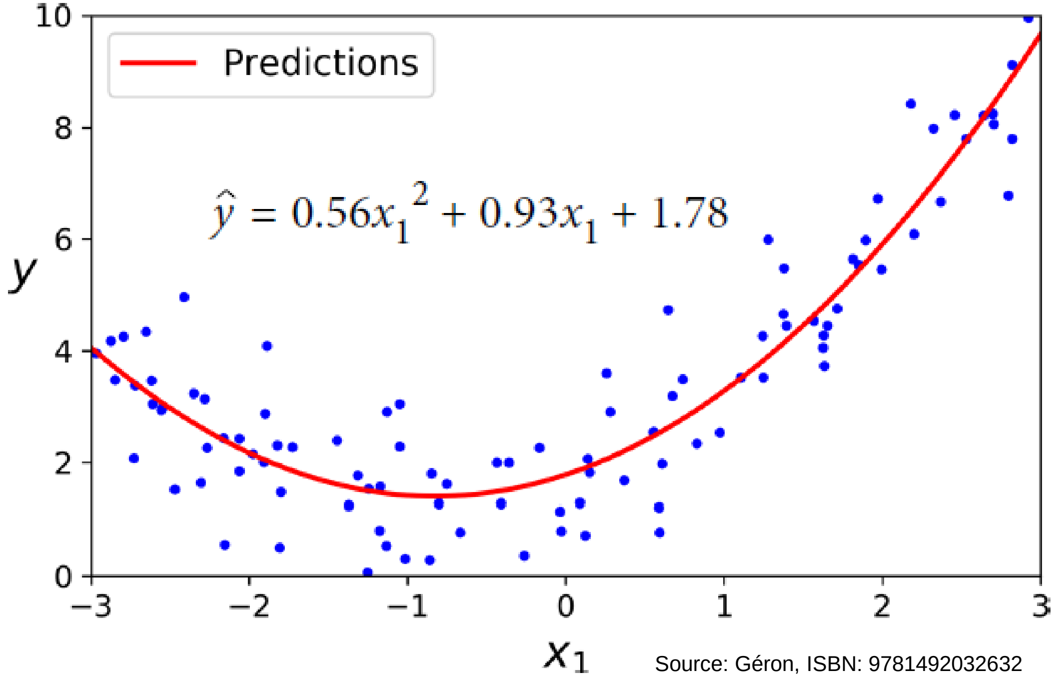
POLYNOMIAL REGRESSION

- Add powers of each feature as new features



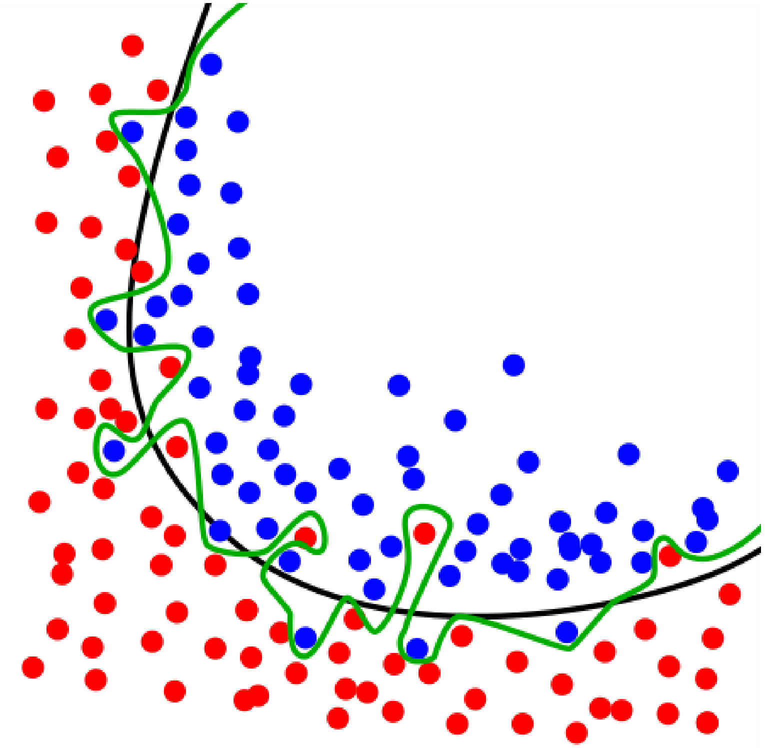
Source: Géron, ISBN: 9781492032632

- Still a linear function of the fitting parameters

PERIL OF OVERFITTING

- An overfit model has more parameters than can be justified by the data.
- Gets a low loss during training but does a poor job predicting new data.
- Results in poor generalization

