QRW-Khánh Linh

1 A

$$\sum_j \hat{c}^\dagger_{j+1,r}\hat{c}_{j,b} = \frac{1}{2\pi}\sum_j \int_{-\pi}^{+\pi} e^{ik_1(j+1)}\hat{c}^\dagger_{k_1,r}dk_1 \int_{-\pi}^{+\pi} e^{-ik_2 j}\hat{c}_{k_2,b}dk_2 = \frac{1}{2\pi}\iint_{-\pi}^{\pi}\sum_j e^{ik_1}e^{i(k_1-k_2)j}\hat{c}^\dagger_{k_1,r}\hat{c}_{k_2,b}dk_1dk_2$$

$$= \iint_{-\pi}^{\pi} e^{ik_1}\delta(k_1-k_2)\hat{c}^\dagger_{k_1,r}\hat{c}_{k_2,b}dk_1dk_2 = \int_{-\pi}^{\pi} dk_1\, e^{ik_1}\hat{c}^\dagger_{k_1,r}\hat{c}_{k_1,b}$$

$$\Rightarrow \sum_j \hat{c}^\dagger_{j+1,r}\hat{c}_{j,b} + \hat{c}^\dagger_{j,r}\hat{c}_{j+1,b} = \int_{-\pi}^{\pi} dk\,(e^{ik}-e^{-ik})\hat{c}^\dagger_{k,r}\hat{c}_{k,b} = \int_{-\pi}^{\pi} dk\, 2i\sin k\,\hat{c}^\dagger_{k,r}\hat{c}_{k,b}$$

$$\Rightarrow H(k) = \begin{bmatrix} 2\cos k & 2i\sin k \\ -2i\sin k & -2\cos k\end{bmatrix}$$

Xét phương trình

$$\begin{vmatrix} 2\cos k - \lambda(k) & 2i\sin k \\ -2i\sin k & -2\cos k - \lambda(k)\end{vmatrix} = 0 \Leftrightarrow -4\cos^2 k + \lambda^2(k) - 4\sin^2 k = 0 \Leftrightarrow \lambda(k) = \pm 2$$

$$\Rightarrow \begin{cases}\lambda_1 = 2 \\ \lambda_2 = -2\end{cases}$$

Ta có

$$\hat{H}(k)\,|g_n(k)\rangle = \lambda_n\,|g_n(k)\rangle$$

$$\Rightarrow \begin{cases}(e^{ik}+e^{-ik})u + (e^{ik}-e^{-ik})v = 2u \\ (-e^{ik}+e^{-ik})u - (e^{ik}+e^{-ik})v = 2v\end{cases} \Leftrightarrow \begin{cases}(e^{2ik}-2e^{ik}+1)u + (e^{2ik}-1)v = 0 \\ (-e^{2ik}+1)u - (e^{2ik}+2e^{ik}+1)v = 0\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}(e^{ik}-1)u + (e^{ik}+1)v = 0 \\ (-e^{ik}+1)u - (e^{ik}+1)v = 0\end{cases} \Leftrightarrow |g_1(k)\rangle = \begin{pmatrix} e^{ik}+1 \\ 1-e^{ik}\end{pmatrix}.$$

Tương tự ta có

$$|g_2(k)\rangle = \begin{pmatrix} 1-e^{ik} \\ 1+e^{ik}\end{pmatrix}.$$

2 B

$$|w_n(j)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} dk\, e^{ik.j}\,|g_n(k)\rangle$$

$$|w_1(j)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} dk\, e^{ikj}\begin{pmatrix}1+e^{ik} \\ 1-e^{ik}\end{pmatrix} = \frac{1}{\sqrt{2\pi}}\begin{pmatrix}\dfrac{e^{i\pi j}-e^{-i\pi j}}{ij} + \dfrac{e^{i\pi(j+1)}-e^{-i\pi(j+1)}}{i(j+1)} \\ \dfrac{e^{i\pi j}-e^{-i\pi j}}{ij} - \dfrac{e^{i\pi(j+1)}-e^{-i\pi(j+1)}}{i(j+1)}\end{pmatrix}$$

