Tai Duc Nguyen - ECEC487 - HW1 - 09/29/2019

Contents

Computer Exercises (CX)

```
seed = 0
randn('seed',seed);
```

```
seed =

0
```

HW CX 3.1

```
clear; close all;

% Prepare the variables:
N = 200;

m1 = [-5;0];
m2 = [5;0];
m = [m1 m2];

S1 = eye(2)*1;
S2 = S1;
S(:,:,1) = S1; S(:,:,2) = S2;

P = [1/2;1/2];

% Generate data set X1, X1, and class assignments y1, y1
```

```
% Generate data set X1, X1_ and class assignments y1, y1_
[X1, y1] = generate_gauss_classes(m, S, P, N);
[X1_, y1_] = generate_gauss_classes(m, S, P, N);

[~,c]=size(X1);
w0 = ones(c,1);
y_temp1 = y1;
y_temp1(y_temp1==2) = -1;

y_temp1_ = y1_;
y_temp1_(y_temp1_==2) = -1;

r_plt = 3;
c_plt = 2;
figure()

% Train the data with the Perception algorithm to create a classifier

w1_per = perceptron_train(X1,y_temp1,w0);
plot_data(X1, y1, m, w1_per, w0, "Data and decision surface for X1 dataset using Perceptr

w1_per = perceptron_train(X1_,y_temp1_,w0);
plot_data(X1_, y1_, m, w1_per, w0, "Data and decision surface for X1' dataset using Perce

% Train the data with the Sum of Error Squares algorithm to create a classifier
w1_ses = SSErr_train(X1,y_temp1);
plot_data(X1, y1, m, w1_ses, w0, "Data and decision surface for X1 dataset using Sum of E

w1_ses = SSErr_train(X1_,y_temp1_);
plot_data(X1_, y1_, m, w1_ses, w0, "Data and decision surface for X1' dataset using Sum c

% Train the data with the LMS algorithm to create a classifier
w1_lms = LMSalg_train(X1,y_temp1,w0);
plot_data(X1, y1, m, w1_lms, w0, "Data and decision surface for X1 dataset using LMS algc

w1_lms = LMSalg_train(X1_,y_temp1_,w0);
plot_data(X1_, y1_, m, w1_lms, w0, "Data and decision surface for X1' dataset using LMS a
```

HW CX 3.2

```
% Prepare the variables:
N = 200;

m1 = [-2;0];
m2 = [2;0];
m = [m1 m2];

S1 = eye(2)*1;
S2 = S1;
S(:,:,1) = S1; S(:,:,2) = S2;

P = [1/2;1/2];
```

```
% Generate data set X2, X2_ and class assignments y2, y2_
[X2, y2] = generate_gauss_classes(m, S, P, N);
[X2_, y2_] = generate_gauss_classes(m, S, P, N);

[~,c]=size(X2);
w0 = ones(c,1);
y_temp2 = y2;
y_temp2(y_temp2==2) = -1;

y_temp2_ = y2_;
y_temp2_(y_temp2_==2) = -1;

r_plt = 3;

c_plt = 2;
figure()

% Train the data with the Perception algorithm to create a classifier
w2_per = perceptron_train(X2,y_temp2,w0);
plot_data(X2, y2, m, w2_per, w0, "Data and decision surface for X2 dataset using Perceptr

w2_per = perceptron_train(X2_,y_temp2_,w0);
plot_data(X2_, y2_, m, w2_per, w0, "Data and decision surface for X2' dataset using Perce


% Train the data with the Sum of Error Squares algorithm to create a classifier
w2_ses = SSErr_train(X2,y_temp2);
plot_data(X2, y2, m, w2_ses, w0, "Data and decision surface for X2 dataset using Sum of E

w2_ses = SSErr_train(X2_,y_temp2_);
plot_data(X2_, y2_, m, w2_ses, w0, "Data and decision surface for X2' dataset using Sum c


% Train the data with the LMS algorithm to create a classifier
w2_lms = LMSalg_train(X2,y_temp2,w0);
plot_data(X2, y2, m, w2_lms, w0, "Data and decision surface for X2 dataset using LMS algc

w2_lms = LMSalg_train(X2_,y_temp2_,w0);
plot_data(X2_, y2_, m, w2_lms, w0, "Data and decision surface for X2' dataset using LMS a
```

Using the data generation procedures from textbook CX 2.3 Page 80 with slight modifications

```
function [X,y] = generate_gauss_classes(m,S,P,N)
[~,c]=size(m);
X=[];
y=[];
    for j=1:c
        % Generating the [p(j)*N)] vectors from each distribution
        t=mvnrnd(m(:,j),S(:,:,j),fix(P(j)*N));
```

```
        % The total number of points may be slightly less than N
        % due to the fix operator
        X=[X; t];
        y=[y ones(1,fix(P(j)*N))*j];
    end
end
```

