Introduction to Robotics
Assignment #1

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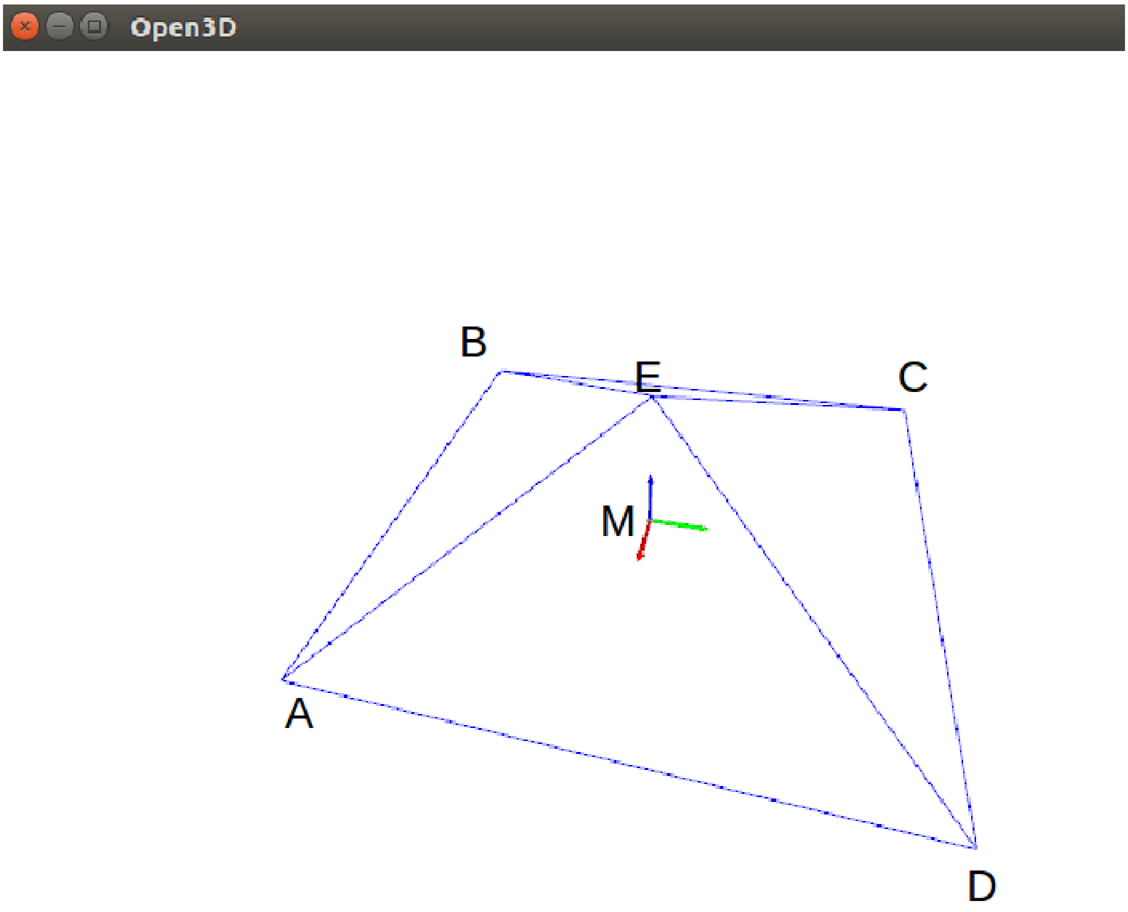
Task 1.1 (8 points) Pyramid:

For this exercise the setup is as follows:

Red - X axis

Green - Y axis

Blue - Z axis



$$T_{u,\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) & 0 \\ 0 & sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{v,\theta} = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{w,\psi} = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 & 0 \\ sin(\psi) & cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.1.1 (4 points):

1. Rotation by $\psi = 50°$ around M_w
2. Rotation by $\phi = -35°$ around M_u
3. Rotation by $\theta = 340°$ around M_v

$$T_{wuv} = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 & 0 \\ sin(\psi) & cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) & 0 \\ 0 & sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} cos(\theta)cos(\psi) - sin(\theta)sin(\phi)sin(\psi) & -cos(\phi)sin(\psi) & cos(\psi)sin(\theta) + cos(\theta)sin(\phi)sin(\psi) & 0 \\ cos(\psi)sin(\theta)sin(\phi) + cos(\theta)sin(\psi) & cos(\phi)cos(\psi) & sin(\theta)sin(\psi) - cos(\theta)cos(\psi)sin(\phi) & 0 \\ -cos(\phi)sin(\theta) & sin(\phi) & cos(\phi)cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{wuv} = \begin{bmatrix} 0.45374424 & -0.62750687 & -0.63273319 & 0. \\ 0.84594497 & 0.52654078 & 0.0844506 & 0. \\ 0.2801665 & -0.57357644 & 0.76975113 & 0. \\ 0. & 0. & 0. & 1. \end{bmatrix}$$

