Introduction to Robotics

Assignment #2

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Task 2.1 (4 points) Planar manipulator:

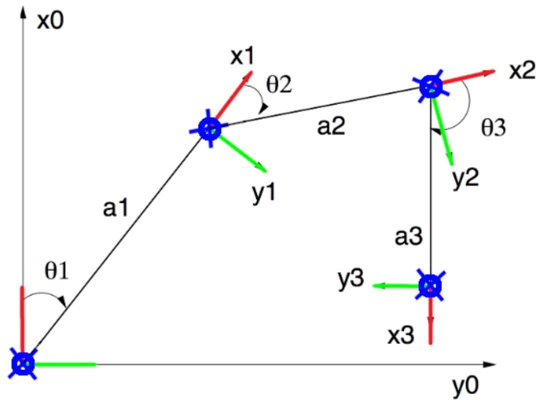

Figure 1: 3-joint planar manipulator.

Note: Z0, Z1 and Z2 point away from the reader (according to the back arrow notation).

2.1.1 (2 points):
The DH-parameters can be found in Table 1:

Link	θ_i	d_i	a_i	α_i
1	θ_1	0	a_1	0
2	θ_2	0	a_2	0
3	$^*\theta_3$	0	a_3	0

Table 1: DH-parameters for the 3-joint planar manipulator. $^*\theta_3 = 180° - \theta_1 - \theta_2$

General homogeneous transformation:

$$\begin{aligned} {}^{i-1}A_i &= Rot_{z_{i-1},\theta_i} Trans_{z_{i-1},d_i} Trans_{x_i,a_i} Rot_{x_i,\alpha_i} \\ &= \begin{bmatrix} cos(\theta_i) & -sin(\theta_i)cos(\alpha_i) & sin(\theta_i)sin(\alpha_i) & a_i cos(\theta_i) \\ sin(\theta_i) & cos(\theta_i)cos(\alpha_i) & -cos(\theta_i)sin(\alpha_i) & a_i sin(\theta_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{1}$$

Partial homogeneous transformations determined by filling (1) with the DH parameters from Table 1:

$${}^0A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^1A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^2A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from base to first frame:

$${}^0T_1 = {}^0A_1 \tag{2}$$

Transformation from base to second frame:

$$
\begin{aligned}
{}^0T_2 &= {}^0A_1{}^1A_2 \\
&= \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1C_1 + a_2C_{12} \\ S_{12} & C_{12} & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{3}
$$

Transformation from base to third frame:

$$
\begin{aligned}
{}^0T_3 &= {}^0A_1{}^1A_2{}^2A_3 \\
&= \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1C_1 + a_2C_{12} + a_3C_{123} \\ S_{123} & C_{123} & 0 & a_1S_1 + a_2S_{12} + a_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 & 0 & a_1C_1 - a_2C_3 - a_3 \\ 0 & -1 & 0 & a_1S_1 + a_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{4}
$$

The final transformation gives us the pose of the end effector with reference to the base link. We can interpret that the orientation of the X_3 and Y_3 axes will always point in the opposite direction of X_0 and Y_0, while Z_3 will remain the same (along Z_0). This is due to the constraint on θ_3.

Finally we can also see that the position is only translated along the X and Y axis.

2.1.2 (2 points):

$$
Rot_{x_0,\theta_0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_0 & -S_0 & 0 \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
Rot_{x_3,\theta_4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_4 & -S_4 & 0 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Task 2.2 (3 points) DH-Parameter parallel joints:

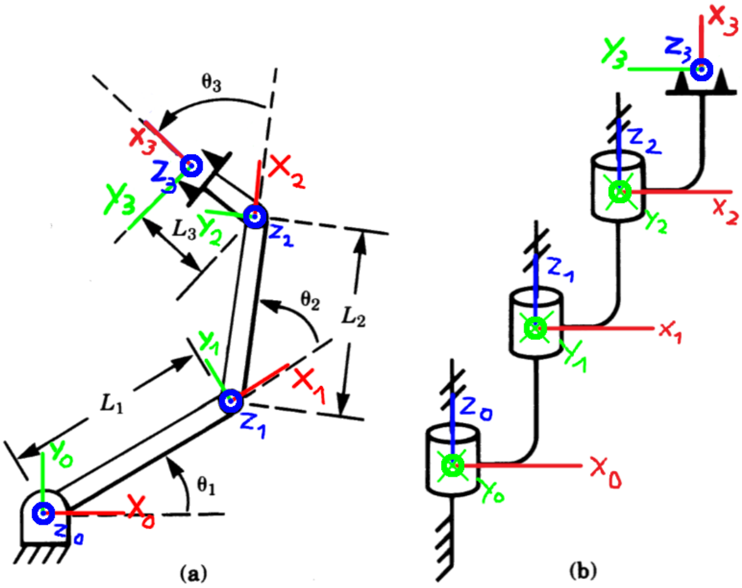

Figure 2: Coordinate frames of the 3-joint planar manipulator

The DH-parameters can be found in Table 2:

Link	θ_i	d_i	a_i	α_i
1	θ_1	0	L_1	0
2	θ_2	0	L_2	0
3	θ_3	0	L_3	0

Table 2: DH-parameters for the 3-joint planar manipulator

Task 2.3 (3 points) DH-Parameter Stanford manipulator:

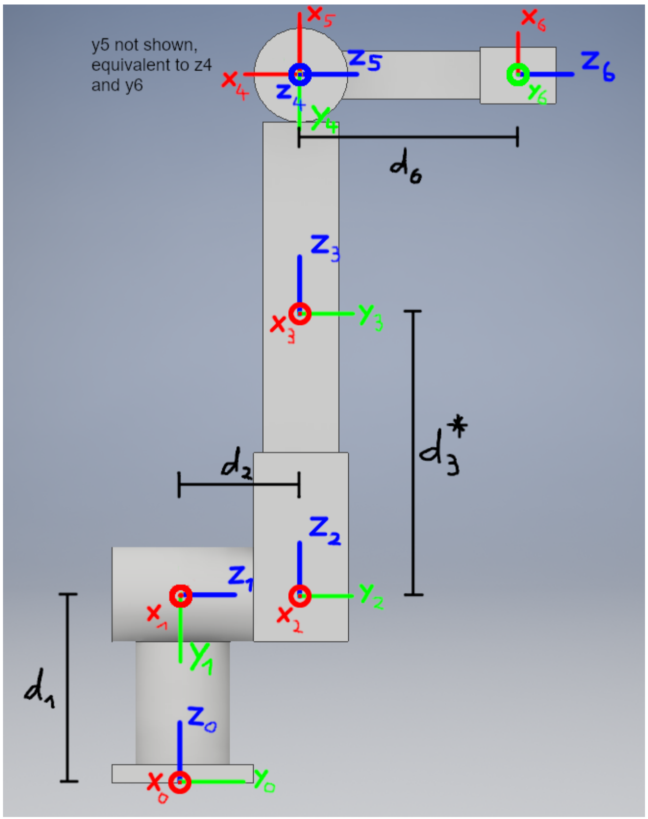

Figure 3: Coordinate frames for the Stanford Manipulator. $x_0, x_1, x_2, x_3, z_4, (y_5)$ & y_6 point towards the reader