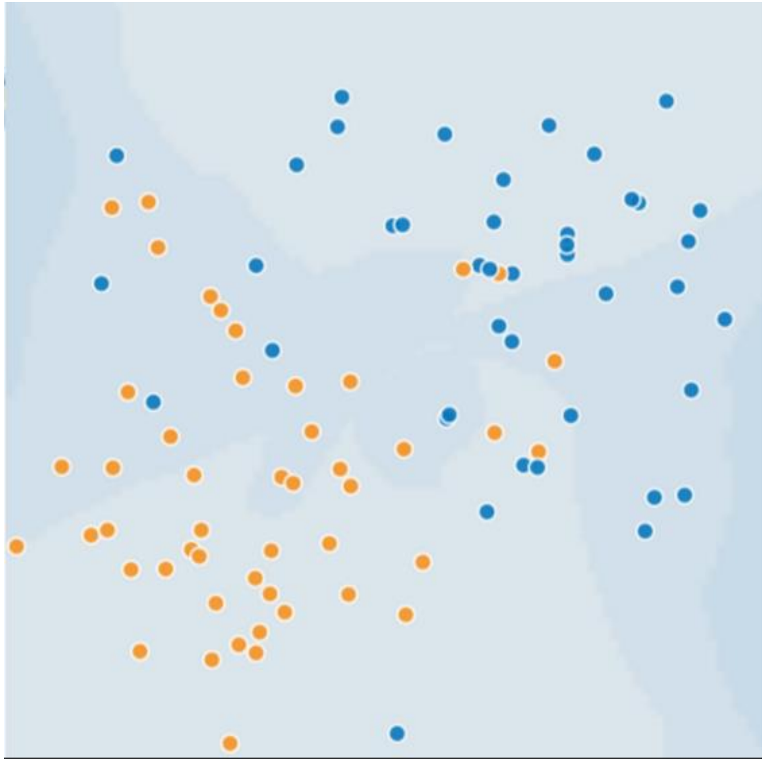
With four parameters I can fit an elephant, and with five I can make him wiggle his trunk – John von Neumann



Source: https://en.wikipedia.org/wiki/Overfitting

SIMPLE EXAMPLE

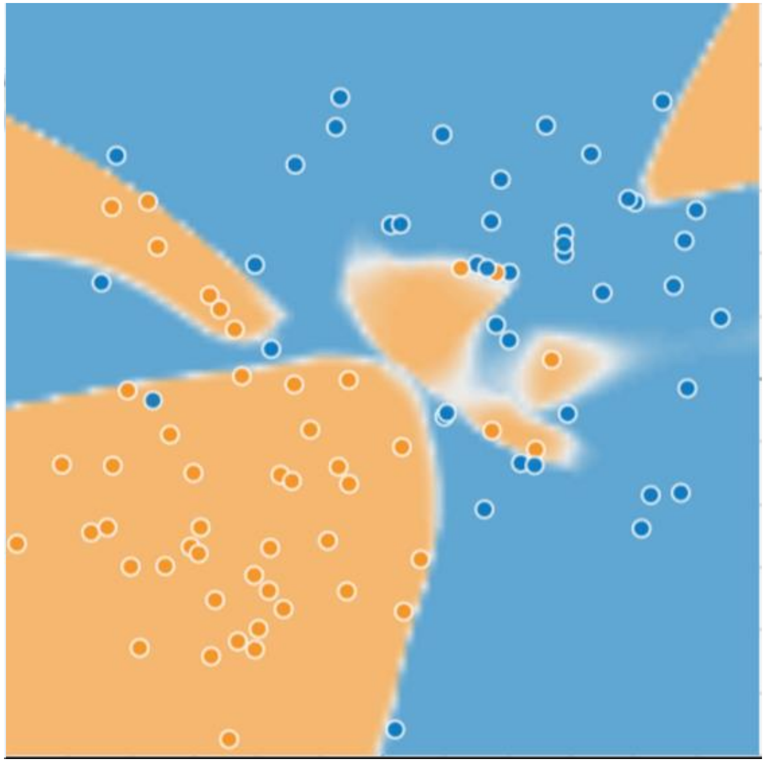
- Sick (blue) and healthy (orange) trees in a forest.



https://developers.google.com/machine-learning/crash-course/generalization/peril-of-overfitting

