Kalman Filter Notes Project 3

11/20/2018

General Kalman Filter EQs

Model $\quad x(k+1) = \phi\, x(k) + \psi\, w(k)$

$\quad\quad\; y(k) = h\, x(k) + n(k)$

For α-KF $\quad \phi = 1 \quad \psi = \frac{T^2}{2} \quad h = 1$

Let x = position

$x(k+1) = x(k) + \left[\frac{T^2}{2}\right] w(k) \leftarrow N(0, \sigma_w^2)$

$y(k) = x(k) + n(k) \leftarrow N(0, \sigma_N^2)$

So we can think of $w(k)$, acceleration input changing position. It has zero mean

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- We are measuring statistics of $n(k)$ from ultrasonic transducer so we know σ_N^2

- We Need σ_w^2
  - Assume we are at some nominal position say 0" (which would also be initial condition)
  - If $w(k)$ is applied we expect x to move
    - Back out for a motion of $\frac{1}{4}$"

$$x(k+1) = x(k) + \frac{T^2}{2} w(k)$$

$$\Rightarrow x(k+1) - x(k) = \frac{T^2}{2} w(k) = \frac{1}{4}"$$

$$\Rightarrow w(k) = \frac{2}{T^2}\frac{1}{4} = \frac{1}{T^2}\frac{1}{2}$$

Let $T = 1 \quad w(k) = \frac{1}{2}$

- So $w(k)$ worst case $= \left|\frac{1}{2}\right|$
- Assume the $\frac{1}{2} = 3\sigma_w$ of normal distribution

$$\Rightarrow \sigma_w = \frac{1}{6}$$

- **INTERPRETATION**
  - use $\sigma_w = \frac{1}{6}$ $\sigma_w^2 = \frac{1}{36}$ as input to the selection of α
  - place the sensor at some value say 8" again $\pm \frac{1}{4}$"
    Error in placement
  - You can modify numbers if necessary

α FILTER EQUATIONS

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + \alpha\left[z(k+1) - \hat{x}(k+1|k)\right]$$

INITIAL CONDITION
- Nominal location of sensor say 8"

α computes

$$\Lambda = T^2\frac{\sqrt{w}}{\sqrt{v}} \qquad \Lambda^2 = \frac{4\alpha^2}{1-\alpha}$$

$$\alpha = \frac{-\Lambda^2 + \sqrt{\Lambda^4 + 16\Lambda^2}}{8}$$

$$\sigma_{\tilde{x}}^2 = \alpha\sigma_v^2$$

Use this to plot $\pm 3\sigma$ LINES ON ESTIMATE $\hat{x}(k+1|k+1)$

USING FILTER

STATIC LOCATION

- Pick NOMINAL LOCATION SAY 10"
  this is IC for FILTER
- place SENSOR at $10 \pm \frac{1}{4}$" use $10\frac{1}{4}$" first shot
- RUN KF SAY 100 UPDATES IC at 0 ↓
- plot $\hat{x}(k|k)$ vs UPDATE $n = 0 \to 100$
  plot $\pm 3\Delta$ LINES, discuss RESULTS
  COMPARE TO JUST AVERAGING READING FROM SENSOR
- Redo with placement at $10 - \frac{1}{4} = 9\frac{3}{4}$"
  - See what is different
- TRY some other NOMINAL LOCATIONS
  6, 12, 18, 24"
  ONE POSITION IS FINE use something IN band $[-\frac{1}{4}, \frac{1}{4}]$ of NOMINAL

DYNAMIC LOCATION (drifting about NOMINAL)

FOR SOME NOMINAL VALUE

- MOVE SENSOR to $\pm\frac{1}{4}$) GET A SINGLE READING from ULTRASONIC

N TIMES [- REPEAT FOR SAY $-\frac{1}{4}, \pm\frac{1}{8}, -\frac{1}{8}$ SOME other VALUES
YOU don't NEED to ACCURATELY MEASURE

- RUN KF & FILTER FOR N DATA SETS
- PLOT RESULTS