TH1. $j \notin \{0,-1\}$

$$|w_1(j)\rangle = \begin{pmatrix}0\\0\end{pmatrix}$$

TH2. $j=0$

$$|w_1(0)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} dk \begin{pmatrix} 1+e^{ik} \\ 1-e^{ik} \end{pmatrix} = \begin{pmatrix} \sqrt{2\pi} \\ \sqrt{2\pi} \end{pmatrix}$$

TH3. $j=-1$

$$|w_1(0)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} dk \begin{pmatrix} 1+e^{-ik} \\ 1-e^{-ik} \end{pmatrix} = \begin{pmatrix} \sqrt{2\pi} \\ \sqrt{2\pi} \end{pmatrix}$$

Tương tự ta có

$$|w_2(j)\rangle = \begin{cases} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & , x \notin \{0,-1\} \\ \begin{pmatrix} \sqrt{2\pi} \\ \sqrt{2\pi} \end{pmatrix} & , x \in \{0,-1\} \end{cases}$$

Vậy "the electrons will also be localized on the Creutz ladder".

3 C

$$\sum_j |j-1,3R\rangle\langle j,1R| = \sum_j \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(j-1)k_1}|k_1,3R\rangle dk_1 \int_{-\pi}^{\pi} e^{ijk_2k_1}\langle k_2,1R| dk_2$$

$$= \frac{1}{2\pi}\iint_{-\pi}^{\pi}\sum_j |k_1,3R\rangle\langle k_2,1R| e^{i(j-1)k_1}e^{ijk_2}dk_1\,dk_2 = \iint_{-\pi}^{\pi} |k_1,3R\rangle\langle k_2,1R| e^{-ik_1}\delta(k_2-k_1)dk_1dk_2$$

$$= \int_{-\pi}^{\pi} e^{-ik_1}|k_1,3R\rangle\langle k_1,1R| dk_1$$

Biến đổi Fourier Transform của Id

$\sum_{j=1}^{4}\sum_{i'}|i',j,R/B\rangle\langle i',j,R/B| = \sum_{j=1}^{4}\int_{-\infty}^{\infty} dk\,|k,jR/B\rangle\langle k,jR/B|$

$$\begin{aligned} S = \int_{-\pi}^{\pi} dk\,(&-e^{-ik}|k,3R\rangle\langle k,1R| - e^{ik}|k,1R\rangle\langle k,3R| - e^{-ik}|k,4B\rangle\langle k,2R| && (1)\\ &+ e^{ik}|k,2B\rangle\langle k,4R| + e^{-ik}|k,3B\rangle\langle k,1B| + e^{-ik}|k,1B\rangle\langle k,3B| && (2)\\ &+ e^{-ik}|k,3B\rangle\langle k,1B| - e^{ik}|k,2R\rangle\langle k,4B|) && (3) \end{aligned}$$

$$\begin{aligned} W = SC = \int_{-\pi}^{\pi} dk\,(&-e^{-ik}|k,3R\rangle\langle k,1R| - e^{ik}|k,1R\rangle\langle k,3R| - e^{-ik}|k,4B\rangle\langle k,2R| && (4)\\ &+ e^{ik}|k,2B\rangle\langle k,4R| + e^{-ik}|k,3B\rangle\langle k,1B| + e^{-ik}|k,1B\rangle\langle k,3B| && (5)\\ &+ e^{-ik}|k,3B\rangle\langle k,1B| - e^{ik}|k,2R\rangle\langle k,4B|)C && (6)\\ = \int_{-\pi}^{\pi} dk\,(&e^{-ik}|k,3R\rangle\langle k,1R| + e^{ik}|k,1R\rangle\langle k,3R| + e^{-ik}|k,4B\rangle\langle k,2R| && (7)\\ &+ e^{ik}|k,2B\rangle\langle k,4R| - e^{-ik}|k,3B\rangle\langle k,1B| - e^{-ik}|k,1B\rangle\langle k,3B| && (8)\\ &- e^{-ik}|k,3B\rangle\langle k,1B| - e^{ik}|k,2R\rangle\langle k,4B|) && (9) \end{aligned}$$

Chưa xong!

