Computer Vision 1: Homework 8

Deadline 20.12. 12:15

Important: Submit your programming solutions through Moodle. The deadline for submitting your work is always on Thursday, at 12.15, the week after handing out the homework.
For other, non-programming homework, bring your solution with you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

A second order polynomial follows the functional form

$$f(x) = a_3 x^2 + a_2 x + a_1,$$

where the three parameters $a_i \in \mathbb{R}$.

- Load the two files `x.npy` and `y.npy` from Moodle using `numpy.load`. These depict the x and y values of the dataset shown in Figure 1. The values contain inliers that are close to the polynomial, and other outliers that are at random locations.

- Implement RANSAC for fitting a second order polynomial to the data. Use an inlier threshold of 0.05 for the Euclidean distance, and run 1000 iterations. Draw a plot similar to Figure 1 using the best parameters you found. You can check your work by comparing the values you find to the true values $a_3 = 2.0, a_2 = -4.2, a_1 = 1.0$.

Hints: Given the three randomly sampled points (x_1, y_1), (x_2, y_2), (x_3, y_3), create a coefficient matrix

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}.$$

The estimated model coefficients $\hat{a}_i$ are then obtained by solving the linear group of equations

$$X \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$

You may use `numpy.linalg.lstsq` or another similar tool to solve the least squares problems for finding the model parameters.

Other tasks.

1. Suppose $\epsilon \in (0, 1)$ is the probability that any single randomly drawn point is an outlier. We need to draw a sample of $m \in \mathbb{N}$ points to fit our model. Find the minimum number of iterations $k \in \mathbb{N}$ required for RANSAC such that with probability p at least one sample is good[1].

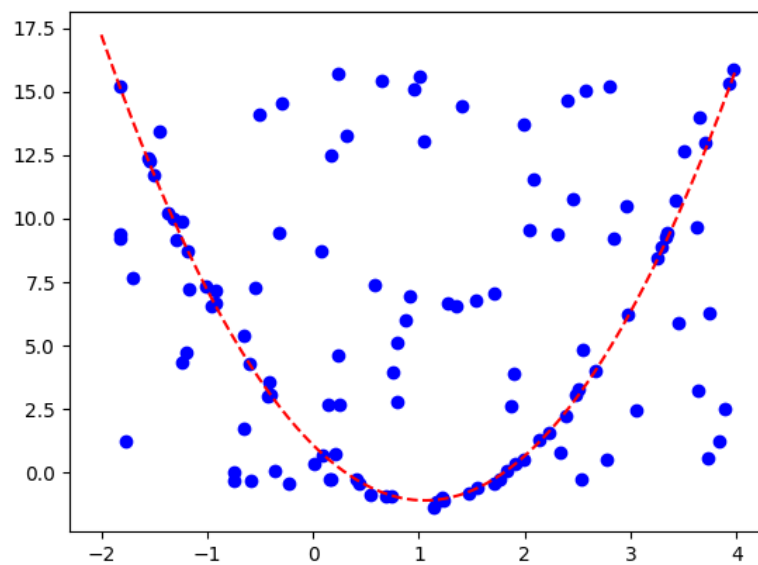[1]In a "good" sample, all of the m randomly drawn points are inliers.

Figure 1: Dataset shown by blue dots. Second order polynomial fit found by RANSAC shown in red.