

Laboration report in Computational Statistics

Computer lab 1 block 1

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1 Assignment 3. Logistic regression and basis function expansion

1.1 Data

The data contains information about the onset of diabetes within 5 years in Pima Indians given medical details. The variables are:

- Number of times pregnant
- Plasma glucose concentration a 2 hours in an oral glucose tolerance test.
- Diastolic blood pressure (mm Hg).
- Triceps skinfold thickness (mm).
- 2-Hour serum insulin (μ U/ml).
- Body mass index (weight in kg/(height in m)²).
- Diabetes pedigree function.
- Age (years).
- Diabetes (0=no or 1=yes).

```
diabetes_df <- read.csv("pima-indians-diabetes.csv", header=FALSE)

colnames(diabetes_df) <- c("times_pregnant", "plasma_glucose_conc",
                           "diastolic_blood_pressure", "triceps_skinfold_thickness",
                           "serum_insulin", "body_mass_index", "diabetes_pedigree",
                           "age", "diabetes")

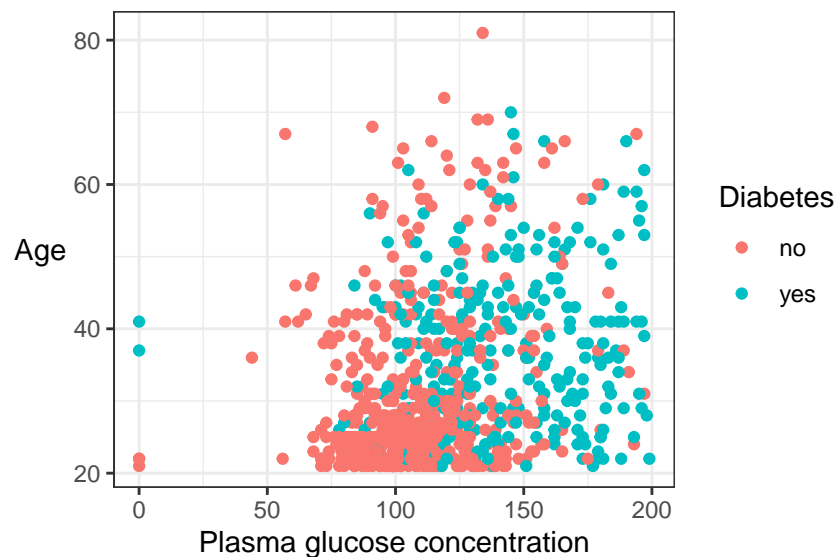
diabetes_df$diabetes <- ifelse(diabetes_df$diabetes == 0, "no", "yes")
diabetes_df$diabetes <- as.factor(diabetes_df$diabetes)
```

1.2 3.1

Question: Make a scatterplot showing a Plasma glucose concentration on Age where observations are colored by Diabetes levels.

```
library(ggplot2)

ggplot(diabetes_df, aes(x = plasma_glucose_conc, y = age, color = diabetes)) +
  geom_point() +
  theme_bw() +
  theme(axis.title.y = element_text(angle = 0, vjust = 0.5)) +
  labs(colour = "Diabetes",
       x = "Plasma glucose concentration",
       y = "Age")
```



Question: Do you think that Diabetes is easy to classify by a standard logistic regression model that uses these two variables as features? Motivate your answer.

We don't think these two variables are good variables to classify Diabetes because there is no clear relationship between age, plasma glucose concentration with Diabetes.

1.3 3.2

Question:

Train a logistic regression model with $y = \text{Diabetes}$ as target $x_1 = \text{Plasma glucose concentration}$ and $x_2 = \text{Age}$ as features and make a prediction for all observations by using $r = 0.5$ as the classification threshold. Report the probabilistic equation of the estimated model and compute also the training misclassification error.

The probabilistic equation:

\$\$\$\$

```
model <- glm(diabetes ~ plasma_glucose_conc + age, data = diabetes_df,
             family = "binomial")

pred <- predict(model, newdata = diabetes_df, type = "response")

# Using 0.5 as the classification threshold
pred <- ifelse(pred > 0.5, "yes", "no")

confusion <- table(diabetes_df$diabetes, pred)
misclass_rate <- (confusion[1,2] + confusion[2,1]) / sum(confusion)

knitr::kable(as.data.frame(round(misclass_rate,2)), col.names = "Misclassification error",
             caption = "Misclassification error")
```

Table 1: Misclassification error

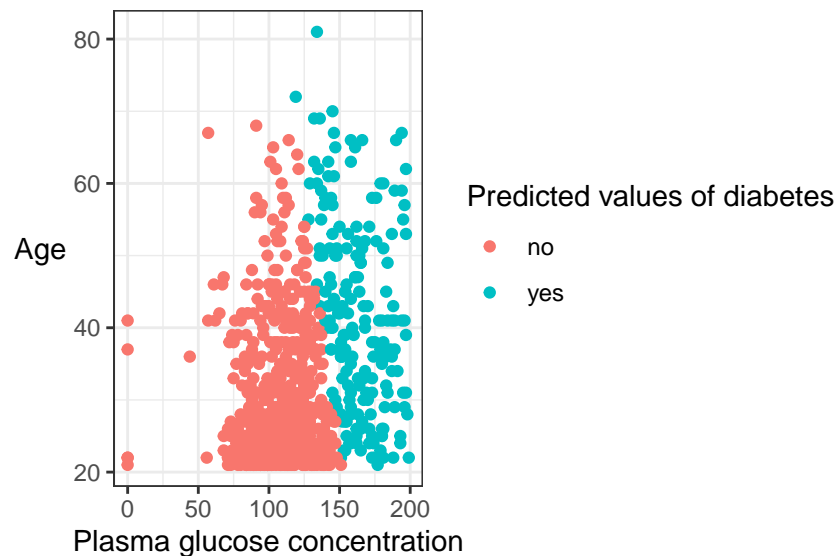
Misclassification error
0.26

Question:

Make a scatter plot of the same kind as in step 1 but showing the predicted values of Diabetes as a color instead.

```
diabetes_df_pred <- diabetes_df
diabetes_df_pred$pred <- pred

ggplot(diabetes_df_pred, aes(x = plasma_glucose_conc, y = age, color = pred)) +
  geom_point() +
  theme_bw() +
  theme(axis.title.y = element_text(angle = 0, vjust = 0.5)) +
  labs(colour = "Predicted values of diabetes",
       x = "Plasma glucose concentration",
       y = "Age")
```



Question:

Comment on the quality of the classification by using these results.

1.4 3.3

Question:

Use the model estimated in step 2 to a) report the equation of the decision boundary between the two classes
b) add a curve showing this boundary to the scatter plot in step 2.

The decision boundary equation:

$$\beta_0 + \beta_1 \cdot \text{plasma_glucose_conc}$$

```
summary(model)
```

```
##
## Call:
## glm(formula = diabetes ~ plasma_glucose_conc + age, family = "binomial",
##      data = diabetes_df)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.3367  -0.7775  -0.5087   0.8367   3.1630
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -5.912449    0.462620  -12.78  < 2e-16 ***
## plasma_glucose_conc  0.035644    0.003290   10.83  < 2e-16 ***
## age              0.024778    0.007374    3.36 0.000778 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 993.48  on 767  degrees of freedom
## Residual deviance: 797.36  on 765  degrees of freedom
## AIC: 803.36
##
## Number of Fisher Scoring iterations: 4
```

```
slope <- coef(model)[2]/(-coef(model)[3])
intercept <- coef(model)[1]/(-coef(model)[3])
```

```
diabetes_df
```

```
##      times_pregnant plasma_glucose_conc diastolic_blood_pressure
## 1                6                148                72
## 2                1                 85                66
## 3                8                183                64
## 4                1                 89                66
## 5                0                137                40
## 6                5                116                74
## 7                3                 78                50
```

## 8	10	115	0
## 9	2	197	70
## 10	8	125	96
## 11	4	110	92
## 12	10	168	74
## 13	10	139	80
## 14	1	189	60
## 15	5	166	72
## 16	7	100	0
## 17	0	118	84
## 18	7	107	74
## 19	1	103	30
## 20	1	115	70
## 21	3	126	88
## 22	8	99	84
## 23	7	196	90
## 24	9	119	80
## 25	11	143	94
## 26	10	125	70
## 27	7	147	76
## 28	1	97	66
## 29	13	145	82
## 30	5	117	92
## 31	5	109	75
## 32	3	158	76
## 33	3	88	58
## 34	6	92	92
## 35	10	122	78
## 36	4	103	60
## 37	11	138	76
## 38	9	102	76
## 39	2	90	68
## 40	4	111	72
## 41	3	180	64
## 42	7	133	84
## 43	7	106	92
## 44	9	171	110
## 45	7	159	64
## 46	0	180	66
## 47	1	146	56
## 48	2	71	70
## 49	7	103	66
## 50	7	105	0
## 51	1	103	80
## 52	1	101	50
## 53	5	88	66
## 54	8	176	90
## 55	7	150	66
## 56	1	73	50

## 57	7	187	68
## 58	0	100	88
## 59	0	146	82
## 60	0	105	64
## 61	2	84	0
## 62	8	133	72
## 63	5	44	62
## 64	2	141	58
## 65	7	114	66
## 66	5	99	74
## 67	0	109	88
## 68	2	109	92
## 69	1	95	66
## 70	4	146	85
## 71	2	100	66
## 72	5	139	64
## 73	13	126	90
## 74	4	129	86
## 75	1	79	75
## 76	1	0	48
## 77	7	62	78
## 78	5	95	72
## 79	0	131	0
## 80	2	112	66
## 81	3	113	44
## 82	2	74	0
## 83	7	83	78
## 84	0	101	65
## 85	5	137	108
## 86	2	110	74
## 87	13	106	72
## 88	2	100	68
## 89	15	136	70
## 90	1	107	68
## 91	1	80	55
## 92	4	123	80
## 93	7	81	78
## 94	4	134	72
## 95	2	142	82
## 96	6	144	72
## 97	2	92	62
## 98	1	71	48
## 99	6	93	50
## 100	1	122	90
## 101	1	163	72
## 102	1	151	60
## 103	0	125	96
## 104	1	81	72
## 105	2	85	65