4 D

$$|\psi_n,t\rangle = F|\psi_{n-1},t\rangle + B|\psi_{n+1},t\rangle$$

$$F = \int_{-\pi}^{\pi} dk\,(-e^{-ik}|k,3R\rangle\langle k,1R| - e^{-ik}|k,4B\rangle\langle k,2R| + e^{-ik}|k,3B\rangle\langle k,1B| + e^{-ik}|k,3B\rangle\langle k,1B|)C$$

$$B = \int_{-\pi}^{\pi} dk\, (-e^{ik} |k, 1R\rangle \langle k, 3R| + e^{ik} |k, 2B\rangle \langle k, 4R| + e^{-ik} |k, 1B\rangle \langle k, 3B| - e^{ik} |k, 2R\rangle \langle k, 4B|)C$$

$$|\psi_3, t=3\rangle = F|\psi_2, t=2\rangle + B|\psi_4, t=2\rangle = F|\psi_1, t=1\rangle + F|\psi_3, t=1\rangle + B|\psi_3, t=1\rangle + B|\psi_5, t=1\rangle \tag{10}$$

$$= F|\psi_0, t=0\rangle + F|\psi_2, t=0\rangle + F|\psi_2, t=0\rangle + F|\psi_4, t=0\rangle + \tag{11}$$

$$F' = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = G_4 \oplus G_4 = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 \end{bmatrix}$$

$F = F'G_4 \oplus G_4 =] [$

$$F = F'C = \begin{pmatrix} -1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = B'C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$J_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = J_F$$

	FF	FB	BF	BB
FF	0	*	0	*
FB	0	*	0	*
BF	*	0	*	0
BB	*	0	*	0

biến đổi fourier

$$|i> \longrightarrow \frac{1}{\sqrt{2\pi}} \sum_k e^{ikj} |k\rangle$$

$$|i-1> \longrightarrow \frac{1}{\sqrt{2\pi}} \sum_k e^{ik(j-1)} |k\rangle$$

$$|i+1> \longrightarrow \frac{1}{\sqrt{2\pi}} \sum_k e^{ik(j+1)} |k\rangle$$

$$\text{chuyển đổi}: \ -|i+1, 1R> < i, 3R| = -e^{ik} |k, 1R> < k, 3R|$$

$$\text{chuyển đổi}: \ -|i+1, 3R> < i, 1R| = -e^{ik} |k, 3R> < k, 1R|$$

Biến đổi tương tự đối với các cặp trạng thái khác, ta nhận được một biểu diễn $\hat{S}$ mới là:

$$\hat{S} = -e^{-ik} |k, 3R\rangle \langle k, 1R| - e^{ik} |k, 1R\rangle \langle k, 3R| - e^{-ik} |k, 4B\rangle \langle k, 2R| + e^{ik} |k, 2B\rangle \langle k, 4R| \tag{12}$$

$$+e^{-ik} |k, 3B\rangle \langle k, 1B| + e^{ik} |k, 1B\rangle \langle k, 3B| + e^{-ik} |k, 4R\rangle \langle k, 2B| - e^{ik} |k, 2R\rangle \langle k, 4B|) \tag{13}$$

Từ đó ta có thể rút ra các trạng thái dưới dạng một ma trận là:

$$\begin{bmatrix} 0 & 0 & -e^{ik} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{ik} \\ -e^{-ik} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-ik} & 0 & 0 \\ 0 & 0 & 0 & e^{ik} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-ik} & 0 & 0 & 0 \\ 0 & -e^{-ik} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

xét các biểu hiện đặc trưng của hệ, ta có thể chia làm hai trường hợp:

4.0.1 B-Backward

tương ứng với chuyển dộng của hạt từ vị trí $(i+1)$ tới i nên có thể xác định rằng:

$$B = -\sum_i \left(|i+1, 1R\rangle \langle 1, 3R| + |i+1, 2B\rangle \langle i, 4R| + |i+1, 1B\rangle \langle i, 3B| - |i+1, 2R\rangle \langle i, 4B| \right)$$

4.0.2 F-Forward

tương ứng với chuyển động đi từ điểm $i-1$ đến vị trí i nên có thể xác định rằng :

$$F = \sum_i -|i-1, 3R\rangle \langle i, 1R| - |i-1, 4B\rangle \langle i, 2R| + |i-1, 3B\rangle \langle i, 1B| + |i-1, 4R\rangle \langle i, 2B|$$

từ đó ta có thể chuyển hóa thành dạng ma trận như sau :