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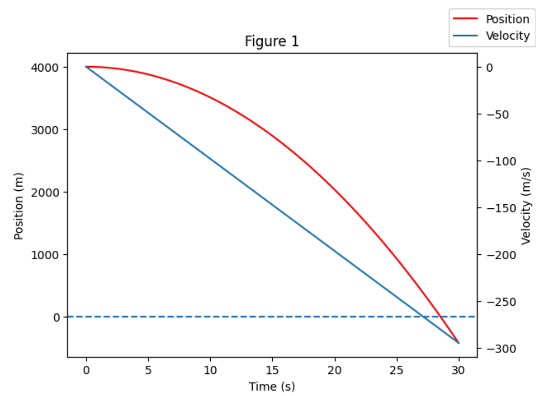
Lab 2: Mine Crafting

I Introduction

In this lab, I calculated the vertical depth of the vertical mine that the mining company operates at the Earth's equator. This mine is about 4 kilometers to the bottom of the shaft. To do this, I'll begin by calculating the ideal case for calculating the vertical depth, which neglects drag and assumes the gravitational force to be constant. Then, I'll recalculate those values, introducing drag and a variable gravitational force. The next step is incorporating the Coriolis Force into my calculations since we must account for the Earth's rotation as the test mass falls. Finally, we introduce the fact that Earth is non-uniform since we know that the density increases toward the center and decreases near the surface.

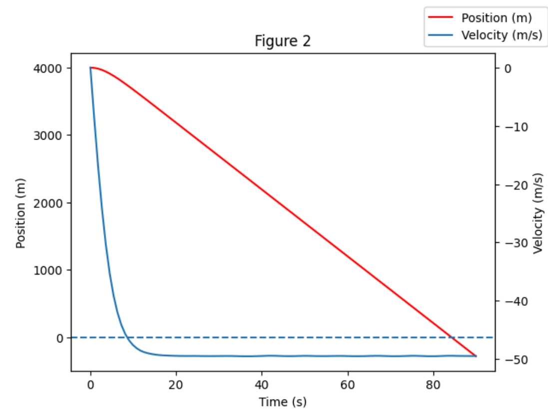
II Calculation of Fall Time



First, we calculated the theoretical free-fall time using the simple free-fall algebraic expression, $y = \frac{1}{2}gt^2$, which can be reorganized to $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 4000m}{9.81\ m/s^2}} = 28.6s$. This equation incorporates y, which is the assumed depth of the shaft, and a constant gravitational force, g. We then confirm this information by solving and plotting the velocity and position of a projectile experiencing constant gravitational force and drag force by setting v = dy/dt and solving the differential equation $\frac{dv}{dt} = -g + \alpha v^{\gamma}$. The resulting plot, created using the SciPy library's solve_ivp functionality, with an added line where the position is at 0 m, shows that the mass hits the bottom of the shaft at about 28 seconds, which is confirmed by solve_ivp using events to keep track of the moment it hits 0 m, calculated to be 28.6 seconds as well, which matches the data calculated earlier.

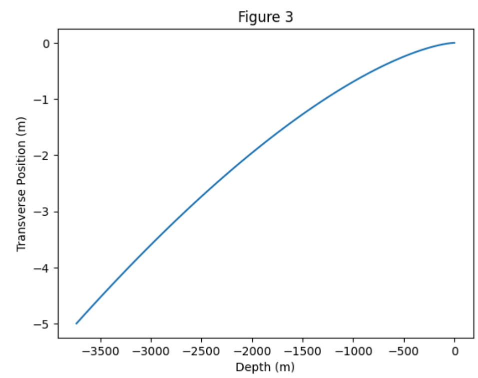
Next, we accounted for the fact that gravitational force changes depending on the distance *r* from the center of the Earth using the equation $g(r) = g_0(\frac{r}{Re})$ where Re is the radius of the Earth and g_0 is the gravity at the surface. Recreating the above graph with the height-dependent g using solve_ivp returns 28.6 seconds, showing consistency with the previous values. However, on further analysis, it's noticeable that the value with a variable gravity is higher than the last value at the 10^{-6} level, likely due to the increasing gravitational force contributing to the change in velocity decreasing, resulting in it taking longer to reach the shaft.