## 106	1	126	56
## 107	1	96	122
## 108	4	144	58
## 109	3	83	58
## 110	0	95	85
## 111	3	171	72
## 112	8	155	62
## 113	1	89	76
## 114	4	76	62
## 115	7	160	54
## 116	4	146	92
## 117	5	124	74
## 118	5	78	48
## 119	4	97	60
## 120	4	99	76
## 121	0	162	76
## 122	6	111	64
## 123	2	107	74
## 124	5	132	80
## 125	0	113	76
## 126	1	88	30
## 127	3	120	70
## 128	1	118	58
## 129	1	117	88
## 130	0	105	84
## 131	4	173	70
## 132	9	122	56
## 133	3	170	64
## 134	8	84	74
## 135	2	96	68
## 136	2	125	60
## 137	0	100	70
## 138	0	93	60
## 139	0	129	80
## 140	5	105	72
## 141	3	128	78
## 142	5	106	82
## 143	2	108	52
## 144	10	108	66
## 145	4	154	62
## 146	0	102	75
## 147	9	57	80
## 148	2	106	64
## 149	5	147	78
## 150	2	90	70
## 151	1	136	74
## 152	4	114	65
## 153	9	156	86
## 154	1	153	82

## 155	8	188	78
## 156	7	152	88
## 157	2	99	52
## 158	1	109	56
## 159	2	88	74
## 160	17	163	72
## 161	4	151	90
## 162	7	102	74
## 163	0	114	80
## 164	2	100	64
## 165	0	131	88
## 166	6	104	74
## 167	3	148	66
## 168	4	120	68
## 169	4	110	66
## 170	3	111	90
## 171	6	102	82
## 172	6	134	70
## 173	2	87	0
## 174	1	79	60
## 175	2	75	64
## 176	8	179	72
## 177	6	85	78
## 178	0	129	110
## 179	5	143	78
## 180	5	130	82
## 181	6	87	80
## 182	0	119	64
## 183	1	0	74
## 184	5	73	60
## 185	4	141	74
## 186	7	194	68
## 187	8	181	68
## 188	1	128	98
## 189	8	109	76
## 190	5	139	80
## 191	3	111	62
## 192	9	123	70
## 193	7	159	66
## 194	11	135	0
## 195	8	85	55
## 196	5	158	84
## 197	1	105	58
## 198	3	107	62
## 199	4	109	64
## 200	4	148	60
## 201	0	113	80
## 202	1	138	82
## 203	0	108	68

## 204	2	99	70
## 205	6	103	72
## 206	5	111	72
## 207	8	196	76
## 208	5	162	104
## 209	1	96	64
## 210	7	184	84
## 211	2	81	60
## 212	0	147	85
## 213	7	179	95
## 214	0	140	65
## 215	9	112	82
## 216	12	151	70
## 217	5	109	62
## 218	6	125	68
## 219	5	85	74
## 220	5	112	66
## 221	0	177	60
## 222	2	158	90
## 223	7	119	0
## 224	7	142	60
## 225	1	100	66
## 226	1	87	78
## 227	0	101	76
## 228	3	162	52
## 229	4	197	70
## 230	0	117	80
## 231	4	142	86
## 232	6	134	80
## 233	1	79	80
## 234	4	122	68
## 235	3	74	68
## 236	4	171	72
## 237	7	181	84
## 238	0	179	90
## 239	9	164	84
## 240	0	104	76
## 241	1	91	64
## 242	4	91	70
## 243	3	139	54
## 244	6	119	50
## 245	2	146	76
## 246	9	184	85
## 247	10	122	68
## 248	0	165	90
## 249	9	124	70
## 250	1	111	86
## 251	9	106	52
## 252	2	129	84

## 253	2	90	80
## 254	0	86	68
## 255	12	92	62
## 256	1	113	64
## 257	3	111	56
## 258	2	114	68
## 259	1	193	50
## 260	11	155	76
## 261	3	191	68
## 262	3	141	0
## 263	4	95	70
## 264	3	142	80
## 265	4	123	62
## 266	5	96	74
## 267	0	138	0
## 268	2	128	64
## 269	0	102	52
## 270	2	146	0
## 271	10	101	86
## 272	2	108	62
## 273	3	122	78
## 274	1	71	78
## 275	13	106	70
## 276	2	100	70
## 277	7	106	60
## 278	0	104	64
## 279	5	114	74
## 280	2	108	62
## 281	0	146	70
## 282	10	129	76
## 283	7	133	88
## 284	7	161	86
## 285	2	108	80
## 286	7	136	74
## 287	5	155	84
## 288	1	119	86
## 289	4	96	56
## 290	5	108	72
## 291	0	78	88
## 292	0	107	62
## 293	2	128	78
## 294	1	128	48
## 295	0	161	50
## 296	6	151	62
## 297	2	146	70
## 298	0	126	84
## 299	14	100	78
## 300	8	112	72
## 301	0	167	0

## 302	2	144	58
## 303	5	77	82
## 304	5	115	98
## 305	3	150	76
## 306	2	120	76
## 307	10	161	68
## 308	0	137	68
## 309	0	128	68
## 310	2	124	68
## 311	6	80	66
## 312	0	106	70
## 313	2	155	74
## 314	3	113	50
## 315	7	109	80
## 316	2	112	68
## 317	3	99	80
## 318	3	182	74
## 319	3	115	66
## 320	6	194	78
## 321	4	129	60
## 322	3	112	74
## 323	0	124	70
## 324	13	152	90
## 325	2	112	75
## 326	1	157	72
## 327	1	122	64
## 328	10	179	70
## 329	2	102	86
## 330	6	105	70
## 331	8	118	72
## 332	2	87	58
## 333	1	180	0
## 334	12	106	80
## 335	1	95	60
## 336	0	165	76
## 337	0	117	0
## 338	5	115	76
## 339	9	152	78
## 340	7	178	84
## 341	1	130	70
## 342	1	95	74
## 343	1	0	68
## 344	5	122	86
## 345	8	95	72
## 346	8	126	88
## 347	1	139	46
## 348	3	116	0
## 349	3	99	62
## 350	5	0	80

## 351	4	92	80
## 352	4	137	84
## 353	3	61	82
## 354	1	90	62
## 355	3	90	78
## 356	9	165	88
## 357	1	125	50
## 358	13	129	0
## 359	12	88	74
## 360	1	196	76
## 361	5	189	64
## 362	5	158	70
## 363	5	103	108
## 364	4	146	78
## 365	4	147	74
## 366	5	99	54
## 367	6	124	72
## 368	0	101	64
## 369	3	81	86
## 370	1	133	102
## 371	3	173	82
## 372	0	118	64
## 373	0	84	64
## 374	2	105	58
## 375	2	122	52
## 376	12	140	82
## 377	0	98	82
## 378	1	87	60
## 379	4	156	75
## 380	0	93	100
## 381	1	107	72
## 382	0	105	68
## 383	1	109	60
## 384	1	90	62
## 385	1	125	70
## 386	1	119	54
## 387	5	116	74
## 388	8	105	100
## 389	5	144	82
## 390	3	100	68
## 391	1	100	66
## 392	5	166	76
## 393	1	131	64
## 394	4	116	72
## 395	4	158	78
## 396	2	127	58
## 397	3	96	56
## 398	0	131	66
## 399	3	82	70

## 400	3	193	70
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## 402	6	137	61
## 403	5	136	84
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## 405	5	168	64
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## 408	0	101	62
## 409	8	197	74
## 410	1	172	68
## 411	6	102	90
## 412	1	112	72
## 413	1	143	84
## 414	1	143	74
## 415	0	138	60
## 416	3	173	84
## 417	1	97	68
## 418	4	144	82
## 419	1	83	68
## 420	3	129	64
## 421	1	119	88
## 422	2	94	68
## 423	0	102	64
## 424	2	115	64
## 425	8	151	78
## 426	4	184	78
## 427	0	94	0
## 428	1	181	64
## 429	0	135	94
## 430	1	95	82
## 431	2	99	0
## 432	3	89	74
## 433	1	80	74
## 434	2	139	75
## 435	1	90	68
## 436	0	141	0
## 437	12	140	85
## 438	5	147	75
## 439	1	97	70
## 440	6	107	88
## 441	0	189	104
## 442	2	83	66
## 443	4	117	64
## 444	8	108	70
## 445	4	117	62
## 446	0	180	78
## 447	1	100	72
## 448	0	95	80

## 449	0	104	64
## 450	0	120	74
## 451	1	82	64
## 452	2	134	70
## 453	0	91	68
## 454	2	119	0
## 455	2	100	54
## 456	14	175	62
## 457	1	135	54
## 458	5	86	68
## 459	10	148	84
## 460	9	134	74
## 461	9	120	72
## 462	1	71	62
## 463	8	74	70
## 464	5	88	78
## 465	10	115	98
## 466	0	124	56
## 467	0	74	52
## 468	0	97	64
## 469	8	120	0
## 470	6	154	78
## 471	1	144	82
## 472	0	137	70
## 473	0	119	66
## 474	7	136	90
## 475	4	114	64
## 476	0	137	84
## 477	2	105	80
## 478	7	114	76
## 479	8	126	74
## 480	4	132	86
## 481	3	158	70
## 482	0	123	88
## 483	4	85	58
## 484	0	84	82
## 485	0	145	0
## 486	0	135	68
## 487	1	139	62
## 488	0	173	78
## 489	4	99	72
## 490	8	194	80
## 491	2	83	65
## 492	2	89	90
## 493	4	99	68
## 494	4	125	70
## 495	3	80	0
## 496	6	166	74
## 497	5	110	68

