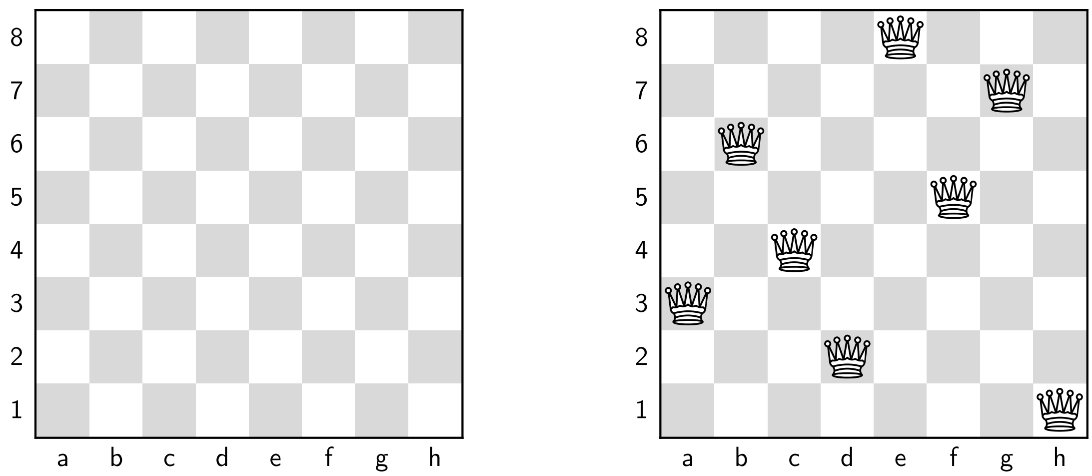
Modeling (Exercices)

For each problem, you have to conceive a model, which can then be written with the library PyCSP^3.

1 Queens

We consider the problem of putting 8 queens on a chessboard.



(a) Puzzle (b) Solution

Figure 1: Putting 8 queens on a chessboard

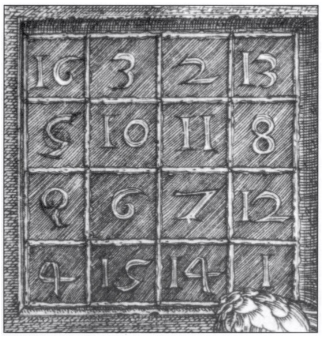
Of course, two queens cannot be put:

- on the same row
- on the same column
- on the same diagonal

1. Propose a model for the generalization of this problem to any number n of queens.
2. Compute the number of solutions for $n = 8$, $n = 10$ and $n = 12$.

2 Magic Square

A magic square of order n is a n by n matrix containing the numbers 1 to n^2, with each row, column and main diagonal being equal to the same sum. As an example, you find below a solution for $n = 4$.



1. Find the mathematical expression that gives the sum value to reach for any order n.
2. Propose a model for this problem.

3 Golomb Ruler

This problem (and its variants) is said to have many practical applications including sensor placements for x-ray crystallography and radio astronomy. A Golomb ruler may be defined as a set of n integers $0 = a_1 < a_2 < ... < a_n$ such that the $n(n-1)/2$ differences $a_j - a_i$, $1 \leq i < j \leq n$ are distinct. Such a ruler is said to contain n marks and is of length a_n. The objective is to find optimal (minimum length) or near optimal rulers. An optimal ruler for $n = 4$ is illustrated below:

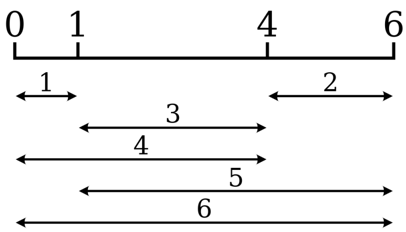


Figure 2: An Optimal Golomb Ruler with 4 Ticks. (image from commons.wikimedia.org)

Dimitromanolakis has computed relatively short Golomb rulers and thus showed with computer aid that the optimal ruler for $n \leq 65,000$ has length less than n^2.

1. Build a first model where we find quaternary constraints.
2. Build a second model where we find ternary constraints (after adding auxiliary variables).

4 Social Golfers

The coordinator of a local golf club has come to you with the following problem. In their club, there are 32 social golfers, each of whom play golf once a week, and always in groups of 4. They would like you to come up with a schedule of play for these golfers, to last as many weeks as possible, such that no golfer plays in the same group as any other golfer on more than one occasion.

Figure 3: A golfer who apparently needs socialization. (image from www.publicdomainpictures.net)

The problem can easily be generalized to that of scheduling G groups of K golfers over at most W weeks, such that no golfer plays in the same group as any other golfer twice (i.e. maximum socialisation is achieved). For the original problem, the values of G and K are respectively 8 and 4.