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Answer to homework 3

1.1

`Theory`

Code

a)

First, we find the determinant of $C - \lambda I$ to get the 2 eigen values from the solution of the quadratic equation:

```
eig_vals = solv_eig_val_2x2(data_C)
Eigen values: (1.6142729428888276, 0.6079492793333952)
```

Second, we use the 2 eigen values to solve for 2 eigen vectors. Here, it's stacked column-wise:

```
eig_vec = solv_eig_vec(data_C, eig_vals)
```

Then finally, we normalize both eigen vectors:

```
eig_vec[:,0] = norm_vec(eig_vec[:,0])
eig_vec[:,1] = norm_vec(eig_vec[:,1])
```

Which, we get:

```
Eigen vectors:
[[ 0.6992951  0.7148331]
 [-0.7148331  0.6992951]]
```

Here are the function used to solve the steps above:

```
solv_quad = lambda a, b, c: ((-b + np.sqrt(b**2 - 4*a*c))/2*a, (-b - np.sqrt(b**2 -
4*a*c))/2*a)
solv_eig_val_2x2 = lambda A: solv_quad(1, -(A[0,0] + A[1,1]), (A[0,0]*A[1,1] - A[0,1]*A[1,0]))
solv_eig_vec = lambda A, V: np.array([[1, -(A[0,0] - V[0])/(A[0,1])], [1, -(A[0,0] -
V[1])/(A[0,1])]]).T
norm_vec = lambda V: V/np.sqrt(V.T @ V)
```

b)

Projecting the data on the the primary principal component by:

```
proj_1D = data @ eig_vec[:,0]
```

Which we get:

```
array([-0.50302024, -0.15100073, -0.6697674 , -0.51043032, -3.20802512,
       -1.18482902,  0.16767341,  1.00511423,  0.34553569,  0.83095699])
```

1.2

`Theory`

Code

a)

Info gains={'x1': 0.8, 'x2': 0.7245112497836532}

b)

Feature 1 (x1) is more discriminative

c)

The steps are:

1. Get mean of each feature for class 0
2. Get mean of each feature for class 1
3. Calculate the covariance matrix for class 0
4. Calculdate the covariance matrix for class 1
5. Calculate the scatter matrix for class 0 from its covariance matrix
6. Calculate the scatter matrix for class 1 from its covariance matrix
7. Get the within-class-scatter matrix by summing the 2 scatter matrices above
8. Get the between-class-scatter matrix using the 2 means matrices above

```
mean_0 = np.mean(data_df[data_df['Y'] == 0].drop(['Y'], axis=1))
mean_1 = np.mean(data_df[data_df['Y'] == 1].drop(['Y'], axis=1))
cov_mat_0 = cov_mat(data_df[data_df['Y'] == 0].drop(['Y'], axis=1))
cov_mat_1 = cov_mat(data_df[data_df['Y'] == 1].drop(['Y'], axis=1))
scatter_mat_0 = (len(data_df[data_df['Y'] == 0]) - 1) * cov_mat_0
scatter_mat_1 = (len(data_df[data_df['Y'] == 1]) - 1) * cov_mat_1
scatter_within = scatter_mat_0 + scatter_mat_1
scatter_between = np.array(mean_0 - mean_1).reshape(2,1) @ np.array(mean_0 -
mean_1).reshape(1,2)
```

```
Scatter between matrix:
           x1         x2
x1  10.000000  -4.082622
x2  -4.082622  10.000000

Scatter within matrix:
[[ 1.81230578 -0.6640409 ]
 [-0.6640409   0.243309  ]]
```

1. Calculate the product of the inverse within-class-scatter matrix with the between-class-scatter matrix
2. Calculate the eigen values and vectors using the product matrix above

```
E, V = np.linalg.eig(np.linalg.inv(scatter_within) @ scatter_between)
print(V[:,0])
```

```
[0.99879099 0.04915855]
```

d)

Projecting the data on to the primary eigen vector:

```
proj_data = data_df.drop(['Y'], axis=1) @ V[:,0]
```

Which we get:

```
0   -0.278748
1   -1.086363
2   -0.527746
3    0.243497
4   -1.651498
5   -0.230252
6    0.456123
7    1.439993
8   -0.078246
9    1.713239
dtype: float64
```

e)

The plot of the projected data is here: Scatter plot of projected data

This plot shows that the projection does not provide good class separation because there are 2 points of each class which essentially overlaps on one another and the `within class distance` after projection is still large, almost as large as the distance `between` classes.