SIMPLE EXAMPLE

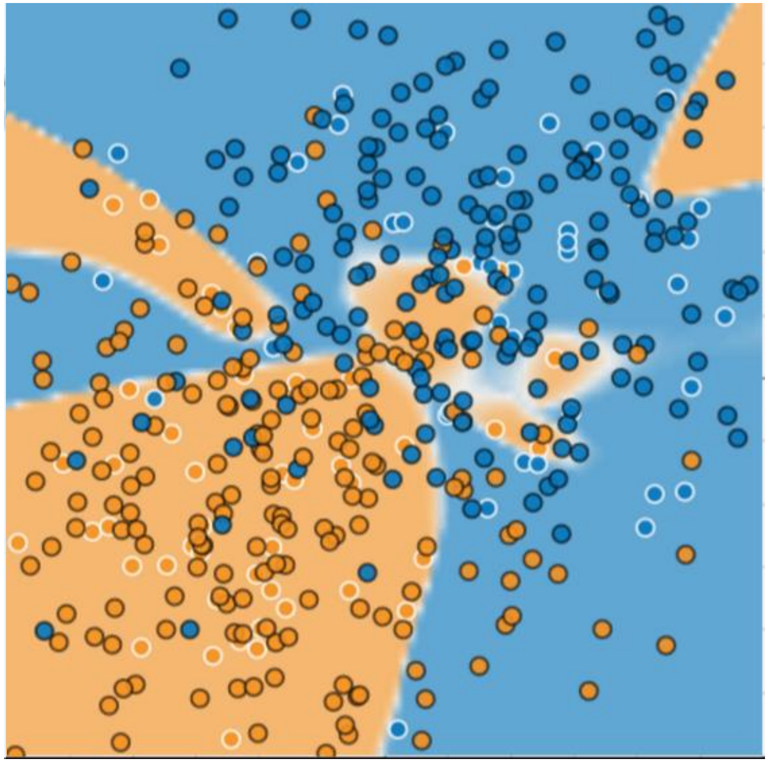
- Sick (blue) and healthy (orange) trees in a forest.
- Low loss, excellent model?



https://developers.google.com/machine-learning/crash-course/generalization/peril-of-overfitting

SIMPLE EXAMPLE

- Sick (blue) and healthy (orange) trees in a forest.
- Low loss, excellent model?
- Poor generalization!
- Model is more complex than necessary



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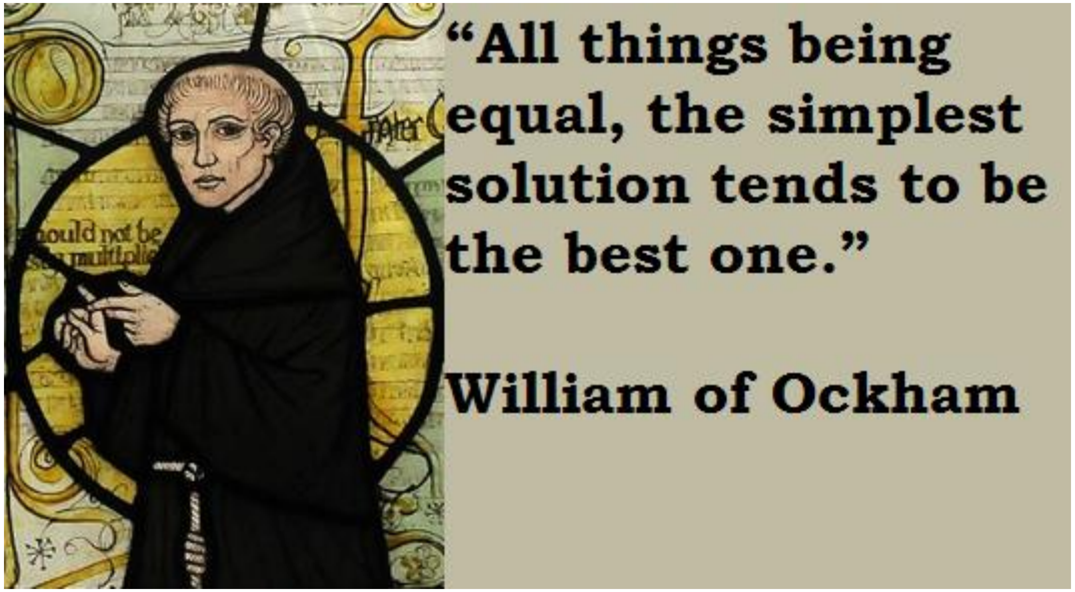
OCKHAM'S RAZOR