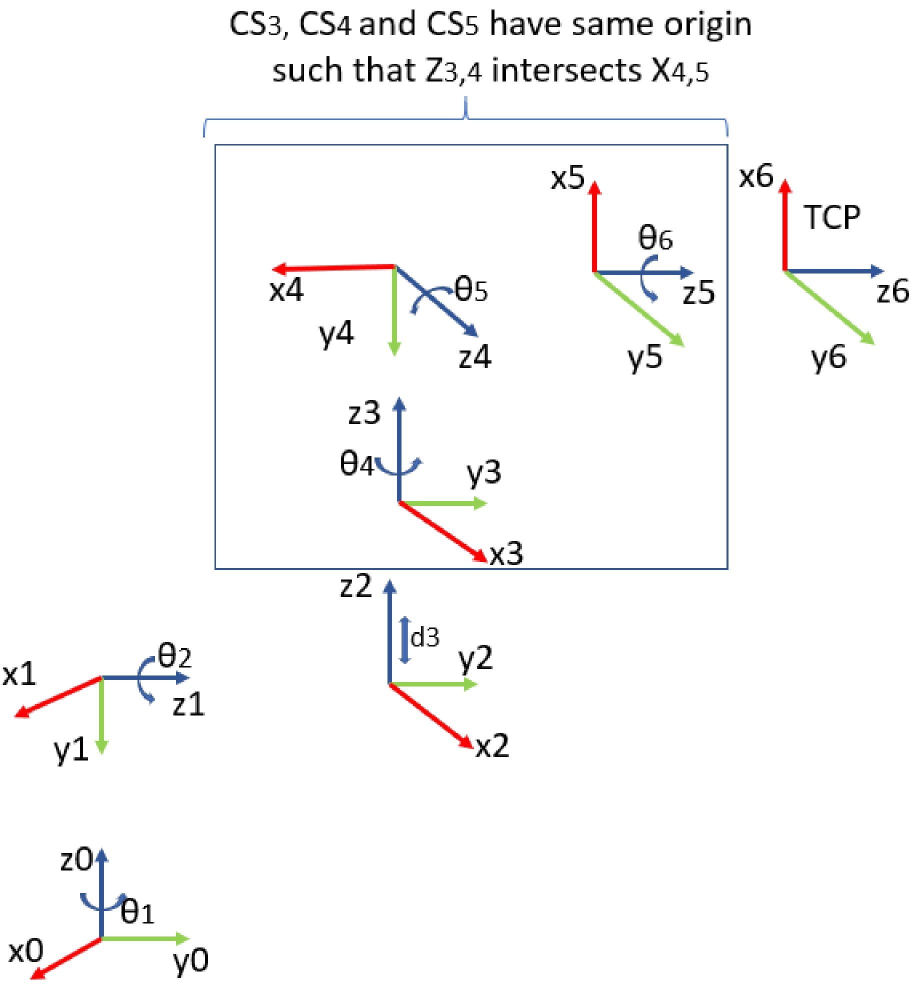

Figure 4: Separated coordinate frames for better visibility

Link	θ_i	d_i	a_i	α_i
1	θ_1*	d_1	0	$-90°$
2	θ_2*	d_2	0	$90°$
3	0	d_3*	0	$0°$
4	θ_4*	0	0	$-90°$
5	θ_5*	0	0	$90°$
6	θ_6*	d_6	0	$0°$

Table 3: DH-parameters for the Stanford Manipulator

Task 2.4 (3 points) DH-Parameter from URDF:

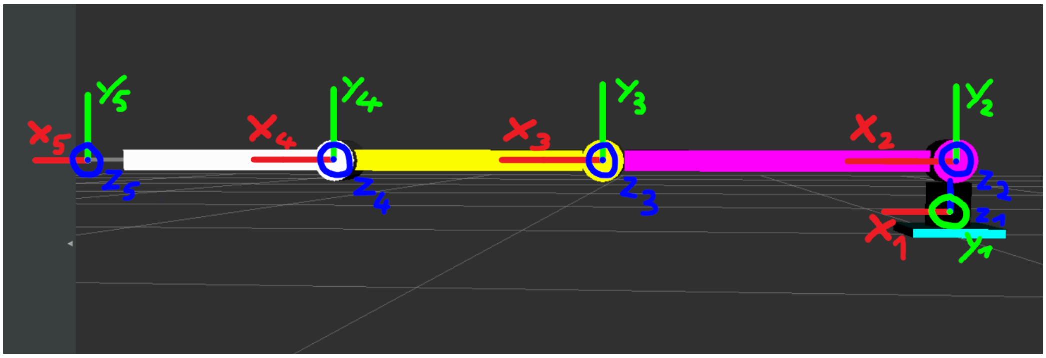

Figure 5: 4-DOF non-planar manipulator. The base frame is inside the `base_link` with the center coordinates (0, 0, 0.01).

The DH-parameters can be found in Table 4:

Link	θ_i	d_i	a_i	α_i
1	θ_1	0.06	0	0
2	θ_2	0.1	0	90
3	θ_3	0	0.8	0
4	θ_4	0	0.65	0
5	0	0	0.65	0

Table 4: DH Parameters extracted from URDF file of 4-DOF non-planar manipulator.

The base frame for the DH-parameters is the center point of the `base_link`.

Task 2.5 (7 points) DH-Parameter SCARA:

2.5.1 (2 points):

Precondition 1: x^i is perpendicular to z^{i-1}

- x^1 is perpendicular to z^0
- x^2 is perpendicular to z^1
- x^3 is perpendicular to z^2

Precondition 2: x^i intersects z^{i-1}

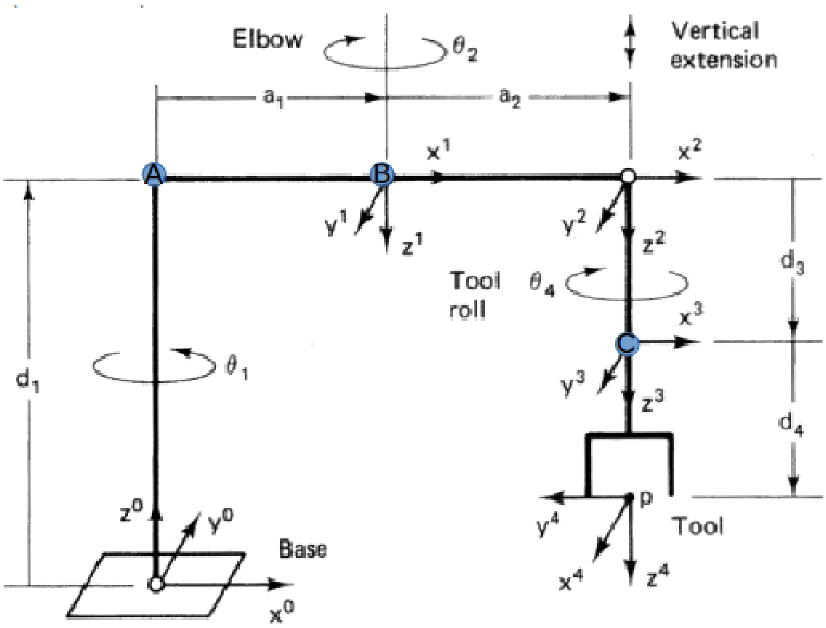

Figure 6: SCARA manipulator (Adept One)

- x^1 intersects z^0 in point A
- x^2 intersects z^1 in point B
- x^3 intersects z^2 in point C

Rules:

1. **CS_0 is the stationary origin at the base of the manipulator:**

   As figure 6 shows, the "Base" is the anchored/mounting point of the SCARA manipulator and that is where CS_0 is attached.