## 498	2	81	72
## 499	7	195	70
## 500	6	154	74
## 501	2	117	90
## 502	3	84	72
## 503	6	0	68
## 504	7	94	64
## 505	3	96	78
## 506	10	75	82
## 507	0	180	90
## 508	1	130	60
## 509	2	84	50
## 510	8	120	78
## 511	12	84	72
## 512	0	139	62
## 513	9	91	68
## 514	2	91	62
## 515	3	99	54
## 516	3	163	70
## 517	9	145	88
## 518	7	125	86
## 519	13	76	60
## 520	6	129	90
## 521	2	68	70
## 522	3	124	80
## 523	6	114	0
## 524	9	130	70
## 525	3	125	58
## 526	3	87	60
## 527	1	97	64
## 528	3	116	74
## 529	0	117	66
## 530	0	111	65
## 531	2	122	60
## 532	0	107	76
## 533	1	86	66
## 534	6	91	0
## 535	1	77	56
## 536	4	132	0
## 537	0	105	90
## 538	0	57	60
## 539	0	127	80
## 540	3	129	92
## 541	8	100	74
## 542	3	128	72
## 543	10	90	85
## 544	4	84	90
## 545	1	88	78
## 546	8	186	90

## 547	5	187	76
## 548	4	131	68
## 549	1	164	82
## 550	4	189	110
## 551	1	116	70
## 552	3	84	68
## 553	6	114	88
## 554	1	88	62
## 555	1	84	64
## 556	7	124	70
## 557	1	97	70
## 558	8	110	76
## 559	11	103	68
## 560	11	85	74
## 561	6	125	76
## 562	0	198	66
## 563	1	87	68
## 564	6	99	60
## 565	0	91	80
## 566	2	95	54
## 567	1	99	72
## 568	6	92	62
## 569	4	154	72
## 570	0	121	66
## 571	3	78	70
## 572	2	130	96
## 573	3	111	58
## 574	2	98	60
## 575	1	143	86
## 576	1	119	44
## 577	6	108	44
## 578	2	118	80
## 579	10	133	68
## 580	2	197	70
## 581	0	151	90
## 582	6	109	60
## 583	12	121	78
## 584	8	100	76
## 585	8	124	76
## 586	1	93	56
## 587	8	143	66
## 588	6	103	66
## 589	3	176	86
## 590	0	73	0
## 591	11	111	84
## 592	2	112	78
## 593	3	132	80
## 594	2	82	52
## 595	6	123	72

## 596	0	188	82
## 597	0	67	76
## 598	1	89	24
## 599	1	173	74
## 600	1	109	38
## 601	1	108	88
## 602	6	96	0
## 603	1	124	74
## 604	7	150	78
## 605	4	183	0
## 606	1	124	60
## 607	1	181	78
## 608	1	92	62
## 609	0	152	82
## 610	1	111	62
## 611	3	106	54
## 612	3	174	58
## 613	7	168	88
## 614	6	105	80
## 615	11	138	74
## 616	3	106	72
## 617	6	117	96
## 618	2	68	62
## 619	9	112	82
## 620	0	119	0
## 621	2	112	86
## 622	2	92	76
## 623	6	183	94
## 624	0	94	70
## 625	2	108	64
## 626	4	90	88
## 627	0	125	68
## 628	0	132	78
## 629	5	128	80
## 630	4	94	65
## 631	7	114	64
## 632	0	102	78
## 633	2	111	60
## 634	1	128	82
## 635	10	92	62
## 636	13	104	72
## 637	5	104	74
## 638	2	94	76
## 639	7	97	76
## 640	1	100	74
## 641	0	102	86
## 642	4	128	70
## 643	6	147	80
## 644	4	90	0

## 645	3	103	72
## 646	2	157	74
## 647	1	167	74
## 648	0	179	50
## 649	11	136	84
## 650	0	107	60
## 651	1	91	54
## 652	1	117	60
## 653	5	123	74
## 654	2	120	54
## 655	1	106	70
## 656	2	155	52
## 657	2	101	58
## 658	1	120	80
## 659	11	127	106
## 660	3	80	82
## 661	10	162	84
## 662	1	199	76
## 663	8	167	106
## 664	9	145	80
## 665	6	115	60
## 666	1	112	80
## 667	4	145	82
## 668	10	111	70
## 669	6	98	58
## 670	9	154	78
## 671	6	165	68
## 672	1	99	58
## 673	10	68	106
## 674	3	123	100
## 675	8	91	82
## 676	6	195	70
## 677	9	156	86
## 678	0	93	60
## 679	3	121	52
## 680	2	101	58
## 681	2	56	56
## 682	0	162	76
## 683	0	95	64
## 684	4	125	80
## 685	5	136	82
## 686	2	129	74
## 687	3	130	64
## 688	1	107	50
## 689	1	140	74
## 690	1	144	82
## 691	8	107	80
## 692	13	158	114
## 693	2	121	70

## 694	7	129	68
## 695	2	90	60
## 696	7	142	90
## 697	3	169	74
## 698	0	99	0
## 699	4	127	88
## 700	4	118	70
## 701	2	122	76
## 702	6	125	78
## 703	1	168	88
## 704	2	129	0
## 705	4	110	76
## 706	6	80	80
## 707	10	115	0
## 708	2	127	46
## 709	9	164	78
## 710	2	93	64
## 711	3	158	64
## 712	5	126	78
## 713	10	129	62
## 714	0	134	58
## 715	3	102	74
## 716	7	187	50
## 717	3	173	78
## 718	10	94	72
## 719	1	108	60
## 720	5	97	76
## 721	4	83	86
## 722	1	114	66
## 723	1	149	68
## 724	5	117	86
## 725	1	111	94
## 726	4	112	78
## 727	1	116	78
## 728	0	141	84
## 729	2	175	88
## 730	2	92	52
## 731	3	130	78
## 732	8	120	86
## 733	2	174	88
## 734	2	106	56
## 735	2	105	75
## 736	4	95	60
## 737	0	126	86
## 738	8	65	72
## 739	2	99	60
## 740	1	102	74
## 741	11	120	80
## 742	3	102	44

## 743	1	109	58	
## 744	9	140	94	
## 745	13	153	88	
## 746	12	100	84	
## 747	1	147	94	
## 748	1	81	74	
## 749	3	187	70	
## 750	6	162	62	
## 751	4	136	70	
## 752	1	121	78	
## 753	3	108	62	
## 754	0	181	88	
## 755	8	154	78	
## 756	1	128	88	
## 757	7	137	90	
## 758	0	123	72	
## 759	1	106	76	
## 760	6	190	92	
## 761	2	88	58	
## 762	9	170	74	
## 763	9	89	62	
## 764	10	101	76	
## 765	2	122	70	
## 766	5	121	72	
## 767	1	126	60	
## 768	1	93	70	
##	triceps_skinfold_thickness	serum_insulin	body_mass_index	diabetes_pedigree
## 1	35	0	33.6	0.627
## 2	29	0	26.6	0.351
## 3	0	0	23.3	0.672
## 4	23	94	28.1	0.167
## 5	35	168	43.1	2.288
## 6	0	0	25.6	0.201
## 7	32	88	31.0	0.248
## 8	0	0	35.3	0.134
## 9	45	543	30.5	0.158
## 10	0	0	0.0	0.232
## 11	0	0	37.6	0.191
## 12	0	0	38.0	0.537
## 13	0	0	27.1	1.441
## 14	23	846	30.1	0.398
## 15	19	175	25.8	0.587
## 16	0	0	30.0	0.484
## 17	47	230	45.8	0.551
## 18	0	0	29.6	0.254
## 19	38	83	43.3	0.183
## 20	30	96	34.6	0.529
## 21	41	235	39.3	0.704
## 22	0	0	35.4	0.388

## 23	0	0	39.8	0.451
## 24	35	0	29.0	0.263
## 25	33	146	36.6	0.254
## 26	26	115	31.1	0.205
## 27	0	0	39.4	0.257
## 28	15	140	23.2	0.487
## 29	19	110	22.2	0.245
## 30	0	0	34.1	0.337
## 31	26	0	36.0	0.546
## 32	36	245	31.6	0.851
## 33	11	54	24.8	0.267
## 34	0	0	19.9	0.188
## 35	31	0	27.6	0.512
## 36	33	192	24.0	0.966
## 37	0	0	33.2	0.420
## 38	37	0	32.9	0.665
## 39	42	0	38.2	0.503
## 40	47	207	37.1	1.390
## 41	25	70	34.0	0.271
## 42	0	0	40.2	0.696
## 43	18	0	22.7	0.235
## 44	24	240	45.4	0.721
## 45	0	0	27.4	0.294
## 46	39	0	42.0	1.893
## 47	0	0	29.7	0.564
## 48	27	0	28.0	0.586
## 49	32	0	39.1	0.344
## 50	0	0	0.0	0.305
## 51	11	82	19.4	0.491
## 52	15	36	24.2	0.526
## 53	21	23	24.4	0.342
## 54	34	300	33.7	0.467
## 55	42	342	34.7	0.718
## 56	10	0	23.0	0.248
## 57	39	304	37.7	0.254
## 58	60	110	46.8	0.962
## 59	0	0	40.5	1.781
## 60	41	142	41.5	0.173
## 61	0	0	0.0	0.304
## 62	0	0	32.9	0.270
## 63	0	0	25.0	0.587
## 64	34	128	25.4	0.699
## 65	0	0	32.8	0.258
## 66	27	0	29.0	0.203
## 67	30	0	32.5	0.855
## 68	0	0	42.7	0.845
## 69	13	38	19.6	0.334
## 70	27	100	28.9	0.189
## 71	20	90	32.9	0.867

## 72	35	140	28.6	0.411
## 73	0	0	43.4	0.583
## 74	20	270	35.1	0.231
## 75	30	0	32.0	0.396
## 76	20	0	24.7	0.140
## 77	0	0	32.6	0.391
## 78	33	0	37.7	0.370
## 79	0	0	43.2	0.270
## 80	22	0	25.0	0.307
## 81	13	0	22.4	0.140
## 82	0	0	0.0	0.102
## 83	26	71	29.3	0.767
## 84	28	0	24.6	0.237
## 85	0	0	48.8	0.227
## 86	29	125	32.4	0.698
## 87	54	0	36.6	0.178
## 88	25	71	38.5	0.324
## 89	32	110	37.1	0.153
## 90	19	0	26.5	0.165
## 91	0	0	19.1	0.258
## 92	15	176	32.0	0.443
## 93	40	48	46.7	0.261
## 94	0	0	23.8	0.277
## 95	18	64	24.7	0.761
## 96	27	228	33.9	0.255
## 97	28	0	31.6	0.130
## 98	18	76	20.4	0.323
## 99	30	64	28.7	0.356
## 100	51	220	49.7	0.325
## 101	0	0	39.0	1.222
## 102	0	0	26.1	0.179
## 103	0	0	22.5	0.262
## 104	18	40	26.6	0.283
## 105	0	0	39.6	0.930
## 106	29	152	28.7	0.801
## 107	0	0	22.4	0.207
## 108	28	140	29.5	0.287
## 109	31	18	34.3	0.336
## 110	25	36	37.4	0.247
## 111	33	135	33.3	0.199
## 112	26	495	34.0	0.543
## 113	34	37	31.2	0.192
## 114	0	0	34.0	0.391
## 115	32	175	30.5	0.588
## 116	0	0	31.2	0.539
## 117	0	0	34.0	0.220
## 118	0	0	33.7	0.654
## 119	23	0	28.2	0.443
## 120	15	51	23.2	0.223