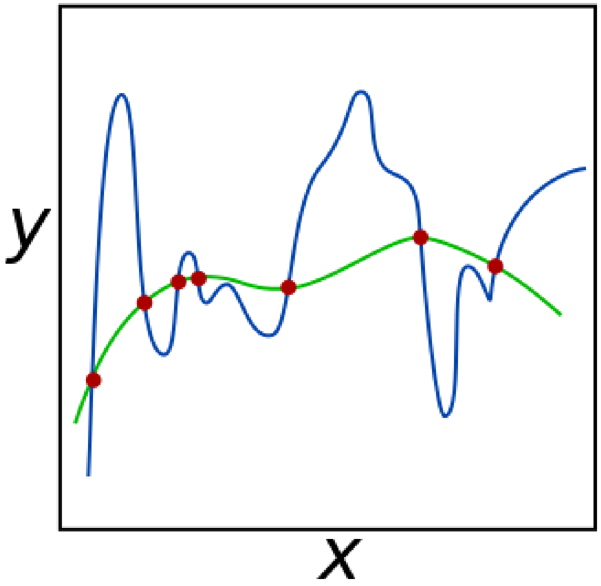
- William of Ockham, a 14th century friar and philosopher, loved simplicity. He believed that scientists should prefer simpler formulas or theories over more complex ones.



Source: https://medium.com/neuralspace/how-bayesian-methods-embody-occams-razor-43f3d0253137

REGULARIZED LINEAR MODELS

- Reduce overfitting by constraining a model
- E.g. limiting the number of polynomial degrees
- Or adding a penalty to the cost function



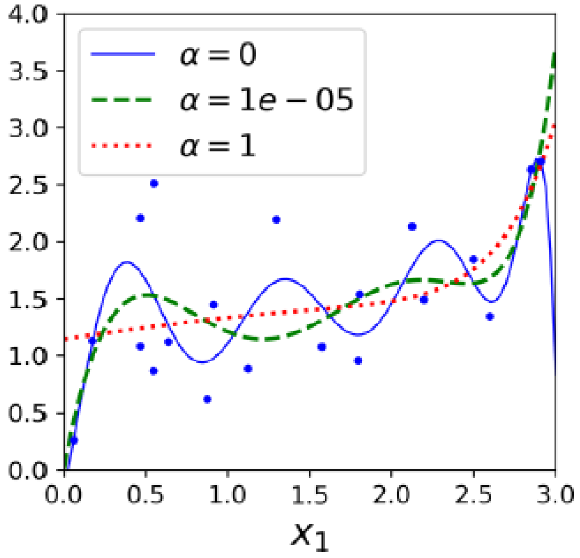
Source: https://en.wikipedia.org/wiki/Regularization_(mathematics)

RIDGE REGRESSION

- L2 regularization

Equation 4-8. Ridge Regression cost function

$$J(\boldsymbol{\theta}) = \mathrm{MSE}(\boldsymbol{\theta}) + \alpha\frac{1}{2}\Sigma_{i=1}^{n}\theta_i^2$$



Source: Géron, ISBN: 9781492032632

- Penalize sum of squared coefficients values

- Fit the data but also keep the model weights as small as possible

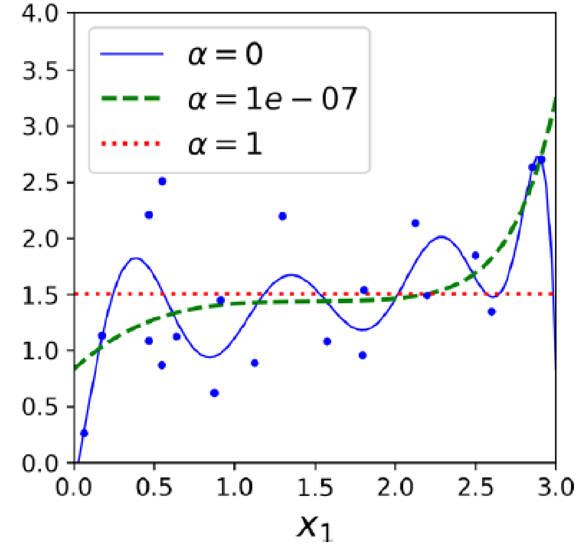
- Keep all features in the model, and balances the contribution

LASSO REGRESSION

- L1 regularization

Equation 4-10. Lasso Regression cost function

$$J(\theta) = \mathrm{MSE}(\theta) + \alpha \sum_{i=1}^{n} \left|\theta_i\right|$$