The next step was to incorporate drag force. Given the assumption that the terminal speed should be near 50 m/s, changing the alpha speed until the graph (Figure 2) came near 50 m/s when the position was 0 m. This value was calibrated to occur when the drag force coefficient was approximately 0.004. Introducing this drag coefficient changed the solution time to 84.3 seconds, which is a significant increase from the earlier values of 28.6 seconds. This makes sense since introducing drag force would result in slower changes in velocity, increasing the time it takes. This data showed that introducing variable gravity and drag force slows the mass down as it approaches the bottom of the mine, as expected.



III Feasibility of Depth Measurement Approach

Plotting the path of the object, both in depth and in the transverse direction as a function of time, depicts that when the drag is 0, the test mass will successfully hit the bottom (shown in Figure 3). However, when the drag force is reintroduced, the depth's range is reduced significantly since the test mass hits the wall and terminates when you drop the mass from the center.



IV Calculation of Crossing Times

Next, we calculated the crossing times of trans-earth and the moon. We started by plotting the depth and velocity as a function of time and saw depth and velocity oscillate as a function of time, with the depth oscillating between 6 and -6 meters and the velocity oscillating between 8000 and -8000 m/s^2. The initial depth line goes from one side of the Earth to the other in 2352 seconds and reaches the center in about 1200 seconds. The speed at this point is about 7.91 x 10^{-3} m/s. Then, using the equation $\frac{v^2}{R} = \frac{GM}{R^2}$ which can be organized as $v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.6743\times10^{-11} m^3/kg/s^2)(5.972\times10^{24} kg)}{(6.3781\times10^{6} m)}} = 7.9052 \times 10^3 m/s$. From this value we can calculate orbital time using the $T = \frac{2\pi R_e}{v} = 5069.42s$. This orbital time is about half of the crossing time we calculated for the crossing-time for the Earth.

Next we incorporated the concept that the Earth isn't a uniform sphere and has varying density. With the density function, $\rho(r) = \rho_n(1 - \frac{r^2}{R_e^2})^n$, the plots showed that the values 0, 1, 2, and 9 for n result in velocities at the center increasing and the time to reach the center decreasing as n increases, and at n = 2, which is considered the closest to the actual value of n for Earth's density distribution, the time to reach the center to be 1035 seconds.

We then calculated the travel time to the center of the moon for a pole-to-pole mine shaft so that we can avoid the Coriolis force, and we assume zero drag. The new calculated g0 value for the moon was 1.6238 m/s^2. The travel time was calculated to be 1624.9 seconds to reach the center of the moon. Then, we calculate the density ratio by plugging in the appropriate values for $\frac{\rho_M}{\rho_E} = \frac{\frac{M_M}{V_M}}{\frac{M_E}{V_E}} = \frac{M_M}{M_E} \times \frac{\frac{4}{3}\pi R_E^3}{\frac{4}{3}\pi R_M^3} = 0.61$. This means that the Moon is about 61% as dense as the Earth. We know from earlier steps that $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = 2\pi\sqrt{\frac{R^3}{GM}}$. Knowing that $M = \rho V = \rho \times \frac{4}{3}\pi R^3$, we can simplify $T = 2\pi\sqrt{\frac{R^3}{G\rho\times\frac{4}{3}\pi R^3}} = \sqrt{\frac{3\pi}{4G\rho}}$. This shows that T is proportional to $1/\sqrt{\rho}$. We can verify this by calculating $\frac{T_M}{T_E} = \frac{5069.42}{6498.9} = 0.78$ and the square root of the ratio of densities $\sqrt{\frac{\rho_M}{\rho_E}} = 0.78$, where both ratios are equal as predicted.

V Discussion and Future Work

In this lab we calculated the vertical depth of the deepest vertical mine that a mining company operates at the Earth's equator, that's about 4 km deep to the bottom of the shaft. We started by calculating the ideal case, neglecting drag and assuming constant gravitational force, and got a time of 28.6 seconds. Incorporating drag force increased the time required to 84.3 seconds since the rate of change of velocity decreased. We then showed that depending on where the test mass is dropped from, it may hit a wall before it hits the bottom of the shaft, due to the Coriolis force, which would alter the data. In Part 4, we calculated the cross-time to be approximately half the orbital time of the Earth, assuming uniform density, and in part 5 we changed it to incorporate the fact that the Earth's density isn't uniform and with the varying density function, the time to reach the center would be about 1035 seconds. Lastly, we compared this data with the density and orbital time of the Moon, showing that there's a relationship between density and orbital time. This information helps understand information that could be useful for a mining company's operations while working in a 4 km deep mine. We could extend this lab by comparing the values to other planets in part 6 and seeing if it verifies the relation between orbital time and density that we calculated.