## 121	56	100	53.2	0.759
## 122	39	0	34.2	0.260
## 123	30	100	33.6	0.404
## 124	0	0	26.8	0.186
## 125	0	0	33.3	0.278
## 126	42	99	55.0	0.496
## 127	30	135	42.9	0.452
## 128	36	94	33.3	0.261
## 129	24	145	34.5	0.403
## 130	0	0	27.9	0.741
## 131	14	168	29.7	0.361
## 132	0	0	33.3	1.114
## 133	37	225	34.5	0.356
## 134	31	0	38.3	0.457
## 135	13	49	21.1	0.647
## 136	20	140	33.8	0.088
## 137	26	50	30.8	0.597
## 138	25	92	28.7	0.532
## 139	0	0	31.2	0.703
## 140	29	325	36.9	0.159
## 141	0	0	21.1	0.268
## 142	30	0	39.5	0.286
## 143	26	63	32.5	0.318
## 144	0	0	32.4	0.272
## 145	31	284	32.8	0.237
## 146	23	0	0.0	0.572
## 147	37	0	32.8	0.096
## 148	35	119	30.5	1.400
## 149	0	0	33.7	0.218
## 150	17	0	27.3	0.085
## 151	50	204	37.4	0.399
## 152	0	0	21.9	0.432
## 153	28	155	34.3	1.189
## 154	42	485	40.6	0.687
## 155	0	0	47.9	0.137
## 156	44	0	50.0	0.337
## 157	15	94	24.6	0.637
## 158	21	135	25.2	0.833
## 159	19	53	29.0	0.229
## 160	41	114	40.9	0.817
## 161	38	0	29.7	0.294
## 162	40	105	37.2	0.204
## 163	34	285	44.2	0.167
## 164	23	0	29.7	0.368
## 165	0	0	31.6	0.743
## 166	18	156	29.9	0.722
## 167	25	0	32.5	0.256
## 168	0	0	29.6	0.709
## 169	0	0	31.9	0.471

## 170	12	78	28.4	0.495
## 171	0	0	30.8	0.180
## 172	23	130	35.4	0.542
## 173	23	0	28.9	0.773
## 174	42	48	43.5	0.678
## 175	24	55	29.7	0.370
## 176	42	130	32.7	0.719
## 177	0	0	31.2	0.382
## 178	46	130	67.1	0.319
## 179	0	0	45.0	0.190
## 180	0	0	39.1	0.956
## 181	0	0	23.2	0.084
## 182	18	92	34.9	0.725
## 183	20	23	27.7	0.299
## 184	0	0	26.8	0.268
## 185	0	0	27.6	0.244
## 186	28	0	35.9	0.745
## 187	36	495	30.1	0.615
## 188	41	58	32.0	1.321
## 189	39	114	27.9	0.640
## 190	35	160	31.6	0.361
## 191	0	0	22.6	0.142
## 192	44	94	33.1	0.374
## 193	0	0	30.4	0.383
## 194	0	0	52.3	0.578
## 195	20	0	24.4	0.136
## 196	41	210	39.4	0.395
## 197	0	0	24.3	0.187
## 198	13	48	22.9	0.678
## 199	44	99	34.8	0.905
## 200	27	318	30.9	0.150
## 201	16	0	31.0	0.874
## 202	0	0	40.1	0.236
## 203	20	0	27.3	0.787
## 204	16	44	20.4	0.235
## 205	32	190	37.7	0.324
## 206	28	0	23.9	0.407
## 207	29	280	37.5	0.605
## 208	0	0	37.7	0.151
## 209	27	87	33.2	0.289
## 210	33	0	35.5	0.355
## 211	22	0	27.7	0.290
## 212	54	0	42.8	0.375
## 213	31	0	34.2	0.164
## 214	26	130	42.6	0.431
## 215	32	175	34.2	0.260
## 216	40	271	41.8	0.742
## 217	41	129	35.8	0.514
## 218	30	120	30.0	0.464

## 219	22	0	29.0	1.224
## 220	0	0	37.8	0.261
## 221	29	478	34.6	1.072
## 222	0	0	31.6	0.805
## 223	0	0	25.2	0.209
## 224	33	190	28.8	0.687
## 225	15	56	23.6	0.666
## 226	27	32	34.6	0.101
## 227	0	0	35.7	0.198
## 228	38	0	37.2	0.652
## 229	39	744	36.7	2.329
## 230	31	53	45.2	0.089
## 231	0	0	44.0	0.645
## 232	37	370	46.2	0.238
## 233	25	37	25.4	0.583
## 234	0	0	35.0	0.394
## 235	28	45	29.7	0.293
## 236	0	0	43.6	0.479
## 237	21	192	35.9	0.586
## 238	27	0	44.1	0.686
## 239	21	0	30.8	0.831
## 240	0	0	18.4	0.582
## 241	24	0	29.2	0.192
## 242	32	88	33.1	0.446
## 243	0	0	25.6	0.402
## 244	22	176	27.1	1.318
## 245	35	194	38.2	0.329
## 246	15	0	30.0	1.213
## 247	0	0	31.2	0.258
## 248	33	680	52.3	0.427
## 249	33	402	35.4	0.282
## 250	19	0	30.1	0.143
## 251	0	0	31.2	0.380
## 252	0	0	28.0	0.284
## 253	14	55	24.4	0.249
## 254	32	0	35.8	0.238
## 255	7	258	27.6	0.926
## 256	35	0	33.6	0.543
## 257	39	0	30.1	0.557
## 258	22	0	28.7	0.092
## 259	16	375	25.9	0.655
## 260	28	150	33.3	1.353
## 261	15	130	30.9	0.299
## 262	0	0	30.0	0.761
## 263	32	0	32.1	0.612
## 264	15	0	32.4	0.200
## 265	0	0	32.0	0.226
## 266	18	67	33.6	0.997
## 267	0	0	36.3	0.933

## 268	42	0	40.0	1.101
## 269	0	0	25.1	0.078
## 270	0	0	27.5	0.240
## 271	37	0	45.6	1.136
## 272	32	56	25.2	0.128
## 273	0	0	23.0	0.254
## 274	50	45	33.2	0.422
## 275	0	0	34.2	0.251
## 276	52	57	40.5	0.677
## 277	24	0	26.5	0.296
## 278	23	116	27.8	0.454
## 279	0	0	24.9	0.744
## 280	10	278	25.3	0.881
## 281	0	0	37.9	0.334
## 282	28	122	35.9	0.280
## 283	15	155	32.4	0.262
## 284	0	0	30.4	0.165
## 285	0	0	27.0	0.259
## 286	26	135	26.0	0.647
## 287	44	545	38.7	0.619
## 288	39	220	45.6	0.808
## 289	17	49	20.8	0.340
## 290	43	75	36.1	0.263
## 291	29	40	36.9	0.434
## 292	30	74	36.6	0.757
## 293	37	182	43.3	1.224
## 294	45	194	40.5	0.613
## 295	0	0	21.9	0.254
## 296	31	120	35.5	0.692
## 297	38	360	28.0	0.337
## 298	29	215	30.7	0.520
## 299	25	184	36.6	0.412
## 300	0	0	23.6	0.840
## 301	0	0	32.3	0.839
## 302	33	135	31.6	0.422
## 303	41	42	35.8	0.156
## 304	0	0	52.9	0.209
## 305	0	0	21.0	0.207
## 306	37	105	39.7	0.215
## 307	23	132	25.5	0.326
## 308	14	148	24.8	0.143
## 309	19	180	30.5	1.391
## 310	28	205	32.9	0.875
## 311	30	0	26.2	0.313
## 312	37	148	39.4	0.605
## 313	17	96	26.6	0.433
## 314	10	85	29.5	0.626
## 315	31	0	35.9	1.127
## 316	22	94	34.1	0.315

## 317	11	64	19.3	0.284
## 318	0	0	30.5	0.345
## 319	39	140	38.1	0.150
## 320	0	0	23.5	0.129
## 321	12	231	27.5	0.527
## 322	30	0	31.6	0.197
## 323	20	0	27.4	0.254
## 324	33	29	26.8	0.731
## 325	32	0	35.7	0.148
## 326	21	168	25.6	0.123
## 327	32	156	35.1	0.692
## 328	0	0	35.1	0.200
## 329	36	120	45.5	0.127
## 330	32	68	30.8	0.122
## 331	19	0	23.1	1.476
## 332	16	52	32.7	0.166
## 333	0	0	43.3	0.282
## 334	0	0	23.6	0.137
## 335	18	58	23.9	0.260
## 336	43	255	47.9	0.259
## 337	0	0	33.8	0.932
## 338	0	0	31.2	0.343
## 339	34	171	34.2	0.893
## 340	0	0	39.9	0.331
## 341	13	105	25.9	0.472
## 342	21	73	25.9	0.673
## 343	35	0	32.0	0.389
## 344	0	0	34.7	0.290
## 345	0	0	36.8	0.485
## 346	36	108	38.5	0.349
## 347	19	83	28.7	0.654
## 348	0	0	23.5	0.187
## 349	19	74	21.8	0.279
## 350	32	0	41.0	0.346
## 351	0	0	42.2	0.237
## 352	0	0	31.2	0.252
## 353	28	0	34.4	0.243
## 354	12	43	27.2	0.580
## 355	0	0	42.7	0.559
## 356	0	0	30.4	0.302
## 357	40	167	33.3	0.962
## 358	30	0	39.9	0.569
## 359	40	54	35.3	0.378
## 360	36	249	36.5	0.875
## 361	33	325	31.2	0.583
## 362	0	0	29.8	0.207
## 363	37	0	39.2	0.305
## 364	0	0	38.5	0.520
## 365	25	293	34.9	0.385