This is verified by the plot of the original data below. By initial scanning, there are essentially no lines which can give good separability after projection

Scatter plot of original data

2.

`Dimensionality Reduction via PCA`

Code

Sk-learn KNN accuracy (k=1) = 0.23256

My implementation of KNN's accuracy = 0.23255813953488372

Applying my KNN implementation on PCA with 100 components gives accuracy = 0.25387596899224807

After whitening, Applying my KNN implementation on PCA with 100 components gives accuracy = 0.3313953488372093

Here is the plot of PC1 and PC2:

PC1-PC2 plot

3.

`Eigenfaces`

Code

Here are the faces that the max and min of the first 2 PCs correspond to (in plot, PC0 is the primary and PC1 is the secondary):

Max and min of PC1 and PC2

It is seen that PC1 captures the overall brightness and PC2 captures if the face is bright on the left half or on the right half

Here is what PC1 captures as a 87x65 image:

What PC1 capture

Here are the reconstruction images. From left to right: Using only PC1, using **188 PCs** (95%), Original

Reconstruction

4.

`Clustering`

Code

The Kmeans algorithm ran for 29 iterations then coverges (Change < 2e-23):

```
Iter: 1, Change: 442.5361633300781
Iter: 2, Change: 128.02601623535156
Iter: 3, Change: 78.1665267944336
Iter: 4, Change: 44.924957275390625
Iter: 5, Change: 29.543842315673828
Iter: 6, Change: 22.499788284301758
Iter: 7, Change: 16.453794479370117
Iter: 8, Change: 14.850354194641113
Iter: 9, Change: 12.765664100646973
Iter: 10, Change: 10.536905288696289
Iter: 11, Change: 9.19281005859375
Iter: 12, Change: 7.387046813964844
Iter: 13, Change: 7.967870235443115
Iter: 14, Change: 7.730221748352051
Iter: 15, Change: 5.344476699829102
Iter: 16, Change: 4.699371337890625
Iter: 17, Change: 4.794102191925049
Iter: 18, Change: 4.1309685707092285
Iter: 19, Change: 4.912514686584473
Iter: 20, Change: 2.7934107780456543
Iter: 21, Change: 1.4903690814971924
Iter: 22, Change: 1.528502345085144
Iter: 23, Change: 1.1923229694366455
Iter: 24, Change: 1.993759274482727
Iter: 25, Change: 2.0061607360839844
Iter: 26, Change: 1.9300038814544678
Iter: 27, Change: 4.196130275726318
Iter: 28, Change: 2.5771408081054688
Iter: 29, Change: 1.7259509563446045
```

Here is the visualization of the algorithm's results:

4. The visualization of k-means cluster centers, and the min and max images

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.cm as cm
```

1.1 Theory

In [2]:
```
data = np.array([
    [-2, 1],
    [-5, -4],
    [-3, 1],
    [0, 3],
    [-8, 11],
    [-2, 5],
    [1, 0],
    [5, -1],
    [-1, -3],
    [6, 1]
])
```

In [3]:
```
standardize = lambda X: (X - np.mean(X))/np.std(X)
```

In [4]:
```
data = standardize(data)
```

In [5]:
```
data_C = (data.T @ data)/(data.shape[0] - 1)
data_C
```

Out[5]:
```
array([[ 1.10005528, -0.50304035],
       [-0.50304035,  1.12216694]])
```

In [6]:
```
solv_quad = lambda a, b, c: ((-b + np.sqrt(b**2 - 4*a*c))/2*a, (-b - np.sqrt(b**2 - 4*a*c))/2*a)
solv_eig_val_2x2 = lambda A: solv_quad(1, -(A[0,0] + A[1,1]), (A[0,0]*A[1,1] - A[0,1]*A[1,0]))
solv_eig_vec = lambda A, V: np.array([[1, -(A[0,0] - V[0])/(A[0,1])], [1, -(A[0,0] - V[1])/(A[0,1])]]).T
norm_vec = lambda V: V/np.sqrt(V.T @ V)
```

In [7]:
```
eig_vals = solv_eig_val_2x2(data_C)
print(f"Eigen values: {eig_vals}")
eig_vec = solv_eig_vec(data_C, eig_vals)
eig_vec[:,0] = norm_vec(eig_vec[:,0])
eig_vec[:,1] = norm_vec(eig_vec[:,1])
print(f"Eigen vectors: \n{eig_vec}")
```