- Penalize sum of absolute values

- Removes highly correlated features by nulling their coefficient

Source: Géron, ISBN: 9781492032632

- Solution instability, impose learning schedule

ELASTIC NETS

- Middle ground between Ridge Regression and Lasso Regression

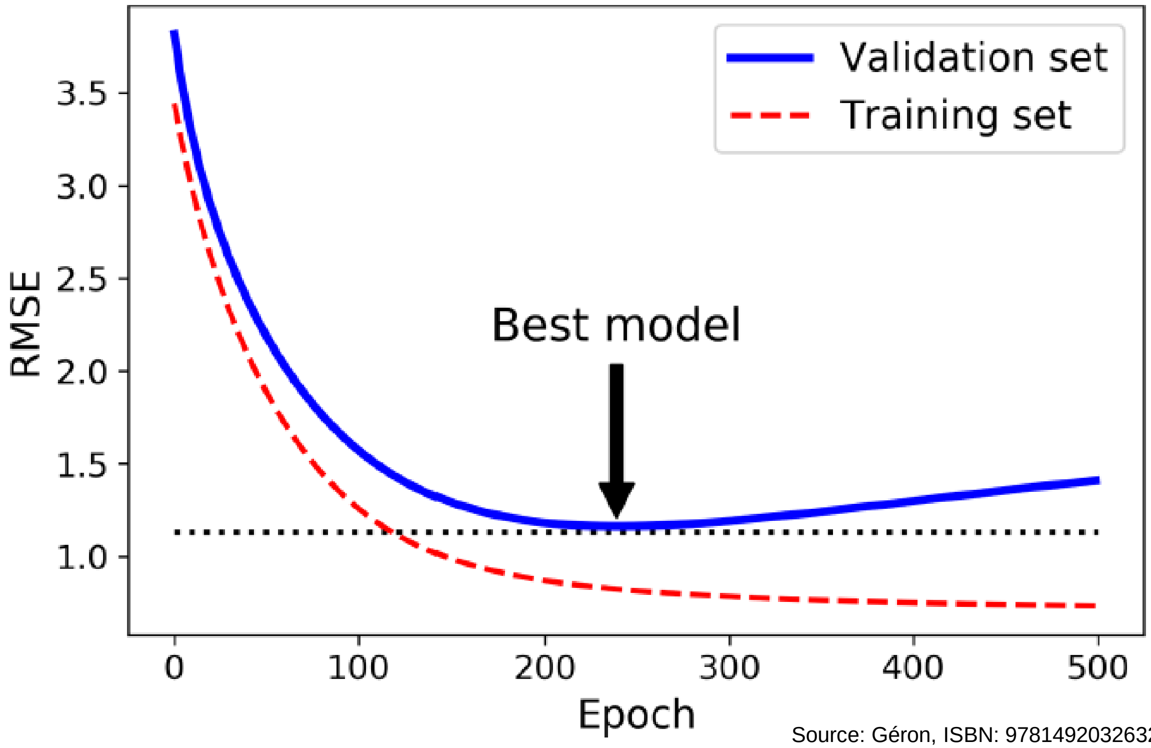
Equation 4-12. Elastic Net cost function

$$J(\boldsymbol{\theta}) = \mathrm{MSE}(\boldsymbol{\theta}) + r\alpha\sum_{i=1}^{n}\left|\theta_i\right| + \frac{1-r}{2}\alpha\sum_{i=1}^{n}\theta_i^2$$

- Ridge is a good default, but if you suspect that only a few features are useful Elastic Net is preferred