## 366	28	83	34.0	0.499
## 367	0	0	27.6	0.368
## 368	17	0	21.0	0.252
## 369	16	66	27.5	0.306
## 370	28	140	32.8	0.234
## 371	48	465	38.4	2.137
## 372	23	89	0.0	1.731
## 373	22	66	35.8	0.545
## 374	40	94	34.9	0.225
## 375	43	158	36.2	0.816
## 376	43	325	39.2	0.528
## 377	15	84	25.2	0.299
## 378	37	75	37.2	0.509
## 379	0	0	48.3	0.238
## 380	39	72	43.4	1.021
## 381	30	82	30.8	0.821
## 382	22	0	20.0	0.236
## 383	8	182	25.4	0.947
## 384	18	59	25.1	1.268
## 385	24	110	24.3	0.221
## 386	13	50	22.3	0.205
## 387	29	0	32.3	0.660
## 388	36	0	43.3	0.239
## 389	26	285	32.0	0.452
## 390	23	81	31.6	0.949
## 391	29	196	32.0	0.444
## 392	0	0	45.7	0.340
## 393	14	415	23.7	0.389
## 394	12	87	22.1	0.463
## 395	0	0	32.9	0.803
## 396	24	275	27.7	1.600
## 397	34	115	24.7	0.944
## 398	40	0	34.3	0.196
## 399	0	0	21.1	0.389
## 400	31	0	34.9	0.241
## 401	0	0	32.0	0.161
## 402	0	0	24.2	0.151
## 403	41	88	35.0	0.286
## 404	25	0	31.6	0.280
## 405	0	0	32.9	0.135
## 406	32	165	42.1	0.520
## 407	0	0	28.9	0.376
## 408	0	0	21.9	0.336
## 409	0	0	25.9	1.191
## 410	49	579	42.4	0.702
## 411	39	0	35.7	0.674
## 412	30	176	34.4	0.528
## 413	23	310	42.4	1.076
## 414	22	61	26.2	0.256

## 415	35	167	34.6	0.534
## 416	33	474	35.7	0.258
## 417	21	0	27.2	1.095
## 418	32	0	38.5	0.554
## 419	0	0	18.2	0.624
## 420	29	115	26.4	0.219
## 421	41	170	45.3	0.507
## 422	18	76	26.0	0.561
## 423	46	78	40.6	0.496
## 424	22	0	30.8	0.421
## 425	32	210	42.9	0.516
## 426	39	277	37.0	0.264
## 427	0	0	0.0	0.256
## 428	30	180	34.1	0.328
## 429	46	145	40.6	0.284
## 430	25	180	35.0	0.233
## 431	0	0	22.2	0.108
## 432	16	85	30.4	0.551
## 433	11	60	30.0	0.527
## 434	0	0	25.6	0.167
## 435	8	0	24.5	1.138
## 436	0	0	42.4	0.205
## 437	33	0	37.4	0.244
## 438	0	0	29.9	0.434
## 439	15	0	18.2	0.147
## 440	0	0	36.8	0.727
## 441	25	0	34.3	0.435
## 442	23	50	32.2	0.497
## 443	27	120	33.2	0.230
## 444	0	0	30.5	0.955
## 445	12	0	29.7	0.380
## 446	63	14	59.4	2.420
## 447	12	70	25.3	0.658
## 448	45	92	36.5	0.330
## 449	37	64	33.6	0.510
## 450	18	63	30.5	0.285
## 451	13	95	21.2	0.415
## 452	0	0	28.9	0.542
## 453	32	210	39.9	0.381
## 454	0	0	19.6	0.832
## 455	28	105	37.8	0.498
## 456	30	0	33.6	0.212
## 457	0	0	26.7	0.687
## 458	28	71	30.2	0.364
## 459	48	237	37.6	1.001
## 460	33	60	25.9	0.460
## 461	22	56	20.8	0.733
## 462	0	0	21.8	0.416
## 463	40	49	35.3	0.705

## 464	30	0	27.6	0.258
## 465	0	0	24.0	1.022
## 466	13	105	21.8	0.452
## 467	10	36	27.8	0.269
## 468	36	100	36.8	0.600
## 469	0	0	30.0	0.183
## 470	41	140	46.1	0.571
## 471	40	0	41.3	0.607
## 472	38	0	33.2	0.170
## 473	27	0	38.8	0.259
## 474	0	0	29.9	0.210
## 475	0	0	28.9	0.126
## 476	27	0	27.3	0.231
## 477	45	191	33.7	0.711
## 478	17	110	23.8	0.466
## 479	38	75	25.9	0.162
## 480	31	0	28.0	0.419
## 481	30	328	35.5	0.344
## 482	37	0	35.2	0.197
## 483	22	49	27.8	0.306
## 484	31	125	38.2	0.233
## 485	0	0	44.2	0.630
## 486	42	250	42.3	0.365
## 487	41	480	40.7	0.536
## 488	32	265	46.5	1.159
## 489	17	0	25.6	0.294
## 490	0	0	26.1	0.551
## 491	28	66	36.8	0.629
## 492	30	0	33.5	0.292
## 493	38	0	32.8	0.145
## 494	18	122	28.9	1.144
## 495	0	0	0.0	0.174
## 496	0	0	26.6	0.304
## 497	0	0	26.0	0.292
## 498	15	76	30.1	0.547
## 499	33	145	25.1	0.163
## 500	32	193	29.3	0.839
## 501	19	71	25.2	0.313
## 502	32	0	37.2	0.267
## 503	41	0	39.0	0.727
## 504	25	79	33.3	0.738
## 505	39	0	37.3	0.238
## 506	0	0	33.3	0.263
## 507	26	90	36.5	0.314
## 508	23	170	28.6	0.692
## 509	23	76	30.4	0.968
## 510	0	0	25.0	0.409
## 511	31	0	29.7	0.297
## 512	17	210	22.1	0.207

## 513	0	0	24.2	0.200
## 514	0	0	27.3	0.525
## 515	19	86	25.6	0.154
## 516	18	105	31.6	0.268
## 517	34	165	30.3	0.771
## 518	0	0	37.6	0.304
## 519	0	0	32.8	0.180
## 520	7	326	19.6	0.582
## 521	32	66	25.0	0.187
## 522	33	130	33.2	0.305
## 523	0	0	0.0	0.189
## 524	0	0	34.2	0.652
## 525	0	0	31.6	0.151
## 526	18	0	21.8	0.444
## 527	19	82	18.2	0.299
## 528	15	105	26.3	0.107
## 529	31	188	30.8	0.493
## 530	0	0	24.6	0.660
## 531	18	106	29.8	0.717
## 532	0	0	45.3	0.686
## 533	52	65	41.3	0.917
## 534	0	0	29.8	0.501
## 535	30	56	33.3	1.251
## 536	0	0	32.9	0.302
## 537	0	0	29.6	0.197
## 538	0	0	21.7	0.735
## 539	37	210	36.3	0.804
## 540	49	155	36.4	0.968
## 541	40	215	39.4	0.661
## 542	25	190	32.4	0.549
## 543	32	0	34.9	0.825
## 544	23	56	39.5	0.159
## 545	29	76	32.0	0.365
## 546	35	225	34.5	0.423
## 547	27	207	43.6	1.034
## 548	21	166	33.1	0.160
## 549	43	67	32.8	0.341
## 550	31	0	28.5	0.680
## 551	28	0	27.4	0.204
## 552	30	106	31.9	0.591
## 553	0	0	27.8	0.247
## 554	24	44	29.9	0.422
## 555	23	115	36.9	0.471
## 556	33	215	25.5	0.161
## 557	40	0	38.1	0.218
## 558	0	0	27.8	0.237
## 559	40	0	46.2	0.126
## 560	0	0	30.1	0.300
## 561	0	0	33.8	0.121

## 562	32	274	41.3	0.502
## 563	34	77	37.6	0.401
## 564	19	54	26.9	0.497
## 565	0	0	32.4	0.601
## 566	14	88	26.1	0.748
## 567	30	18	38.6	0.412
## 568	32	126	32.0	0.085
## 569	29	126	31.3	0.338
## 570	30	165	34.3	0.203
## 571	0	0	32.5	0.270
## 572	0	0	22.6	0.268
## 573	31	44	29.5	0.430
## 574	17	120	34.7	0.198
## 575	30	330	30.1	0.892
## 576	47	63	35.5	0.280
## 577	20	130	24.0	0.813
## 578	0	0	42.9	0.693
## 579	0	0	27.0	0.245
## 580	99	0	34.7	0.575
## 581	46	0	42.1	0.371
## 582	27	0	25.0	0.206
## 583	17	0	26.5	0.259
## 584	0	0	38.7	0.190
## 585	24	600	28.7	0.687
## 586	11	0	22.5	0.417
## 587	0	0	34.9	0.129
## 588	0	0	24.3	0.249
## 589	27	156	33.3	1.154
## 590	0	0	21.1	0.342
## 591	40	0	46.8	0.925
## 592	50	140	39.4	0.175
## 593	0	0	34.4	0.402
## 594	22	115	28.5	1.699
## 595	45	230	33.6	0.733
## 596	14	185	32.0	0.682
## 597	0	0	45.3	0.194
## 598	19	25	27.8	0.559
## 599	0	0	36.8	0.088
## 600	18	120	23.1	0.407
## 601	19	0	27.1	0.400
## 602	0	0	23.7	0.190
## 603	36	0	27.8	0.100
## 604	29	126	35.2	0.692
## 605	0	0	28.4	0.212
## 606	32	0	35.8	0.514
## 607	42	293	40.0	1.258
## 608	25	41	19.5	0.482
## 609	39	272	41.5	0.270
## 610	13	182	24.0	0.138