```
Eigen values: (1.6142729428888276, 0.6079492793333952)
```

```
Eigen vectors:
[[ 0.6992951  0.7148331]
 [-0.7148331  0.6992951]]
```

In [9]:

```
proj_1D = data @ eig_vec[:,0]
proj_1D
```

Out[9]:

```
array([-0.50302024, -0.15100073, -0.6697674 , -0.51043032, -3.20802512,
       -1.18482902,  0.16767341,  1.00511423,  0.34553569,  0.83095699])
```

1.2 Theory

In [10]:

```
data = np.array([
    [-2, 1, 0],
    [-5, -4, 0],
    [-3, 1, 0],
    [0, 3, 0],
    [-8, 11, 0],
    [-2, 5, 1],
    [1, 0, 1],
    [5, -1, 1],
    [-1, -3, 1],
    [6, 1, 1]
])

data_df = pd.DataFrame(data, columns=['x1', 'x2', 'Y'])
data_df
```

Out[10]:

	x1	x2	Y
0	-2	1	0
1	-5	-4	0
2	-3	1	0
3	0	3	0
4	-8	11	0
5	-2	5	1
6	1	0	1
7	5	-1	1
8	-1	-3	1

	x1	x2	Y
9	6	1	1

In [11]:
```
data_df['x1'] = standardize(data_df['x1'])
data_df['x2'] = standardize(data_df['x2'])
data_df
```

Out[11]:

	x1	x2	Y
0	-0.274230	-0.098653	0
1	-1.022129	-1.331812	0
2	-0.523530	-0.098653	0
3	0.224370	0.394611	0
4	-1.770029	2.367665	0
5	-0.274230	0.887875	1
6	0.473670	-0.345285	1
7	1.470869	-0.591916	1
8	-0.024930	-1.085180	1
9	1.720169	-0.098653	1

In [13]:
```
cross_entropy = lambda x: -x*np.log2(x)
```

In [55]:
```
total = len(data_df)
E_Y_pos = cross_entropy(len(data_df[data_df['Y'] == 1])/total)
E_Y_neg = cross_entropy(len(data_df[data_df['Y'] == 0])/total)
E_Y = E_Y_pos + E_Y_neg
print(f"Sample Entropy={E_Y:.4f}")
```

```
Sample Entropy=1.0000
```

In [56]:
```
info_gains = {'x1': 0, 'x2': 0}
for feature in info_gains:
    entropy = 0
    for val in set(data_df[feature]):
        t = len(data_df[data_df[feature] == val])
        p = len(data_df[(data_df[feature] == val) & (data_df['Y'] == 1)])
        n = t-p
```

```python
        if t > 0 and p*n > 0:
            entropy += (t/total)*(cross_entropy(p/t) + cross_entropy(n/t))
    info_gains[feature] = E_Y - entropy

print(f"Info gains={info_gains}")
```

```
Info gains={'x1': 0.8, 'x2': 0.7245112497836532}
```

In [16]:
```python
cov_mat = lambda M: (M.T @ M)/(M.shape[0] - 1)
```

In [57]:
```python
mean_0 = np.mean(data_df[data_df['Y'] == 0].drop(['Y'], axis=1))
mean_1 = np.mean(data_df[data_df['Y'] == 1].drop(['Y'], axis=1))
cov_mat_0 = cov_mat(data_df[data_df['Y'] == 0].drop(['Y'], axis=1))
cov_mat_1 = cov_mat(data_df[data_df['Y'] == 1].drop(['Y'], axis=1))
scatter_mat_0 = (len(data_df[data_df['Y'] == 0]) - 1) * cov_mat_0
scatter_mat_1 = (len(data_df[data_df['Y'] == 1]) - 1) * cov_mat_1
scatter_within = scatter_mat_0 + scatter_mat_1
scatter_between = np.array(mean_0 - mean_1).reshape(2,1) @ np.array(mean_0 - mean_1).reshape(1,2)

print(scatter_within)
print(scatter_between)
```

```
           x1         x2
x1  10.000000  -4.082622
x2  -4.082622  10.000000
[[ 1.81230578 -0.6640409 ]
 [-0.6640409   0.243309  ]]
```

In [58]:
```python
E, V = np.linalg.eig(np.linalg.inv(scatter_within) @ scatter_between)
print(V[:,0])