EARLY STOPPING

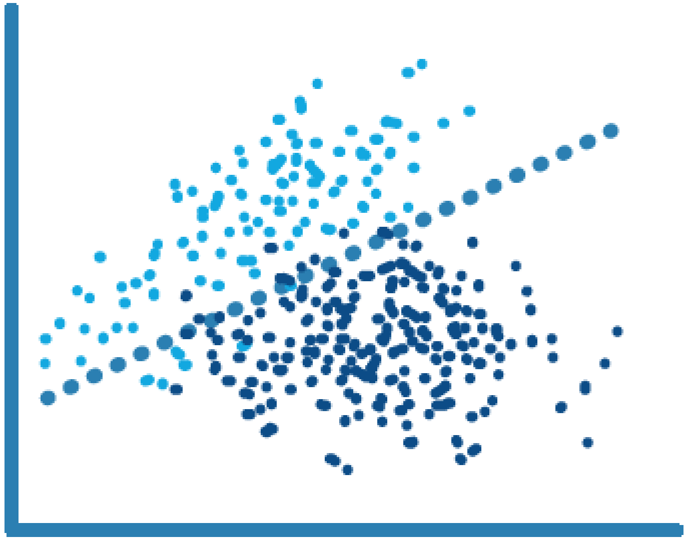
- Interpretation of learning curves



Source: Géron, ISBN: 9781492032632

LOGISTIC REGRESSION

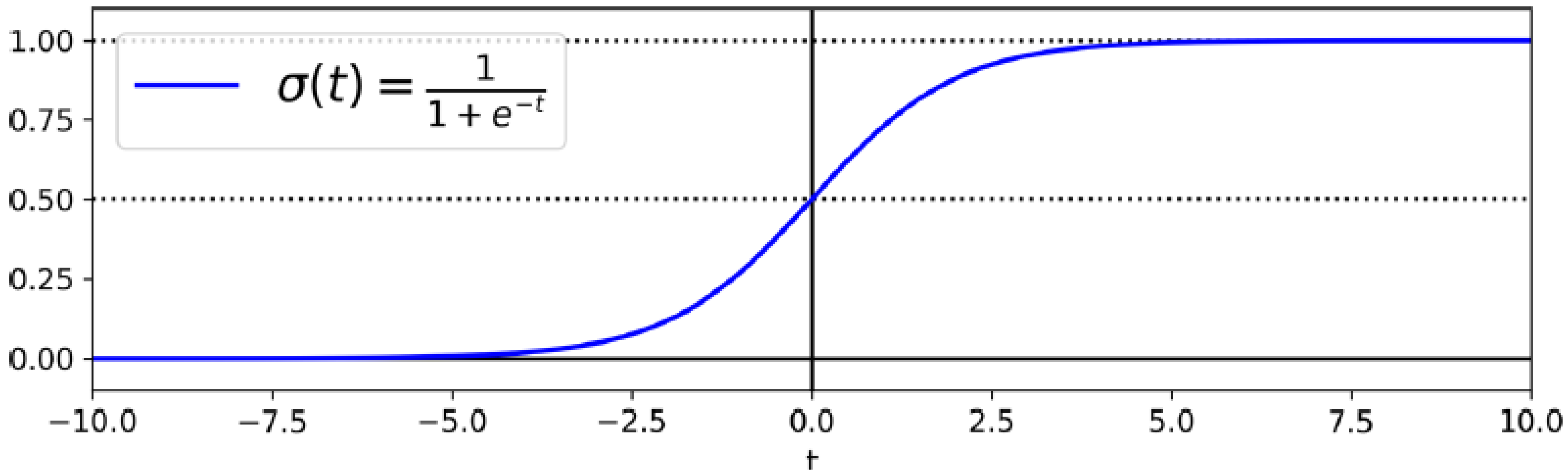
- Estimate the probability that an instance belongs to a particular class
- Binary classifier
- Baseline for evaluating more complex classification methods



Source: Mathworks, Applying Supervised Learning

ESTIMATING PROBABILITY

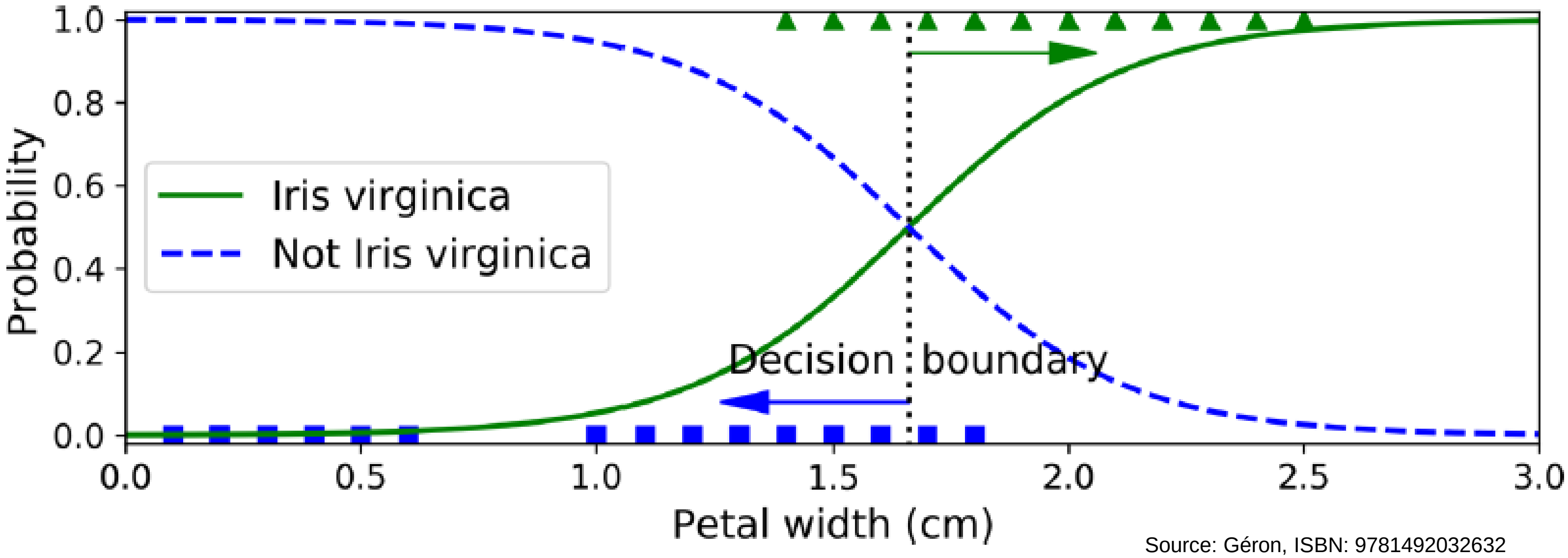
- Logistic functions maps prediction result to probability
- Sigmoid function