## 611	21	158	30.9	0.292
## 612	22	194	32.9	0.593
## 613	42	321	38.2	0.787
## 614	28	0	32.5	0.878
## 615	26	144	36.1	0.557
## 616	0	0	25.8	0.207
## 617	0	0	28.7	0.157
## 618	13	15	20.1	0.257
## 619	24	0	28.2	1.282
## 620	0	0	32.4	0.141
## 621	42	160	38.4	0.246
## 622	20	0	24.2	1.698
## 623	0	0	40.8	1.461
## 624	27	115	43.5	0.347
## 625	0	0	30.8	0.158
## 626	47	54	37.7	0.362
## 627	0	0	24.7	0.206
## 628	0	0	32.4	0.393
## 629	0	0	34.6	0.144
## 630	22	0	24.7	0.148
## 631	0	0	27.4	0.732
## 632	40	90	34.5	0.238
## 633	0	0	26.2	0.343
## 634	17	183	27.5	0.115
## 635	0	0	25.9	0.167
## 636	0	0	31.2	0.465
## 637	0	0	28.8	0.153
## 638	18	66	31.6	0.649
## 639	32	91	40.9	0.871
## 640	12	46	19.5	0.149
## 641	17	105	29.3	0.695
## 642	0	0	34.3	0.303
## 643	0	0	29.5	0.178
## 644	0	0	28.0	0.610
## 645	30	152	27.6	0.730
## 646	35	440	39.4	0.134
## 647	17	144	23.4	0.447
## 648	36	159	37.8	0.455
## 649	35	130	28.3	0.260
## 650	25	0	26.4	0.133
## 651	25	100	25.2	0.234
## 652	23	106	33.8	0.466
## 653	40	77	34.1	0.269
## 654	0	0	26.8	0.455
## 655	28	135	34.2	0.142
## 656	27	540	38.7	0.240
## 657	35	90	21.8	0.155
## 658	48	200	38.9	1.162
## 659	0	0	39.0	0.190

## 660	31	70	34.2	1.292
## 661	0	0	27.7	0.182
## 662	43	0	42.9	1.394
## 663	46	231	37.6	0.165
## 664	46	130	37.9	0.637
## 665	39	0	33.7	0.245
## 666	45	132	34.8	0.217
## 667	18	0	32.5	0.235
## 668	27	0	27.5	0.141
## 669	33	190	34.0	0.430
## 670	30	100	30.9	0.164
## 671	26	168	33.6	0.631
## 672	10	0	25.4	0.551
## 673	23	49	35.5	0.285
## 674	35	240	57.3	0.880
## 675	0	0	35.6	0.587
## 676	0	0	30.9	0.328
## 677	0	0	24.8	0.230
## 678	0	0	35.3	0.263
## 679	0	0	36.0	0.127
## 680	17	265	24.2	0.614
## 681	28	45	24.2	0.332
## 682	36	0	49.6	0.364
## 683	39	105	44.6	0.366
## 684	0	0	32.3	0.536
## 685	0	0	0.0	0.640
## 686	26	205	33.2	0.591
## 687	0	0	23.1	0.314
## 688	19	0	28.3	0.181
## 689	26	180	24.1	0.828
## 690	46	180	46.1	0.335
## 691	0	0	24.6	0.856
## 692	0	0	42.3	0.257
## 693	32	95	39.1	0.886
## 694	49	125	38.5	0.439
## 695	0	0	23.5	0.191
## 696	24	480	30.4	0.128
## 697	19	125	29.9	0.268
## 698	0	0	25.0	0.253
## 699	11	155	34.5	0.598
## 700	0	0	44.5	0.904
## 701	27	200	35.9	0.483
## 702	31	0	27.6	0.565
## 703	29	0	35.0	0.905
## 704	0	0	38.5	0.304
## 705	20	100	28.4	0.118
## 706	36	0	39.8	0.177
## 707	0	0	0.0	0.261
## 708	21	335	34.4	0.176

## 709	0	0	32.8	0.148
## 710	32	160	38.0	0.674
## 711	13	387	31.2	0.295
## 712	27	22	29.6	0.439
## 713	36	0	41.2	0.441
## 714	20	291	26.4	0.352
## 715	0	0	29.5	0.121
## 716	33	392	33.9	0.826
## 717	39	185	33.8	0.970
## 718	18	0	23.1	0.595
## 719	46	178	35.5	0.415
## 720	27	0	35.6	0.378
## 721	19	0	29.3	0.317
## 722	36	200	38.1	0.289
## 723	29	127	29.3	0.349
## 724	30	105	39.1	0.251
## 725	0	0	32.8	0.265
## 726	40	0	39.4	0.236
## 727	29	180	36.1	0.496
## 728	26	0	32.4	0.433
## 729	0	0	22.9	0.326
## 730	0	0	30.1	0.141
## 731	23	79	28.4	0.323
## 732	0	0	28.4	0.259
## 733	37	120	44.5	0.646
## 734	27	165	29.0	0.426
## 735	0	0	23.3	0.560
## 736	32	0	35.4	0.284
## 737	27	120	27.4	0.515
## 738	23	0	32.0	0.600
## 739	17	160	36.6	0.453
## 740	0	0	39.5	0.293
## 741	37	150	42.3	0.785
## 742	20	94	30.8	0.400
## 743	18	116	28.5	0.219
## 744	0	0	32.7	0.734
## 745	37	140	40.6	1.174
## 746	33	105	30.0	0.488
## 747	41	0	49.3	0.358
## 748	41	57	46.3	1.096
## 749	22	200	36.4	0.408
## 750	0	0	24.3	0.178
## 751	0	0	31.2	1.182
## 752	39	74	39.0	0.261
## 753	24	0	26.0	0.223
## 754	44	510	43.3	0.222
## 755	32	0	32.4	0.443
## 756	39	110	36.5	1.057
## 757	41	0	32.0	0.391

## 758	0	0	36.3	0.258
## 759	0	0	37.5	0.197
## 760	0	0	35.5	0.278
## 761	26	16	28.4	0.766
## 762	31	0	44.0	0.403
## 763	0	0	22.5	0.142
## 764	48	180	32.9	0.171
## 765	27	0	36.8	0.340
## 766	23	112	26.2	0.245
## 767	0	0	30.1	0.349
## 768	31	0	30.4	0.315
##	age	diabetes		
## 1	50	yes		
## 2	31	no		
## 3	32	yes		
## 4	21	no		
## 5	33	yes		
## 6	30	no		
## 7	26	yes		
## 8	29	no		
## 9	53	yes		
## 10	54	yes		
## 11	30	no		
## 12	34	yes		
## 13	57	no		
## 14	59	yes		
## 15	51	yes		
## 16	32	yes		
## 17	31	yes		
## 18	31	yes		
## 19	33	no		
## 20	32	yes		
## 21	27	no		
## 22	50	no		
## 23	41	yes		
## 24	29	yes		
## 25	51	yes		
## 26	41	yes		
## 27	43	yes		
## 28	22	no		
## 29	57	no		
## 30	38	no		
## 31	60	no		
## 32	28	yes		
## 33	22	no		
## 34	28	no		
## 35	45	no		
## 36	33	no		
## 37	35	no		

## 38	46	yes
## 39	27	yes
## 40	56	yes
## 41	26	no
## 42	37	no
## 43	48	no
## 44	54	yes
## 45	40	no
## 46	25	yes
## 47	29	no
## 48	22	no
## 49	31	yes
## 50	24	no
## 51	22	no
## 52	26	no
## 53	30	no
## 54	58	yes
## 55	42	no
## 56	21	no
## 57	41	yes
## 58	31	no
## 59	44	no
## 60	22	no
## 61	21	no
## 62	39	yes
## 63	36	no
## 64	24	no
## 65	42	yes
## 66	32	no
## 67	38	yes
## 68	54	no
## 69	25	no
## 70	27	no
## 71	28	yes
## 72	26	no
## 73	42	yes
## 74	23	no
## 75	22	no
## 76	22	no
## 77	41	no
## 78	27	no
## 79	26	yes
## 80	24	no
## 81	22	no
## 82	22	no
## 83	36	no
## 84	22	no
## 85	37	yes
## 86	27	no

## 87	45	no
## 88	26	no
## 89	43	yes
## 90	24	no
## 91	21	no
## 92	34	no
## 93	42	no
## 94	60	yes
## 95	21	no
## 96	40	no
## 97	24	no
## 98	22	no
## 99	23	no
## 100	31	yes
## 101	33	yes
## 102	22	no
## 103	21	no
## 104	24	no
## 105	27	no
## 106	21	no
## 107	27	no
## 108	37	no
## 109	25	no
## 110	24	yes
## 111	24	yes
## 112	46	yes
## 113	23	no
## 114	25	no
## 115	39	yes
## 116	61	yes
## 117	38	yes
## 118	25	no
## 119	22	no
## 120	21	no
## 121	25	yes
## 122	24	no
## 123	23	no
## 124	69	no
## 125	23	yes
## 126	26	yes
## 127	30	no
## 128	23	no
## 129	40	yes
## 130	62	yes
## 131	33	yes
## 132	33	yes
## 133	30	yes
## 134	39	no
## 135	26	no

## 136	31	no
## 137	21	no
## 138	22	no
## 139	29	no
## 140	28	no
## 141	55	no
## 142	38	no
## 143	22	no
## 144	42	yes
## 145	23	no
## 146	21	no
## 147	41	no
## 148	34	no
## 149	65	no
## 150	22	no
## 151	24	no
## 152	37	no
## 153	42	yes
## 154	23	no
## 155	43	yes
## 156	36	yes
## 157	21	no
## 158	23	no
## 159	22	no
## 160	47	yes
## 161	36	no
## 162	45	no
## 163	27	no
## 164	21	no
## 165	32	yes
## 166	41	yes
## 167	22	no
## 168	34	no
## 169	29	no
## 170	29	no
## 171	36	yes
## 172	29	yes
## 173	25	no
## 174	23	no
## 175	33	no
## 176	36	yes
## 177	42	no
## 178	26	yes
## 179	47	no
## 180	37	yes
## 181	32	no
## 182	23	no
## 183	21	no
## 184	27	no

