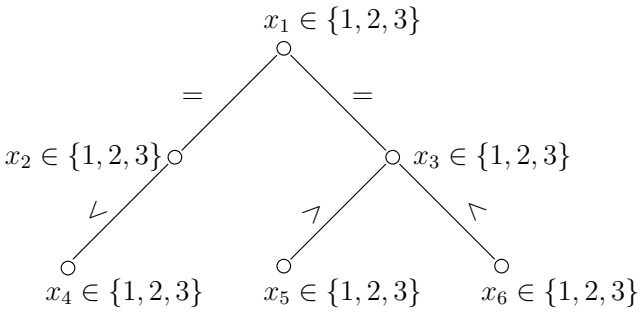
Consistencies (Exercices)

1 Acyclic Constraint Networks

We focus here on binary CNs, i.e., constraint networks only involving binary constraints.

1. Compute the AC-closure of the following CN:



2. Indicate precisely what is the best order in general to conduct constraint propagation on networks having a tree structure?
3. Prove that if a binary constraint network is both acyclic and AC, then a solution can be found by a backtrack-free depth-first search.

2 Singleton Arc Consistency

Let us consider the CN depicted in Figure 1. As in the course slides, show the results of performing:

- the singleton check for $(x, 0)$
- the singleton check for $(x, 1)$

3 Path Consistency

Let P be the following CN:

```
from pycsp3 import *

x = VarArray(size=4, dom=range(4))

satisfy(
    x[i] != x[j] for i, j in combinations(range(4), 2)
)
```

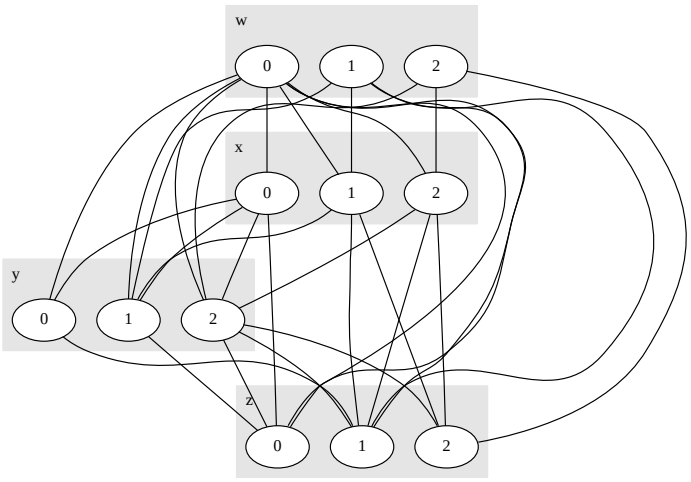
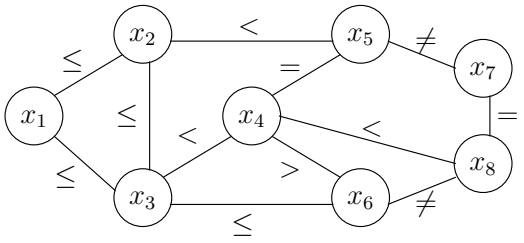


Figure 1: A binary constraint network before enforcing singleton arc consistency.

1. Draw the compatibility graph of this CN. Is this CN arc-consistent (AC)? Is this CN path-consistent (PC)? Is this CN satisfiable ? Now, assuming that the domain of $x[3]$ is $\{0, 1\}$ instead of $\{0, 1, 2\}$, compute the PC-closure of this new CN.

2. Observe the following constraint network P where $dom(x_i) = \{0, 1, 2, 3\}, \forall i \in 1..8$



Compute the AC-closure of P. Compute the PC-closure of P.

4 Formal Proofs

1. Prove that on binary constraint networks, the worst-case time complexity of the algorihtm AC3 is $O(ed^3)$

2. Here is the definition for Restricted Path Consistency (RPC):

> A value (x, a) of a constraint network P is *restricted path-consistent* (RPC) iff for every binary constraint c_{xy} of P involving x and another variable y such that (x, a) has a unique support (y, b) on c_{xy}, then for every additional variable z of P, there exists a value $c \in dom(z)$ guaranteeing that $\{(x, a), (z, c)\}$ and $\{(y, b), (z, c)\}$ are both locally consistent.

Prove that RPC is strictly stronger than AC