Game Tree Searching by Min/Max Approximation – A Review

This summary is a review of the work on "Game Tree Searching by Min/Max Approximation" done by Ronald L. Rivest in 1987 [1].

In this paper by Ronald L. Rivest, the author approximates the non-differentiable min/max operators by generalized and differentiable mean value operators to present an iterative method for searching min/max trees. The author claims to get better results than min/max with alpha-beta pruning, but at the cost of increased CPU and memory utilization. Though min/max algorithm with alpha-beta pruning prunes a large part of game trees, their huge combinatorial explosion requires more techniques to be invented. While exploring partial game trees, the differentiable nature of generalized mean value operators helps find the important line of play in a non-trivial manner, thus producing superior play than minimax (with alpha-beta) for same number of underlying “move” operator.

The iterative heuristic used in the proposed algorithm is different from the static evaluations functions used more often. This iterative heuristic grows the search tree one step at a time, choosing a tip node (or leaf) at each successive step. The successors of the expanded tip node are added to the tree and their static evaluation values are backed up to the ancestors in a min-max fashion. A nonnegative penalty is assigned to every edge in the game tree such that edges representing bad moves are penalized more than edges representing good moves. We define the penalty of a tip to be the sum of penalties of all edges between tip and root. Idea is to always expand the node with least penalty. Each time we expand a tip node with least penalty, its heuristic value is estimated, and subsequently, the penalty of ancestor nodes is updated as well. This is called penalty based iterative search methods. So, we note that penalty based schemes are oriented more towards improving the value of the estimate at the root, rather than towards selecting the best move to make from the root.

The “min/max approximation” heuristic is special case of penalty based search method, where the penalties are defined in terms of the derivatives of the approximating functions. The chain of partial derivatives is used to measure the sensitivity of the root value to changes in the tip value.

Experimental results show that min/max approximation method performs better than minimax with alpha-beta pruning for the same number of calls to “move” operator. However, when CPU is more limiting resource, minimax with alpha-beta seems to perform better. Because all penalties are backed-up right from expanding node to source node, it is able to more accurately predict the most suitable branch. But it’s heavy on computing resources, since it causes large number of traversals back and forth the tree for the updating purpose.

Though the author has presented a novel approach to game tree searching, based on approximating the min/max functions by suitable generalized mean value functions, still there are some open problems. Like, how we should best choose mean value functions, or penalty functions. How we can incorporate the efficiency of depth-first search with min/max approximations, or how well these ideas can be parallelized. If some of its disadvantages are mitigated, this can be the next milestone in game playing. Though the work was conducted long before in 1987, it still can be referenced to develop insight into more efficient ways to searching game trees and to understand difficulties encountered while dealing with their huge size.

References:

1. Rivest, Ronald L. "Game tree searching by min/max approximation." *Artificial Intelligence* 34.1 (1987): 77-96.