##	185	40	no
##	186	41	yes
##	187	60	yes
##	188	33	yes
##	189	31	yes
##	190	25	yes
##	191	21	no
##	192	40	no
##	193	36	yes
##	194	40	yes
##	195	42	no
##	196	29	yes
##	197	21	no
##	198	23	yes
##	199	26	yes
##	200	29	yes
##	201	21	no
##	202	28	no
##	203	32	no
##	204	27	no
##	205	55	no
##	206	27	no
##	207	57	yes
##	208	52	yes
##	209	21	no
##	210	41	yes
##	211	25	no
##	212	24	no
##	213	60	no
##	214	24	yes
##	215	36	yes
##	216	38	yes
##	217	25	yes
##	218	32	no
##	219	32	yes
##	220	41	yes
##	221	21	yes
##	222	66	yes
##	223	37	no
##	224	61	no
##	225	26	no
##	226	22	no
##	227	26	no
##	228	24	yes
##	229	31	no
##	230	24	no
##	231	22	yes
##	232	46	yes
##	233	22	no

##	234	29	no
##	235	23	no
##	236	26	yes
##	237	51	yes
##	238	23	yes
##	239	32	yes
##	240	27	no
##	241	21	no
##	242	22	no
##	243	22	yes
##	244	33	yes
##	245	29	no
##	246	49	yes
##	247	41	no
##	248	23	no
##	249	34	no
##	250	23	no
##	251	42	no
##	252	27	no
##	253	24	no
##	254	25	no
##	255	44	yes
##	256	21	yes
##	257	30	no
##	258	25	no
##	259	24	no
##	260	51	yes
##	261	34	no
##	262	27	yes
##	263	24	no
##	264	63	no
##	265	35	yes
##	266	43	no
##	267	25	yes
##	268	24	no
##	269	21	no
##	270	28	yes
##	271	38	yes
##	272	21	no
##	273	40	no
##	274	21	no
##	275	52	no
##	276	25	no
##	277	29	yes
##	278	23	no
##	279	57	no
##	280	22	no
##	281	28	yes
##	282	39	no

##	283	37	no
##	284	47	yes
##	285	52	yes
##	286	51	no
##	287	34	no
##	288	29	yes
##	289	26	no
##	290	33	no
##	291	21	no
##	292	25	yes
##	293	31	yes
##	294	24	yes
##	295	65	no
##	296	28	no
##	297	29	yes
##	298	24	no
##	299	46	yes
##	300	58	no
##	301	30	yes
##	302	25	yes
##	303	35	no
##	304	28	yes
##	305	37	no
##	306	29	no
##	307	47	yes
##	308	21	no
##	309	25	yes
##	310	30	yes
##	311	41	no
##	312	22	no
##	313	27	yes
##	314	25	no
##	315	43	yes
##	316	26	no
##	317	30	no
##	318	29	yes
##	319	28	no
##	320	59	yes
##	321	31	no
##	322	25	yes
##	323	36	yes
##	324	43	yes
##	325	21	no
##	326	24	no
##	327	30	yes
##	328	37	no
##	329	23	yes
##	330	37	no
##	331	46	no

##	332	25	no
##	333	41	yes
##	334	44	no
##	335	22	no
##	336	26	no
##	337	44	no
##	338	44	yes
##	339	33	yes
##	340	41	yes
##	341	22	no
##	342	36	no
##	343	22	no
##	344	33	no
##	345	57	no
##	346	49	no
##	347	22	no
##	348	23	no
##	349	26	no
##	350	37	yes
##	351	29	no
##	352	30	no
##	353	46	no
##	354	24	no
##	355	21	no
##	356	49	yes
##	357	28	yes
##	358	44	yes
##	359	48	no
##	360	29	yes
##	361	29	yes
##	362	63	no
##	363	65	no
##	364	67	yes
##	365	30	no
##	366	30	no
##	367	29	yes
##	368	21	no
##	369	22	no
##	370	45	yes
##	371	25	yes
##	372	21	no
##	373	21	no
##	374	25	no
##	375	28	no
##	376	58	yes
##	377	22	no
##	378	22	no
##	379	32	yes
##	380	35	no

##	381	24	no
##	382	22	no
##	383	21	no
##	384	25	no
##	385	25	no
##	386	24	no
##	387	35	yes
##	388	45	yes
##	389	58	yes
##	390	28	no
##	391	42	no
##	392	27	yes
##	393	21	no
##	394	37	no
##	395	31	yes
##	396	25	no
##	397	39	no
##	398	22	yes
##	399	25	no
##	400	25	yes
##	401	31	yes
##	402	55	no
##	403	35	yes
##	404	38	no
##	405	41	yes
##	406	26	no
##	407	46	yes
##	408	25	no
##	409	39	yes
##	410	28	yes
##	411	28	no
##	412	25	no
##	413	22	no
##	414	21	no
##	415	21	yes
##	416	22	yes
##	417	22	no
##	418	37	yes
##	419	27	no
##	420	28	yes
##	421	26	no
##	422	21	no
##	423	21	no
##	424	21	no
##	425	36	yes
##	426	31	yes
##	427	25	no
##	428	38	yes
##	429	26	no

##	430	43	yes
##	431	23	no
##	432	38	no
##	433	22	no
##	434	29	no
##	435	36	no
##	436	29	yes
##	437	41	no
##	438	28	no
##	439	21	no
##	440	31	no
##	441	41	yes
##	442	22	no
##	443	24	no
##	444	33	yes
##	445	30	yes
##	446	25	yes
##	447	28	no
##	448	26	no
##	449	22	yes
##	450	26	no
##	451	23	no
##	452	23	yes
##	453	25	no
##	454	72	no
##	455	24	no
##	456	38	yes
##	457	62	no
##	458	24	no
##	459	51	yes
##	460	81	no
##	461	48	no
##	462	26	no
##	463	39	no
##	464	37	no
##	465	34	no
##	466	21	no
##	467	22	no
##	468	25	no
##	469	38	yes
##	470	27	no
##	471	28	no
##	472	22	no
##	473	22	no
##	474	50	no
##	475	24	no
##	476	59	no
##	477	29	yes
##	478	31	no

## 479	39	no
## 480	63	no
## 481	35	yes
## 482	29	no
## 483	28	no
## 484	23	no
## 485	31	yes
## 486	24	yes
## 487	21	no
## 488	58	no
## 489	28	no
## 490	67	no
## 491	24	no
## 492	42	no
## 493	33	no
## 494	45	yes
## 495	22	no
## 496	66	no
## 497	30	no
## 498	25	no
## 499	55	yes
## 500	39	no
## 501	21	no
## 502	28	no
## 503	41	yes
## 504	41	no
## 505	40	no
## 506	38	no
## 507	35	yes
## 508	21	no
## 509	21	no
## 510	64	no
## 511	46	yes
## 512	21	no
## 513	58	no
## 514	22	no
## 515	24	no
## 516	28	yes
## 517	53	yes
## 518	51	no
## 519	41	no
## 520	60	no
## 521	25	no
## 522	26	no
## 523	26	no
## 524	45	yes
## 525	24	no
## 526	21	no
## 527	21	no

## 528	24	no
## 529	22	no
## 530	31	no
## 531	22	no
## 532	24	no
## 533	29	no
## 534	31	no
## 535	24	no
## 536	23	yes
## 537	46	no
## 538	67	no
## 539	23	no
## 540	32	yes
## 541	43	yes
## 542	27	yes
## 543	56	yes
## 544	25	no
## 545	29	no
## 546	37	yes
## 547	53	yes
## 548	28	no
## 549	50	no
## 550	37	no
## 551	21	no
## 552	25	no
## 553	66	no
## 554	23	no
## 555	28	no
## 556	37	no
## 557	30	no
## 558	58	no
## 559	42	no
## 560	35	no
## 561	54	yes
## 562	28	yes
## 563	24	no
## 564	32	no
## 565	27	no
## 566	22	no
## 567	21	no
## 568	46	no
## 569	37	no
## 570	33	yes
## 571	39	no
## 572	21	no
## 573	22	no
## 574	22	no
## 575	23	no
## 576	25	no

##	577	35	no
##	578	21	yes
##	579	36	no
##	580	62	yes
##	581	21	yes
##	582	27	no
##	583	62	no
##	584	42	no
##	585	52	yes
##	586	22	no
##	587	41	yes
##	588	29	no
##	589	52	yes
##	590	25	no
##	591	45	yes
##	592	24	no
##	593	44	yes
##	594	25	no
##	595	34	no
##	596	22	yes
##	597	46	no
##	598	21	no
##	599	38	yes
##	600	26	no
##	601	24	no
##	602	28	no
##	603	30	no
##	604	54	yes
##	605	36	yes
##	606	21	no
##	607	22	yes
##	608	25	no
##	609	27	no
##	610	23	no
##	611	24	no
##	612	36	yes
##	613	40	yes
##	614	26	no
##	615	50	yes
##	616	27	no
##	617	30	no
##	618	23	no
##	619	50	yes
##	620	24	yes
##	621	28	no
##	622	28	no
##	623	45	no
##	624	21	no
##	625	21	no

## 626	29	no
## 627	21	no
## 628	21	no
## 629	45	no
## 630	21	no
## 631	34	yes
## 632	24	no
## 633	23	no
## 634	22	no
## 635	31	no
## 636	38	yes
## 637	48	no
## 638	23	no
## 639	32	yes
## 640	28	no
## 641	27	no
## 642	24	no
## 643	50	yes
## 644	31	no
## 645	27	no
## 646	30	no
## 647	33	yes
## 648	22	yes
## 649	42	yes
## 650	23	no
## 651	23	no
## 652	27	no
## 653	28	no
## 654	27	no
## 655	22	no
## 656	25	yes
## 657	22	no
## 658	41	no
## 659	51	no
## 660	27	yes
## 661	54	no
## 662	22	yes
## 663	43	yes
## 664	40	yes
## 665	40	yes
## 666	24	no
## 667	70	yes
## 668	40	yes
## 669	43	no
## 670	45	no
## 671	49	no
## 672	21	no
## 673	47	no
## 674	22	no