Assuming we define the points in a homogeneous coordinates fashion : $A = \begin{bmatrix}21, -21, 0, 1\end{bmatrix}$ $B = \begin{bmatrix}-21, -21, 0, 1\end{bmatrix}$ $C = \begin{bmatrix}-21, 21, 0, 1\end{bmatrix}$ $D = \begin{bmatrix}21, 21, 0, 1\end{bmatrix}$ $E = \begin{bmatrix}0, 0, 12, 1\end{bmatrix}$ $M = \begin{bmatrix}0, 0, 0, 1\end{bmatrix}$
We get the following results:

$$A = \begin{bmatrix}22.7063 & 6.70749 & 17.9286 & 1\end{bmatrix}$$

$$B = \begin{bmatrix}3.64901529 & -28.82220092 & 6.16160867 & 1\end{bmatrix}$$

$$C = \begin{bmatrix}-22.70627331 & -6.70748797 & -17.92860165 & 1\end{bmatrix}$$

$$D = \begin{bmatrix}-3.64901529 & 28.82220092 & -6.16160867 & 1\end{bmatrix}$$

$$E = \begin{bmatrix}-7.5927983 & 1.0134072 & 9.23701358 & 1\end{bmatrix}$$

$$M = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$$

1.1.2 (4 points):

1. Rotation by $\psi = 50°$ around M_z
2. Rotation by $\phi = -35°$ around M_x
3. Rotation by $\theta = 340°$ around M_y

$$T_{yxz} = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) & 0 \\ 0 & sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 & 0 \\ sin(\psi) & cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} cos(\theta)cos(\psi) + sin(\theta)sin(\phi)sin(\psi) & cos(\psi)sin(\theta)sin(\phi) - cos(\theta)sin(\psi) & cos(\phi)sin(\theta) & 0 \\ cos(\phi)sin(\psi) & cos(\phi)cos(\psi) & -sin(\phi) & 0 \\ cos(\theta)sin(\phi)sin(\psi) - cos(\psi)sin(\theta) & cos(\theta)cos(\psi)sin(\phi) + sin(\theta)sin(\psi) & cos(\theta)cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{yxz} = \begin{bmatrix} 0.75430131 & -0.59374765 & -0.2801665 & 0. \\ 0.62750687 & 0.52654078 & 0.57357644 & 0. \\ -0.19304057 & -0.60845586 & 0.76975113 & 0. \\ 0. & 0. & 0. & 1. \end{bmatrix}$$

Assuming we define the points in a homogeneous coordinates fashion : $A = \begin{bmatrix}21, -21, 0, 1\end{bmatrix}$ $B = \begin{bmatrix}-21, -21, 0, 1\end{bmatrix}$ $C = \begin{bmatrix}-21, 21, 0, 1\end{bmatrix}$ $D = \begin{bmatrix}21, 21, 0, 1\end{bmatrix}$ $E = \begin{bmatrix}0, 0, 12, 1\end{bmatrix}$ $M = \begin{bmatrix}0, 0, 0, 1\end{bmatrix}$
We get the following results:

$$A = \begin{bmatrix}28.30902807 & 2.12028783 & 8.72372107 & 1\end{bmatrix}$$

$$B = \begin{bmatrix}-3.37162689 & -24.23500078 & 16.83142506 & 1\end{bmatrix}$$
$$C = \begin{bmatrix}-28.30902807 & -2.12028783 & -8.72372107 & 1\end{bmatrix}$$
$$D = \begin{bmatrix}3.37162689 & 24.23500078 & -16.83142506 & 1\end{bmatrix}$$
$$E = \begin{bmatrix}-3.361998 & 6.88291724 & 9.23701358 & 1\end{bmatrix}$$
$$M = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$$