Using the data plotting procedures from textbook CX 2.4 Page 80 with slight modifications

```
function plot_data(X,y,m,w,w0,TLE,r_plt,c_plt,iplot)
    [N,~]=size(X); % N=no. of data vectors, l=dimensionality
    [l,c]=size(m); % c=no. of classes
    if(l ~= 2)
        fprintf('NO PLOT CAN BE GENERATED\n')
        return
    else
        pale=['r.'; 'g.'; 'b.'; 'y.'; 'm.'; 'c.'];
        subplot(r_plt,c_plt,iplot);
        % Plot of the data vectors
        hold on
        X1 = X(:,1);
        X2 = X(:,2);
        for j=1:c
            scatter(X1(y == j),X2(y == j),pale(j,:))
        end
        scatter(m(1,:),m(2,:),'k+')
    end

    decision_x = linspace(min(X1), max(X1));
    decision_y = -(w(1)/w(2))*decision_x - (w0/w(2));
    plot(decision_x, decision_y, "k");
    hold off
    title(TLE)
    xlabel("x_{1}")
    ylabel("x_{2}")
    ylim([min(X2) max(X2)])
    legend("class 1", "class 2", "class's mean", "linear decision boundary")
end
```

Using the Perception algorithm from textbook CX 3.1 Page 145 with slight modifications

```
function w = perceptron_train(X,y,w_ini)
    [N,c]=size(X);
    max_iter=10000;     % Maximum allowable number of iterations
    rho=0.05;           % Learning rate
    w=w_ini;            % Initialization of the parameter vector
```

```
    iter=0;             % Iteration counter
    mis_clas=N;         % Number of misclassified vectors
    while (mis_clas>0) && (iter<max_iter)
        iter=iter+1;
        mis_clas=0;
        gradi=zeros(c,1);   % Computation of the "gradient" term
        for i=1:N
            if ((X(i,:)*w)*y(i)>0)
                mis_clas=mis_clas+1;
                gradi=gradi+rho*(-y(i)*X(i,:)');
            end
        end
        w=w-rho*gradi;      % Updating the parameter vector
    end
end
```

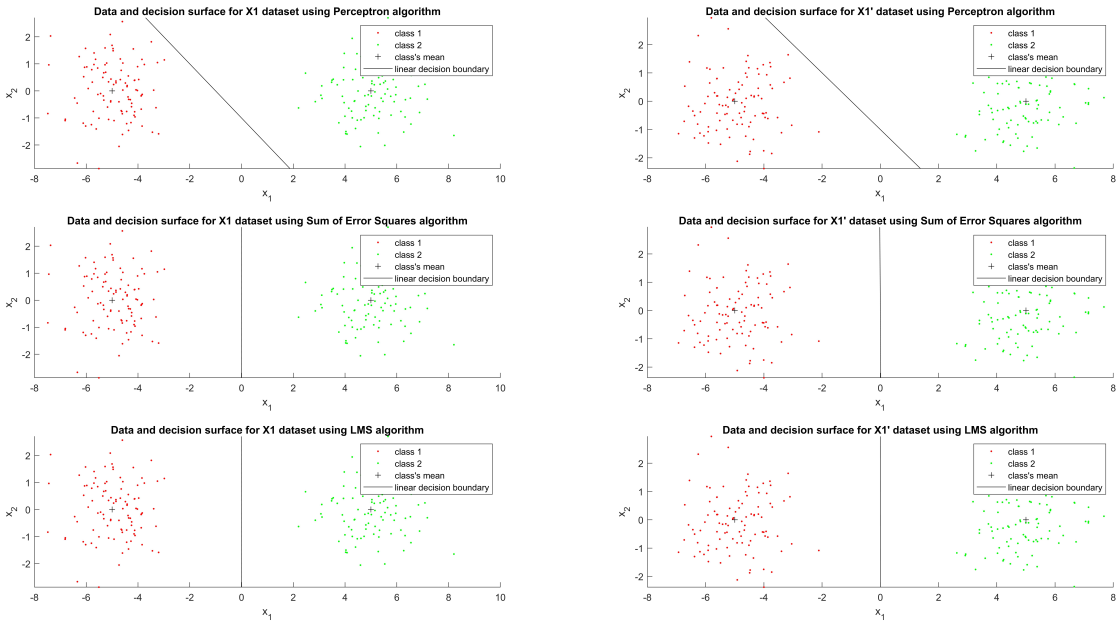
Using the Sum of Error Squares algorithm from textbook CX 3.2 Page 145 with slight modifications