https://developers.google.com/machine-learning/crash-course/logistic-regression/calculating-a-probability

DECISION BOUNDARY

- Aka classification threshold
- Both probabilities are equal to 50% ?



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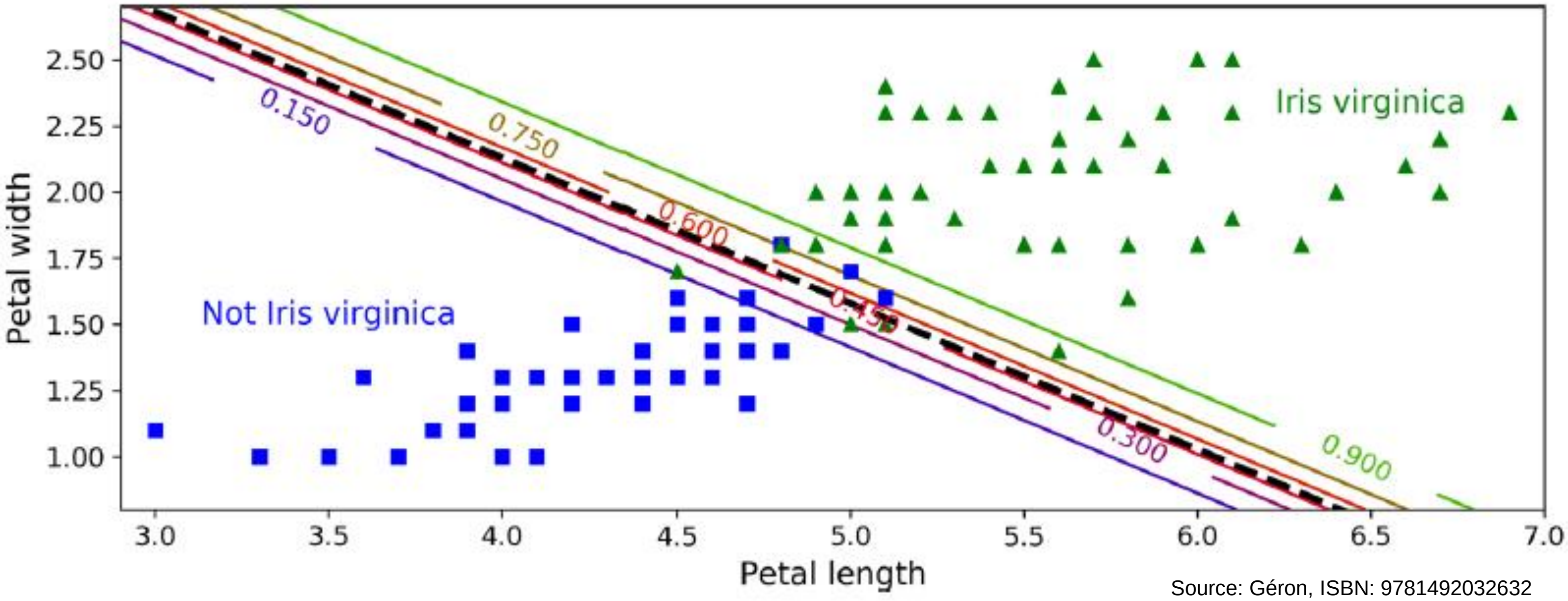
LINEAR DECISION BOUNDARY