##	675	68	no
##	676	31	yes
##	677	53	yes
##	678	25	no
##	679	25	yes
##	680	23	no
##	681	22	no
##	682	26	yes
##	683	22	no
##	684	27	yes
##	685	69	no
##	686	25	no
##	687	22	no
##	688	29	no
##	689	23	no
##	690	46	yes
##	691	34	no
##	692	44	yes
##	693	23	no
##	694	43	yes
##	695	25	no
##	696	43	yes
##	697	31	yes
##	698	22	no
##	699	28	no
##	700	26	no
##	701	26	no
##	702	49	yes
##	703	52	yes
##	704	41	no
##	705	27	no
##	706	28	no
##	707	30	yes
##	708	22	no
##	709	45	yes
##	710	23	yes
##	711	24	no
##	712	40	no
##	713	38	yes
##	714	21	no
##	715	32	no
##	716	34	yes
##	717	31	yes
##	718	56	no
##	719	24	no
##	720	52	yes
##	721	34	no
##	722	21	no
##	723	42	yes

```
## 724 42      no
## 725 45      no
## 726 38      no
## 727 25      no
## 728 22      no
## 729 22      no
## 730 22      no
## 731 34      yes
## 732 22      yes
## 733 24      yes
## 734 22      no
## 735 53      no
## 736 28      no
## 737 21      no
## 738 42      no
## 739 21      no
## 740 42      yes
## 741 48      yes
## 742 26      no
## 743 22      no
## 744 45      yes
## 745 39      no
## 746 46      no
## 747 27      yes
## 748 32      no
## 749 36      yes
## 750 50      yes
## 751 22      yes
## 752 28      no
## 753 25      no
## 754 26      yes
## 755 45      yes
## 756 37      yes
## 757 39      no
## 758 52      yes
## 759 26      no
## 760 66      yes
## 761 22      no
## 762 43      yes
## 763 33      no
## 764 63      no
## 765 27      no
## 766 30      no
## 767 47      yes
## 768 23      no
```

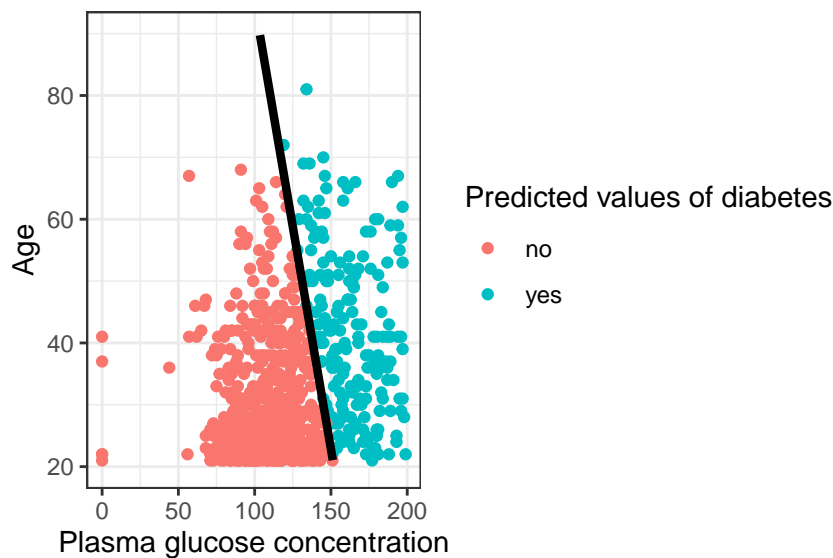
```
#coef(model)[1] + t(as.matrix(coef(model)[2:3])) %*% as.matrix(diabetes_df[,c("plasma_glucose_conc", "age")])
ggplot(diabetes_df_pred, aes(x = plasma_glucose_conc, y = age, color = pred)) +
```

```

geom_point() +
theme_bw() +
stat_function(fun = ({function(x) (-coef(model)[1] - coef(model)[2]*x)/ coef(model)[3] }),
              size=1.5, color = "black") +
ylim(20,90) +
labs(colour = "Predicted values of diabetes",
     x = "Plasma glucose concentration",
     y = "Age")

```

Warning: Removed 76 row(s) containing missing values (geom_path).



Question:

Comment whether the decision boundary seems to catch the data distribution well.

1.5 3.4

Question:

Make same kind of plots as in step 2 but use thresholds $r = 0.2$ and $r = 0.8$. By using these plots

```
library("ggpubr")
```

Warning: package 'ggpubr' was built under R version 4.2.1

```

# Using 0.2 as the classification threshold
pred <- predict(model, newdata = diabetes_df, type = "response")
pred <- ifelse(pred > 0.2, "yes", "no")
diabetes_df_pred$pred <- pred

```

```

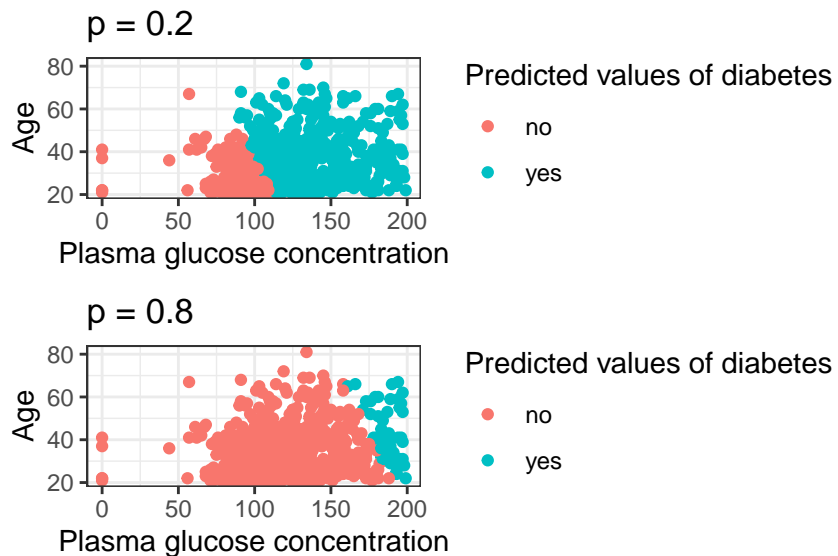
p1 <- ggplot(diabetes_df_pred, aes(x = plasma_glucose_conc, y = age, color = pred)) +
  geom_point() +
  theme_bw() +
  labs(colour = "Predicted values of diabetes",
       x = "Plasma glucose concentration",
       y = "Age") +
  ggtitle("p = 0.2")

# Using 0.8 as the classification threshold
pred <- predict(model, newdata = diabetes_df, type = "response")
pred <- ifelse(pred > 0.8, "yes", "no")
diabetes_df_pred$pred <- pred

p2 <- ggplot(diabetes_df_pred, aes(x = plasma_glucose_conc, y = age, color = pred)) +
  geom_point() +
  theme_bw() +
  labs(colour = "Predicted values of diabetes",
       x = "Plasma glucose concentration",
       y = "Age") +
  ggtitle("p = 0.8")

ggarrange(p1, p2, ncol = 1, nrow = 2)

```



Question:

Comment on what happens with the prediction when r value changes.

1.6 3.5

Question:

Perform a basis function expansion trick by computing new features $z_1 = x_1^4$, $z_2 = x_1^3 x_2$, $z_3 = x_1^2 x_2^2$, $z_4 = x_1 x_2^3$, $z_5 = x_2^4$, adding them to the data set and then computing a logistic regression model with y as target and $x_1, x_2, z_1, \dots, z_5$ as features. Create a scatterplot of the same kind as in step 2 for this model.

```
diabetes_df$z1 <- diabetes_df$times_pregnant^4
diabetes_df$z2 <- diabetes_df$times_pregnant^3 * diabetes_df$plasma_glucose_conc^2
diabetes_df$z3 <- diabetes_df$times_pregnant^2 * diabetes_df$plasma_glucose_conc^2
diabetes_df$z4 <- diabetes_df$times_pregnant * diabetes_df$plasma_glucose_conc^3
diabetes_df$z5 <- diabetes_df$plasma_glucose_conc^4

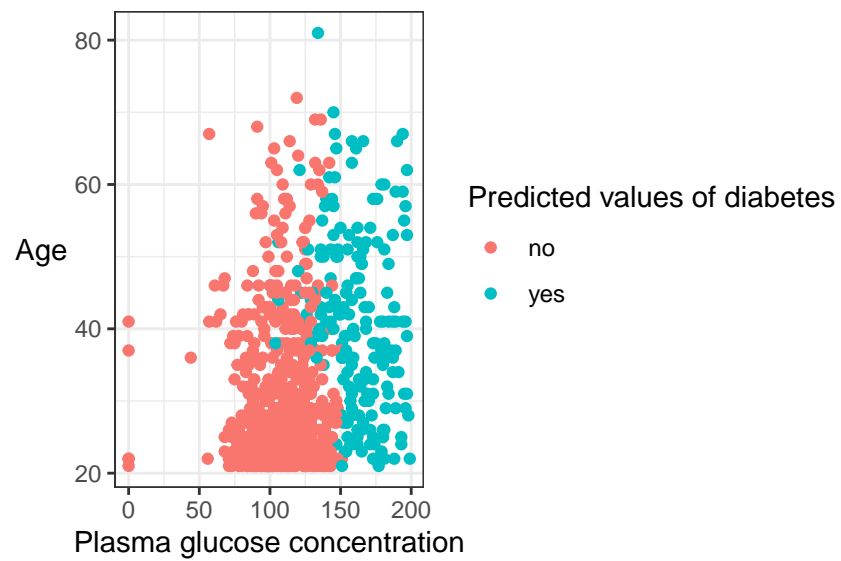
model <- glm(diabetes ~ plasma_glucose_conc + age + z1 + z2 + z3 + z4 + z5, data = diabetes_df,
             family = "binomial")

pred <- predict(model, newdata = diabetes_df, type = "response")

# Using 0.5 as the classification threshold
pred <- ifelse(pred > 0.5, "yes", "no")

diabetes_df_pred <- diabetes_df
diabetes_df_pred$pred <- pred

ggplot(diabetes_df_pred, aes(x = plasma_glucose_conc, y = age, color = pred)) +
  geom_point() +
  theme_bw() +
  theme(axis.title.y = element_text(angle = 0, vjust = 0.5)) +
  labs(colour = "Predicted values of diabetes",
       x = "Plasma glucose concentration",
       y = "Age")
```

Question:

Compute the training misclassification rate. What can you say about the quality of this model compared to the previous logistic regression model? How have the basis expansion trick affected the shape of the decision boundary and the prediction accuracy?