proj_data = data_df.drop(['Y'], axis=1) @ V[:,0]
proj_data
```

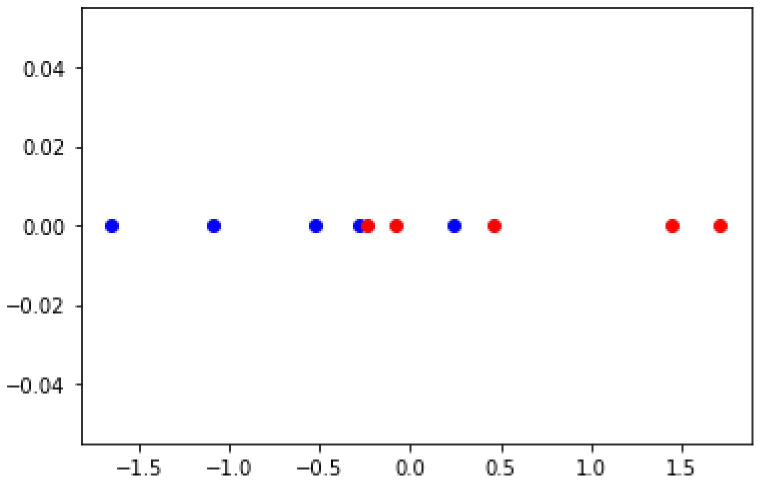
```
[0.99879099 0.04915855]
```

Out[58]:
```
0   -0.278748
1   -1.086363
2   -0.527746
3    0.243497
4   -1.651498
5   -0.230252
6    0.456123
7    1.439993
8   -0.078246
9    1.713239
dtype: float64
```

2.e. Scatter plot projected data

In [59]:
```
plt.scatter(proj_data[:5], np.zeros((1,5)), c='b')
plt.scatter(proj_data[5:], np.zeros((1,5)), c='r')
```

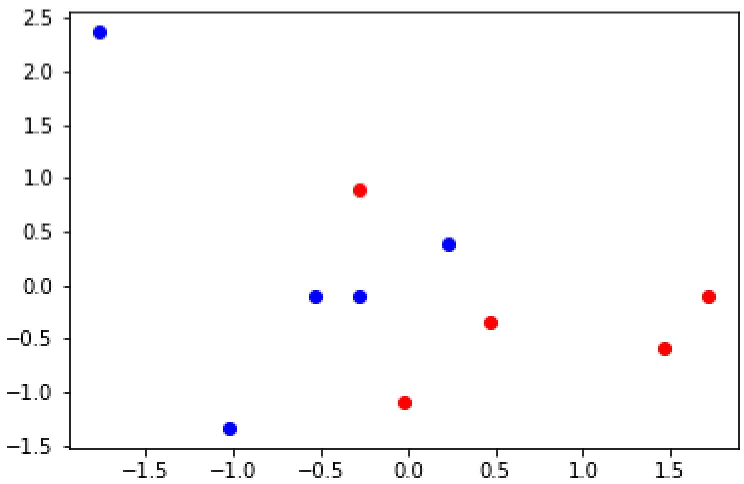
Out[59]: <matplotlib.collections.PathCollection at 0x7fecc73e5eb0>



2.e. Scatter plot original data

In [60]:
```
plt.scatter(data_df.loc[data_df['Y'] == 0, 'x1'], data_df.loc[data_df['Y'] == 0, 'x2'], c = 'b')
plt.scatter(data_df.loc[data_df['Y'] == 1, 'x1'], data_df.loc[data_df['Y'] == 1, 'x2'], c = 'r')
```

Out[60]: <matplotlib.collections.PathCollection at 0x7fecc46c8430>



2. Dimensionality Reduction via PCA

In [21]:
```
from sklearn.datasets import fetch_lfw_people
from sklearn.neighbors import KNeighborsClassifier
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
```

In [22]:
```
people = fetch_lfw_people(min_faces_per_person=20, resize=0.7)
image_shape = people.images[0].shape

fig, axes = plt.subplots(2, 5, figsize=(15,8), subplot_kw={'xticks': (), 'yticks': ()})
for target, image, ax in zip(people.target, people.images, axes.ravel()):
    ax.imshow(image, cmap=cm.gray)
    ax.set_title(people.target_names[target])
```

In [23]:
```python
mask = np.zeros(people.target.shape, dtype=np.bool)
for target in np.unique(people.target):
    mask[np.where(people.target == target)[0][:50]] = 1