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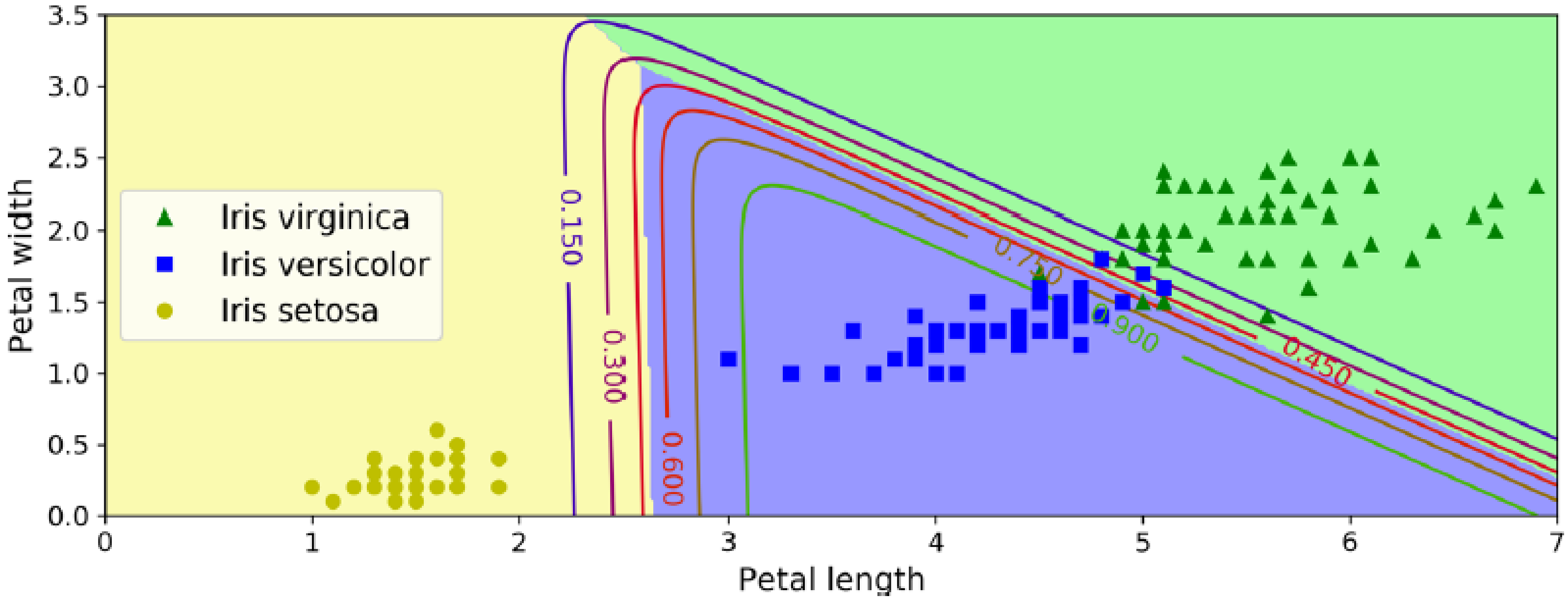
- Logistic Regression models can be regularized

SOFTMAX REGRESSION

- Multinomial Logistic Regression
- Estimate probability of each class



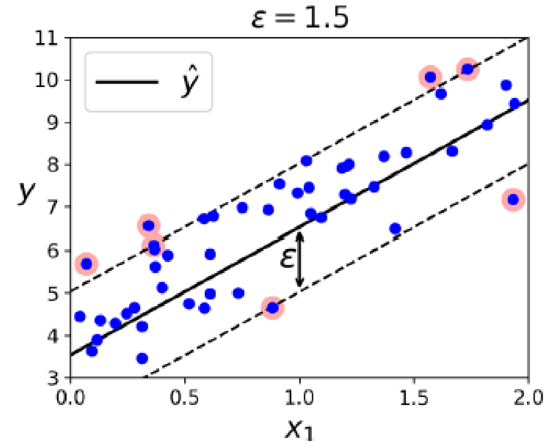
Source: Géron, ISBN: 9781492032632

Equation 4-20. Softmax function

$$\hat{p}_k = \sigma(\mathbf{s}(\mathbf{x}))_k = \frac{\exp\left(s_k(\mathbf{x})\right)}{\sum_{j=1}^{K} \exp\left(s_j(\mathbf{x})\right)}$$

CLASSIFICATION AND REGRESSION

- SVM
  Reverse the training objective



- Regression trees:
  Approximate value
  instead of predict a class