2. **Axis z^{i-1} is set along the axis of motion of the i^{th} joint:**

   - θ_1 rotation around z^0
   - θ_2 rotation around z^1
   - d_3 is variable along z^2
   - θ_4 rotation around z^3

3. **Axis x^i is parallel to the common normal of z^{i-1} and z^i :**

   - x^1 is perpendicular to both z^0 and z^1
   - x^2 is perpendicular to both z^1 and z^2
   - x^3 is perpendicular to both z^2 and z^3

4. **Axis y^i concludes a right-handed coordinate system:**

   - $Y^0 = Z^0 \times X^0$
   - $Y^1 = Z^1 \times X^1$
   - $Y^2 = Z^2 \times X^2$

- $Y^3 = Z^3 \times X^3$

5. **For the end-effector set Z equal to the approach vector, along the direction** Z_{i-1}

- Z^4 has same direction and orientation as Z^3
- X^4 is perpendicular to Z^3 and Z^4
- $Y^4 = Z^4 \times X^4$

2.5.2 (3 points):

$$^0A_1 = \begin{bmatrix} cos(q_1) & sin(q_1) & 0 & a_1cos(q_1) \\ sin(q_1) & -cos(q_1) & 0 & a_1sin(q_1) \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1A_2 = \begin{bmatrix} cos(q_2) & -sin(q_2) & 0 & a_2cos(q_2) \\ sin(q_2) & cos(q_2) & 0 & a_2sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3A_4 = \begin{bmatrix} cos(q_4) & -sin(q_4) & 0 & 0 \\ sin(q_4) & cos(q_4) & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation will be:

$$^{Base}T_{Tool} = {}^0A_1 * {}^1A_2 * {}^2A_3 * {}^3A_4$$

$$^0T_2 = {}^0A_1 * {}^1A_2 = \begin{bmatrix} cos(q_1 - q_2) & sin(q_1 - q_2) & 0 & a_1cos(q_1) + a_2cos(q_1 - q_2) \\ sin(q_1 - q_2) & -cos(q_1 - q_2) & 0 & a_1sin(q_1) + a_2sin(q_1 - q_2) \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = {}^0T_2 * {}^2A_3 = \begin{bmatrix} cos(q_1 - q_2) & sin(q_1 - q_2) & 0 & a_1cos(q_1) + a_2cos(q_1 - q_2) \\ sin(q_1 - q_2) & -cos(q_1 - q_2) & 0 & a_1sin(q_1) + a_2sin(q_1 - q_2) \\ 0 & 0 & -1 & d_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4 = {}^0T_3 * {}^3A_4 = \begin{bmatrix} cos(q_1 - q_2 - q_4) & sin(q_1 - q_2 - q_4) & 0 & a_1cos(q_1) + a_2cos(q_1 - q_2) \\ sin(q_1 - q_2 - q_4) & -cos(q_1 - q_2 - q_4) & 0 & a_1sin(q_1) + a_2sin(q_1 - q_2) \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.5.3 (2 points):

q = $[\frac{\pi}{4}, -\frac{\pi}{3}, 120, \frac{\pi}{2}]^T$ d = $[877, 0, d3, 200]^T$ mm a = $[425, 375, 0, 0]^T$ mm
Therefore

$$^0T_4 = \begin{bmatrix} cos(\frac{\pi}{12}) & sin(\frac{\pi}{12}) & 0 & 425cos(\frac{\pi}{4}) + 375cos(\frac{7\pi}{12}) \\ sin(\frac{\pi}{12}) & -cos(\frac{\pi}{12}) & 0 & 425sin(\frac{\pi}{4}) + 375sin(\frac{7\pi}{12}) \\ 0 & 0 & -1 & 557 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We know that:

$$T = \begin{bmatrix} \vec{n} & \vec{o} & \vec{a} & \vec{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence the orientation of the TCP (Tool Center Point) is:

$$\vec{n} = [cos(\frac{\pi}{12}), sin(\frac{\pi}{12}), 0]$$

$$\vec{o} = [sin(\frac{\pi}{12}), -cos(\frac{\pi}{12}), 0]$$

$$\vec{a} = [0, 0, -1]$$

And the position of the TCP is:

$$\vec{p} = [203.46, 662.74, 557]$$