X_people = people.data[mask]
y_people = people.target[mask]

X_people /= 255
```

In [24]:
```python
# See how well KNN does with just the pixels alone
X_train, X_test, y_train, y_test = train_test_split(X_people, y_people, stratify=y_people, random_state=0)
```

In [61]:
```python
knn = KNeighborsClassifier(n_neighbors=1, n_jobs=8)
knn.fit(X_train, y_train)
print(f"Test set score of 1-nn: {knn.score(X_test, y_test):.5f}")
```

```
Test set score of 1-nn: 0.23256
```

In [26]:
```
euclid = lambda a, b: np.linalg.norm(a-b)
```

In [27]:
```
def KNN(X_train, y_train, X_test, num_neighbors):
    distances = []
    predictions = []
    for i, test_row in enumerate(X_test):
        distances.clear()
        for j, train_row in enumerate(X_train):
            dist = euclid(test_row, train_row)
            distances.append((j, dist))
        distances.sort(key=lambda tup: tup[1])
        neighbors = list(list(zip(*distances))[0])[:num_neighbors]
        predictions.append(max(set(neighbors), key=neighbors.count))
    return y_train[np.array(predictions)]
```

In [28]:
```
%time y_test_pred = KNN(X_train, y_train, X_test, 1)
```

```
CPU times: user 6.88 s, sys: 251 µs, total: 6.88 s
Wall time: 6.88 s
```

In [29]:
```
sum(y_test_pred - y_test == 0)/len(y_test)
```

Out[29]:
```
0.23255813953488372
```

In [30]:
```
def PCA(data, n_components=100):
    C = cov_mat(data)
    E, V = np.linalg.eig(C)
    return E[:n_components], V[:,:n_components]
```

In [31]:
```
scaler = StandardScaler().fit(X_train)
X_train_std = scaler.transform(X_train)
X_test_std = scaler.transform(X_test)
```

In [32]:
```
%time E, V = PCA(X_train_std, X_train_std.shape[1])
X_train_proj = X_train_std @ V[:,:100]
X_test_proj = X_test_std @ V[:,:100]
```

```
CPU times: user 12min 4s, sys: 10min 23s, total: 22min 28s
Wall time: 1min 49s
```

In [33]:
```
%time y_test_pred = KNN(X_train_proj, y_train, X_test_proj, 1)
```

```
CPU times: user 5.04 s, sys: 854 ms, total: 5.89 s
Wall time: 4.94 s
```

In [34]:
```python
sum(y_test_pred - y_test == 0)/len(y_test)
```

Out[34]:
```
0.25387596899224807
```

In [35]:
```python
X_train_proj_white = X_train_proj @ np.diag(1/np.sqrt(E[:100]))
X_test_proj_white = X_test_proj @ np.diag(1/np.sqrt(E[:100]))
```

In [36]:
```python
%time y_test_pred = KNN(X_train_proj_white, y_train, X_test_proj_white, 1)
```

```
CPU times: user 5.27 s, sys: 728 ms, total: 5.99 s
Wall time: 5.05 s
```

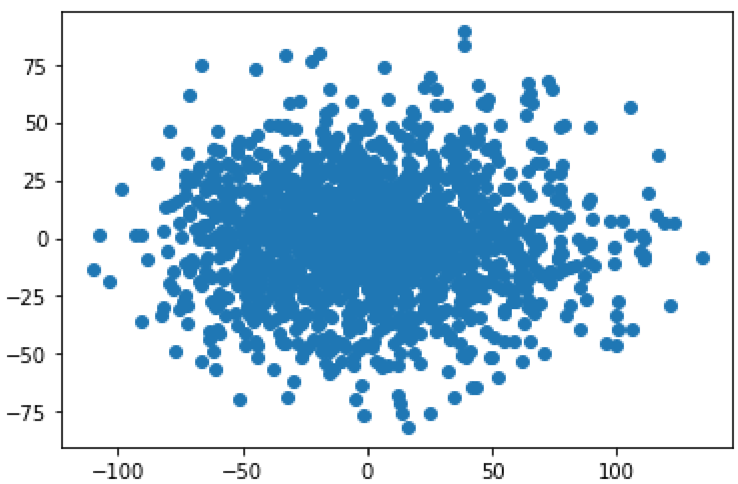
In [37]:
```python
sum(y_test_pred - y_test == 0)/len(y_test)
```

Out[37]:
```
0.3313953488372093
```

2. PC1-PC2 plot

In [38]:
```python
plt.scatter(X_train_proj[:,0], X_train_proj[:,1])
```

Out[38]:
```
<matplotlib.collections.PathCollection at 0x7fecc759ce20>
```



3. Eigenfaces

In [39]:
```
X_train_proj_all = X_train_std @ V
X_test_proj_all = X_test_std @ V
```

In [40]:
```
pc0 = X_train_proj_all[:,0]
pc1 = X_train_proj_all[:,1]

