In [6]:
```
import numpy as np
import matplotlib.pyplot as plt
import sympy as sp
import ipywidgets
```

Endogenous Growth: R&D Model With Capital

Goods Producing Sector

$$Y(t) = [(1 - a_K)K(t)]^{\alpha}[A(t)(1 - a_L)L(t)]^{(1-\alpha)}$$

R&D Sector

$$\dot{A}(t) = B[a_K K(t)]^{\beta}[a_L L(t)]^{\gamma} A(t)^{\theta}$$

Population

$$\dot{L}(t) = nL(t)$$

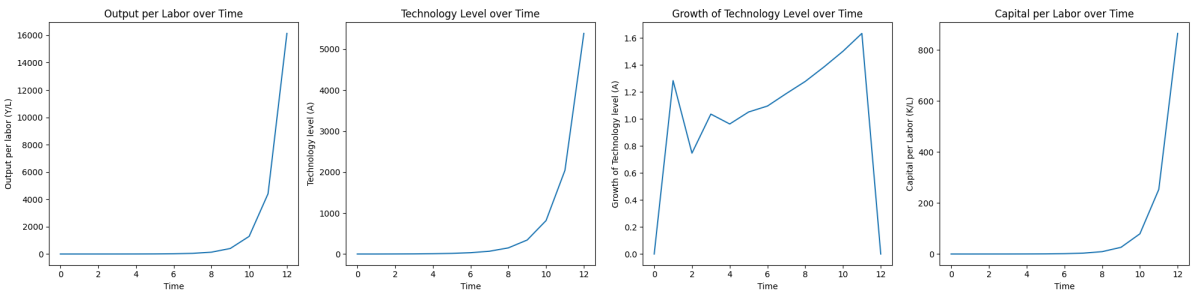
Capital

$$\dot{K}(t) = sY(t)$$

In [9]:
```
def RD_growth_model(n, gamma, theta, beta, alpha, L0, A0, T, s, K0, a):

    Y0 = A0 * (L0*(1 - a))**alpha * (K0*(1 - alpha))**(1-alpha)


    Lpath = np.zeros(T + 1)
    Apath = np.zeros(T + 1)
    Ypath = np.zeros(T + 1)
    Kpath = np.zeros(T + 1)
    YLpath = np.zeros(T + 1)
    AGpath = np.zeros(T + 1)
    KLpath = np.zeros(T + 1)



    Lpath[0] = L0
    Apath[0] = A0
    Ypath[0] = Y0
    Kpath[0] = K0
    YLpath[0] = Y0/L0
    KLpath[0] = K0/L0


    for t in range(T):
        Adot = (a * Lpath[t])**gamma * (Kpath[t-1] * a)**beta * Apath[t]**th
```

```
        Apath[t + 1] = Apath[t] + Adot
        AGpath[t] = Adot / Apath[t]

        Ydot = Apath[t] * (Lpath[t]*(1 - a))**(1-alpha) * (Kpath[t]*(1 - a))
        Lpath[t+1] = (1 + n) * Lpath[t]

        Ypath[t + 1] = Ypath[t] + Ydot
        Kpath[t + 1] = s * Ypath[t]

        YLpath[t + 1] = Ypath[t + 1] / Lpath[t + 1]
        KLpath[t + 1] = Kpath[t + 1] / Lpath[t + 1]



    t = np.arange(T + 1)

    fig, axs = plt.subplots(1, 4, figsize=(20, 5))

    axs[0].plot(t, YLpath)
    axs[0].set_xlabel('Time')
    axs[0].set_ylabel('Output per labor (Y/L)')
    axs[0].set_title('Output per Labor over Time')

    axs[1].plot(t, Apath)
    axs[1].set_xlabel('Time')
    axs[1].set_ylabel('Technology level (A)')
    axs[1].set_title('Technology Level over Time')

    axs[2].plot(t, AGpath)
    axs[2].set_xlabel('Time')
    axs[2].set_ylabel('Growth of Technology level (A)')
    axs[2].set_title('Growth of Technology Level over Time')

    axs[3].plot(t, KLpath)
    axs[3].set_xlabel('Time')
    axs[3].set_ylabel('Capital per Labor (K/L)')
    axs[3].set_title('Capital per Labor over Time')


    plt.tight_layout()
    plt.show()
```

In [22]:
```
mod1 = RD_growth_model(0.02, 0.6, 0.65, 0.35, 0.35, 30, 1, 12, 0.20, 1, 0.15
print(mod1)
```



```
None
```

In [ ]:

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