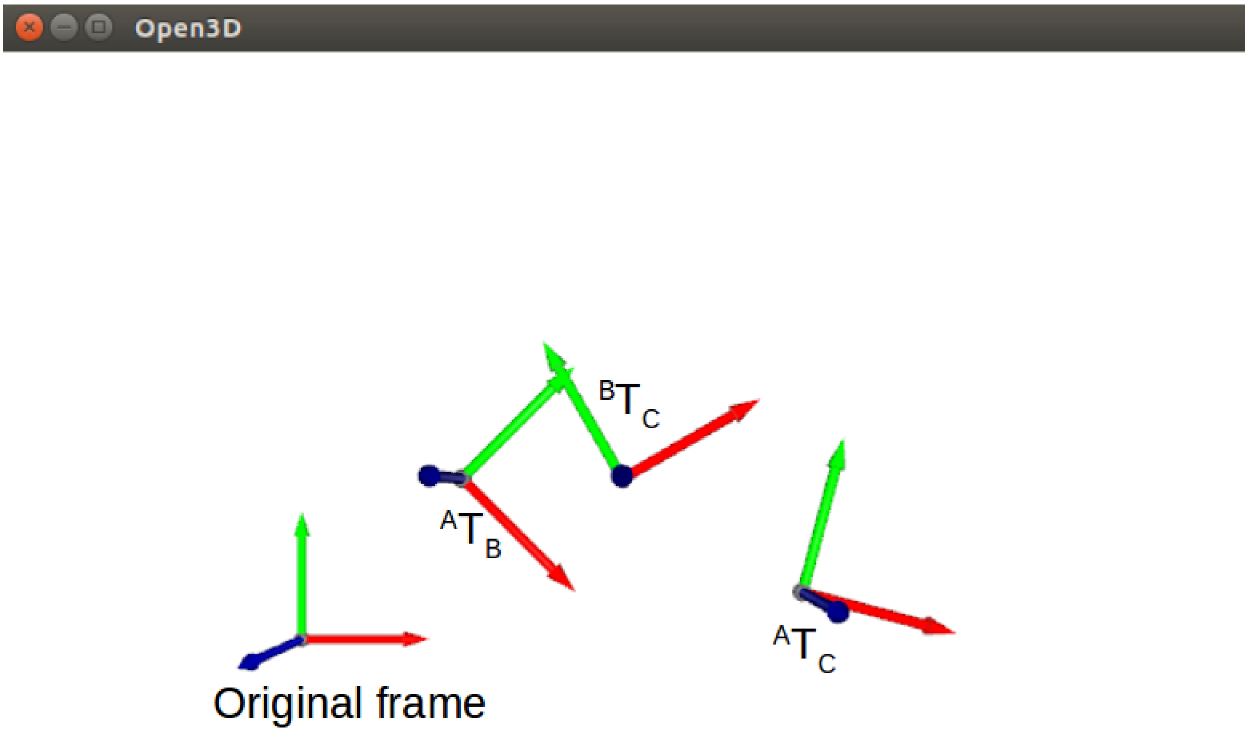
Task 1.2 (6 points) Homogeneous transformations:

$$^{A}T_{B} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$^{B}T_{C} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 2 \\ 1/2 & \sqrt{3}/2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

1.2.1 (3 points): Yes $^{A}T_{C}$ can be considered to be unambiguous because we are saying that we want to rotate frame A into frame C in the following sequence: first rotating and translating from A to B and then rotating and translating from B to C. This basically translates into computing the following multiplication of homogeneous matrices ${}^{A}T_{B} *^{B} T_{C}$

1.2.2 (3 points): We used Open3D for visualizing. According to the documentation of this library, it uses a right-handed coordinate system, with the blue axis corresponding to the Z dimension, the green to Y and the red to X.



Task 1.3 (6 points) Euler Angles:

1.3.1 (4 points):

- 90 degree yaw, followed by a 90 degree roll, followed by a 90 degree pitch is equivalent to a 90 degree roll.
- 180 degree yaw, followed by a 180 degree roll, followed by a 180 degree pitch will result in returning to the original position
- Rotate along the yaw axis 90 degrees, then along the original pitch axis 90 degrees. We obtain a Gimbal lock with a loss of one degree of freedom.
- Rotate along the yaw axis 90 degrees, then along the original pitch axis 90 degrees and then along the original roll axis 90 degrees. We obtain a Gimbal lock with a loss of two degrees of freedom.

1.3.2 (2 points):

There are exactly 12 combinations because we have a permutation of the 3 fixed axes which is 3!(factorial) = 6 and an extra permutation of the already rotated axes which is again 3! = 6. Therefore 3!+3! = 6+6 = 12 The exact list of permutations is: XYZ XZY YXZ YZX ZXY ZYX XY'Z" XZ'Y" YX'Z" YZ'X" ZX'Y" ZY'X" , where the prime means the axis was rotated and double prime means the axis was rotated again.