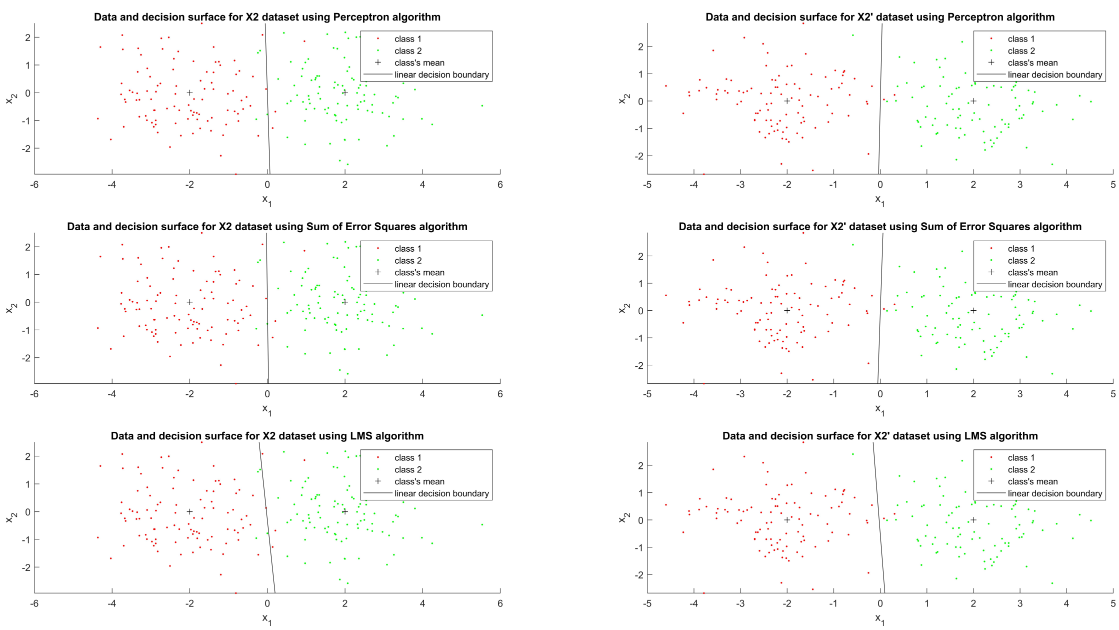
```
function w = SSErr_train(X,y)
    w = (X'*X)'*X'*y';
end
```

Using the LMS algorithm from textbook CX 3.3 Page 146 with slight modifications

```
function w = LMSalg_train(X,y,w_ini)
    [N,~]=size(X);
    rho=0.1; % Learning rate initialization
    w=w_ini; % Initialization of the parameter vector
    for i=1:N
        w=w-(rho/i)*(y(i)-X(i,:)*w)*X(i,:)';
    end
end
```

Conclusion and remarks





From the results of the experiment 1 (CX 3.1), it is observed that all 3 algorithms produces such vector w that has 100% accuracy after training. This is due to the large separation of the data points into their own class ($m1 = [-5, 0]$ & $m2 = [5, 0]$ and $S1 = S2 = I$)

However, in experiment 2 (CX 3.2), $m1 = [-2, 0]$ & $m2 = [2, 0]$ and $S1 = S2 = I$, which makes the data points of both classes overlaps -- there is no clear boundaries separating the two classes. It is apparent that the performance of all 3 algorithms are very similar:

- Perceptron: 6 misses in X2 and 3 misses in X2'
- SES: 7 misses in X2 and 3 misses in X2'
- LMS: 8 misses in X2 and 3 misses in X2'

The differences in the amount of miss classification for each algorithm is statistically small, which means that the performance of all 3 algorithms are the same.

Written Homework

Question

Explain why the perceptron cost function is a continuous piecewise linear function.

Answer

To understand why the perceptron cost function (PCF) is a *continuous* and *piecewise linear* function, we need to revise the Perceptron Algorithm:

- Chose $w(0)$ and ρ_0 randomly (at $t = 0$)
- While Y is not $\emptyset$: (*)
  - Y = $\emptyset$
  - Classify training set using $w(t)$ and ρ_t. (**)
  - Wrong classification will be added to set Y
  - Adjust $w(t+1)$
  - Adjust ρ_t
  - $t = t + 1$
- End While

Assume that the algorithm works and coverges to a solution (a.k.a $Y = \emptyset$ after step (**)), then, naturally, after each iteration of the outer loop (*), the number of elements in set Y will ideally reduce. Hence, while the the number of elements in set Y stay the same, $J(\mathbf{w}) = \sum_{x \in y}(\delta_x \mathbf{w}^T \mathbf{x})$ is a **linear** function. However, when number of elements in set Y changes, $\frac{\partial J(w)}{\partial w}|_{w=w(t)}$ is undefined and therefore, **discontinuous**. Hence, the perceptron cost function is a **continuous piecewise linear** function.