pc0_max, pc0_min = np.argmax(pc0), np.argmin(pc0)
pc1_max, pc1_min = np.argmax(pc1), np.argmin(pc1)
```

3. Visualizing max and min of PC0 and PC1 after projection on X_train

In [41]:
```
fig, axes = plt.subplots(2, 2, figsize=(15,8), subplot_kw={'xticks': (), 'yticks': ()})

for i, t, ax in zip([pc0_max, pc0_min, pc1_max, pc1_min], ["pc0_max", "pc0_min", "pc1_max", "pc1_min"], axes.r
    ax.imshow(X_train[i].reshape(87, 65), cmap=cm.gray)
    ax.set_title(f"{t} {people.target_names[y_train[i]]}")
```

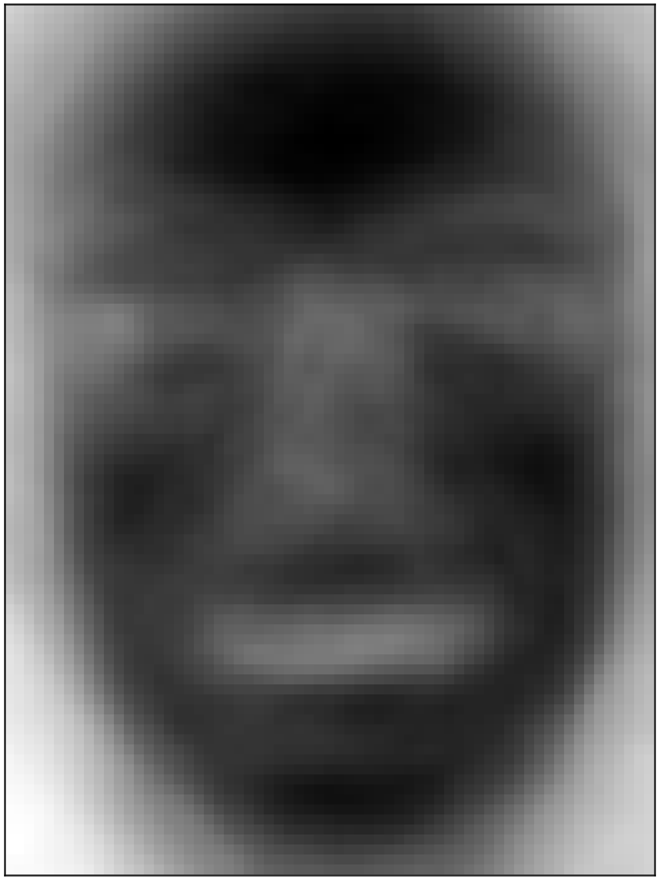
3. Visualize what PC0 captures

In [42]:

```
fig, axes = plt.subplots(1, 1, figsize=(15,8), subplot_kw={'xticks': (), 'yticks': ()})
axes.imshow(V[:,0].reshape(87, 65), cmap=cm.gray)
```

Out[42]:

```
<matplotlib.image.AxesImage at 0x7fecc7420880>
```



In [43]:
```
def PCA_recon(imgs_proj, V, k, variance, mean):
    img = imgs_proj[:,:k].reshape(-1, k)
    recon_img = img @ (V[:,:k].T.reshape(k, -1))
    recon_img = recon_img * np.sqrt(variance) + mean
    return recon_img
```

In [44]:
```
encode_E = 0
sum_E = np.sum(E)
n_components = 0
for n_components in range(len(E)):
    encode_E += E[n_components]
    if encode_E / sum_E > 0.95:
        break
print(n_components)
```

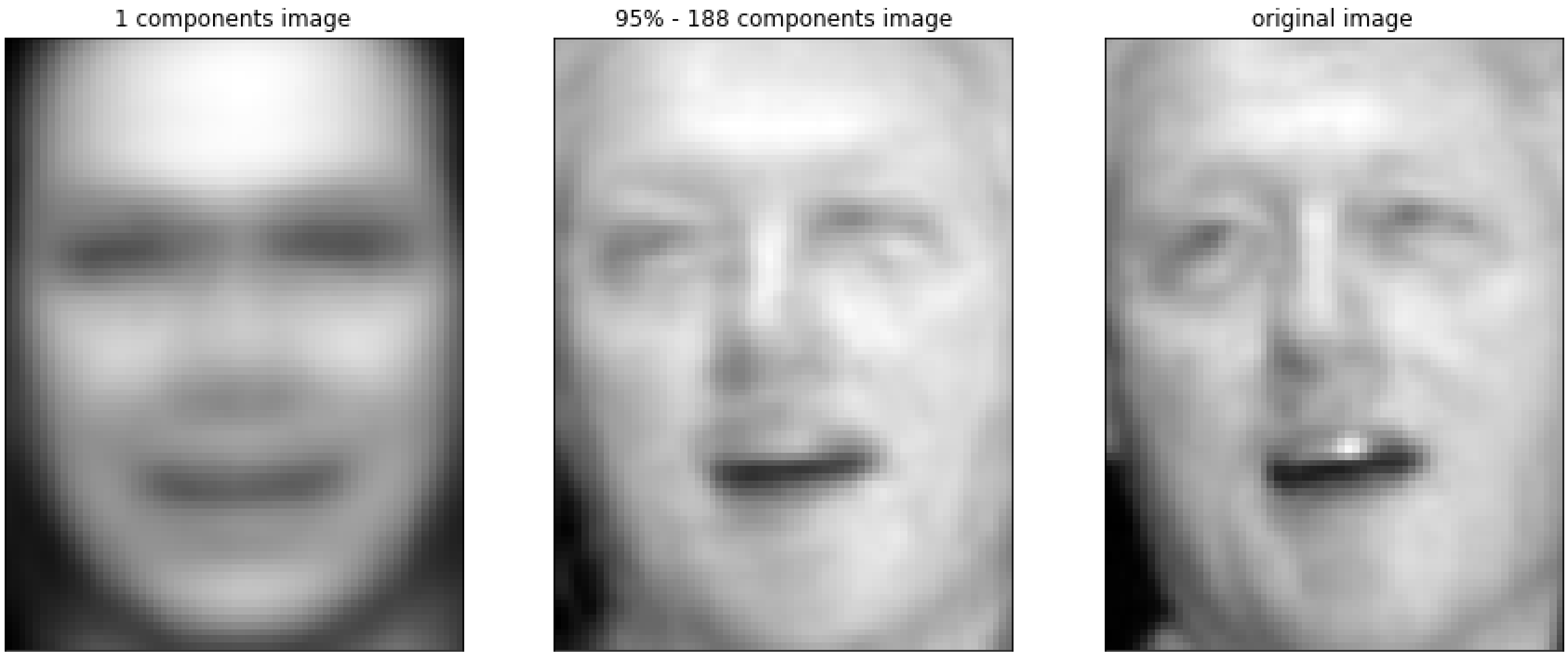
```
188
```

3. Reconstruction

In [45]:

```
recon_img_1_comp = PCA_recon(X_train_proj_all, V, 1, scaler.var_, scaler.mean_)
recon_img_95 = PCA_recon(X_train_proj_all, V, n_components, scaler.var_, scaler.mean_)
original_img = X_train

fig, axes = plt.subplots(1, 3, figsize=(15,8), subplot_kw={'xticks': (), 'yticks': ()})
for i, t, ax in zip([recon_img_1_comp, recon_img_95, original_img], ["1 components", f"95% - {n_components} co
    ax.imshow(i[0,:].reshape(87, 65), cmap=cm.gray)
    ax.set_title(f"{t} image")
```



4. Clustering

In [46]:

```
def assign_points_to_clusters(centroids, dataset):
    clusters = []
    distances = []
    for i, point in enumerate(dataset):
        distances.clear()
        for j, centroid in enumerate(centroids):
            dist = euclid(point, centroid)
```

```
            distances.append((j, dist))
        distances.sort(key=lambda tup: tup[1])
        clusters.append(distances[0][0])
    return np.array(clusters)
```

In [47]:
```
def calc_new_centroids(clusters, dataset):
    new_centroids = []
    for c in set(clusters):
        points_in_cluster = np.where(clusters == c)
        cluster_mean = dataset[points_in_cluster].mean(axis=0)
        new_centroids.append(cluster_mean)
    return np.array(new_centroids)
```

In [48]:
```
def calc_centroids_diff(old_C, new_C):
    dists = []
    for i in range(len(old_C)):
        dists.append(euclid(old_C[i], new_C[i]))
    return np.sum(dists)
```

In [49]:
```
# standardize the data:
scaler_all = StandardScaler().fit(X_people)
X_people_std = scaler_all.transform(X_people)
```

In [50]:
```
%time E, V = PCA(X_people_std, n_components=100)
X_people_proj = X_people_std @ V
```

```
CPU times: user 11min 29s, sys: 11min 11s, total: 22min 41s
Wall time: 1min 54s
```

In [51]:
```
np.random.seed(0)
```

In [52]:
```
# randomly choose k points as k centroids
centroids_change = 100
eps = 2e-23
k = 10
itr = 1
max_itr = 10000
centroids = X_people_proj[np.random.randint(0, X_people_proj.shape[0], size=k)]
new_centroids = None

while True:
    clusters = assign_points_to_clusters(centroids, X_people_proj)
    new_centroids = calc_new_centroids(clusters, X_people_proj)
    centroids_change = calc_centroids_diff(centroids, new_centroids)
```

```python
    if centroids_change < eps or itr > max_itr:
        break

    centroids = new_centroids
    print(f"Iter: {itr}, Change: {centroids_change}")
    itr += 1
```

```
Iter: 1, Change: 442.5361633300781
Iter: 2, Change: 128.02601623535156
Iter: 3, Change: 78.1665267944336
Iter: 4, Change: 44.924957275390625
Iter: 5, Change: 29.543842315673828
Iter: 6, Change: 22.499788284301758
Iter: 7, Change: 16.453794479370117
Iter: 8, Change: 14.850354194641113
Iter: 9, Change: 12.765664100646973
Iter: 10, Change: 10.536905288696289
Iter: 11, Change: 9.19281005859375
Iter: 12, Change: 7.387046813964844
Iter: 13, Change: 7.967870235443115
Iter: 14, Change: 7.730221748352051
Iter: 15, Change: 5.344476699829102
Iter: 16, Change: 4.699371337890625
Iter: 17, Change: 4.794102191925049
Iter: 18, Change: 4.1309685707092285
Iter: 19, Change: 4.912514686584473
Iter: 20, Change: 2.7934107780456543
Iter: 21, Change: 1.4903690814971924
Iter: 22, Change: 1.528502345085144
Iter: 23, Change: 1.1923229694366455
Iter: 24, Change: 1.993759274482727
Iter: 25, Change: 2.0061607360839844
Iter: 26, Change: 1.9300038814544678
Iter: 27, Change: 4.196130275726318
Iter: 28, Change: 2.5771408081054688
Iter: 29, Change: 1.7259509563446045
```

In [53]:

```python
report_imgs = []

for i, C in enumerate(new_centroids):
    report_imgs.append(C)
    points_in_cluster = np.where(clusters == i)[0]
    distances = []
    for j, p in enumerate(points_in_cluster):
        distances.append((j, euclid(X_people_proj[p], C)))
    distances.sort(key=lambda tup: tup[1])
    closest = distances[0][0]
```

```
    farthest = distances[-1][0]
    report_imgs.append(X_people_proj[points_in_cluster[closest]])
    report_imgs.append(X_people_proj[points_in_cluster[farthest]])

report_imgs = np.array(report_imgs)
recon_imgs_100_comp = PCA_recon(report_imgs, V, 100, scaler_all.var_, scaler_all.mean_)
```

4. The visualization of k-means cluster centers, and the min and max images

In [54]:

```
fig, axes = plt.subplots(10, 3, figsize=(30,18), subplot_kw={'xticks': (), 'yticks': ()})


for i, ax in zip(recon_imgs_100_comp, axes.ravel()):
    ax.imshow(i.reshape(87, 65), cmap=cm.gray)

for title, ax in zip(["Centroid", "Closest to centroid in cluster", "Furthest to centroid in cluster"], axes.f
    ax.axis("off")
    ax.set_title(f"{title}", fontweight='bold')
```


Centroid
Closest to centroid in cluster
Furthest to centroid in cluster

In [ ]:

```

```