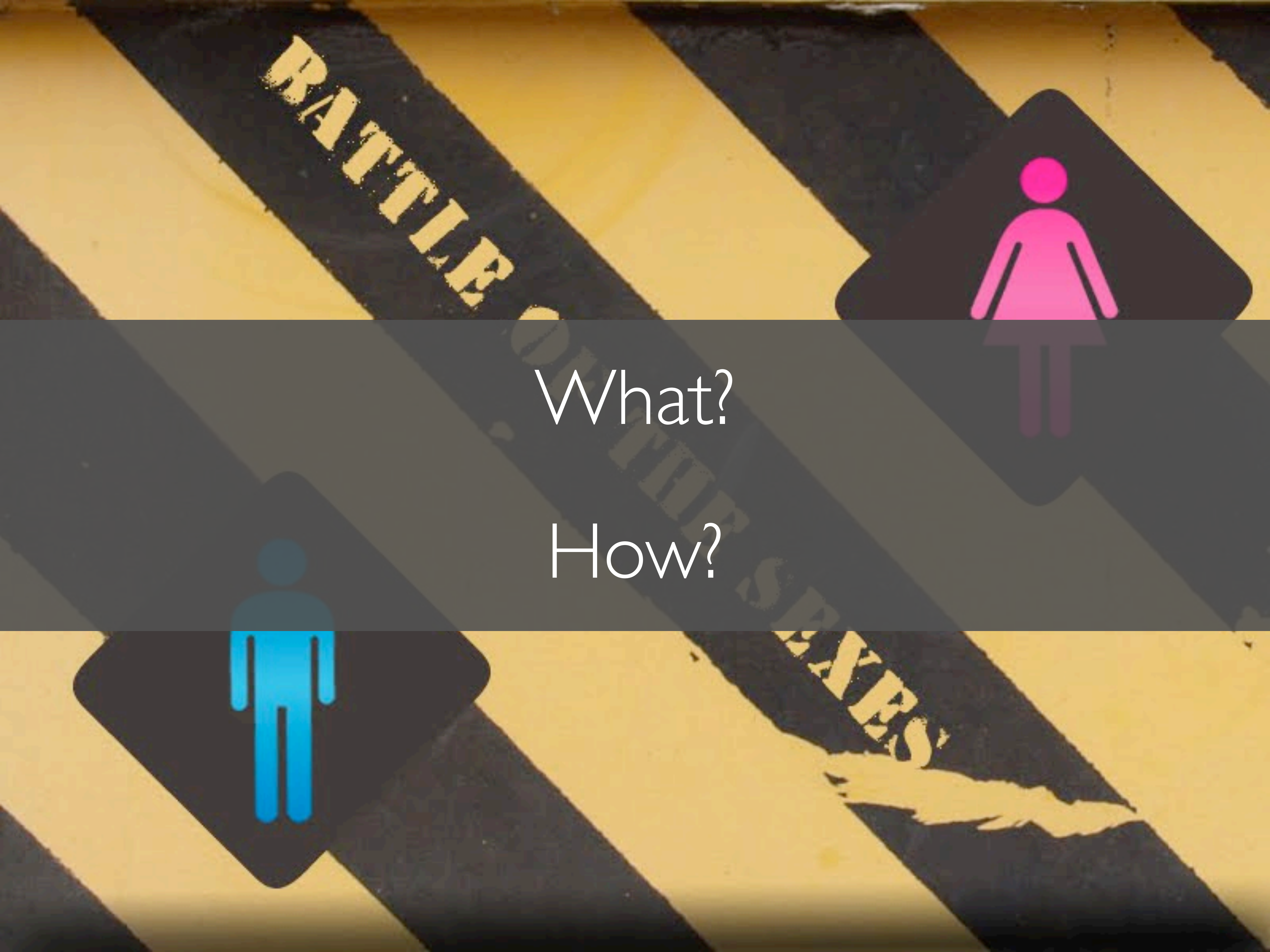
Empirical Evidence Equilibria

Nicolas Dudebout

Game Theory


BATTLE OF THE SEXES

What?



What?

How?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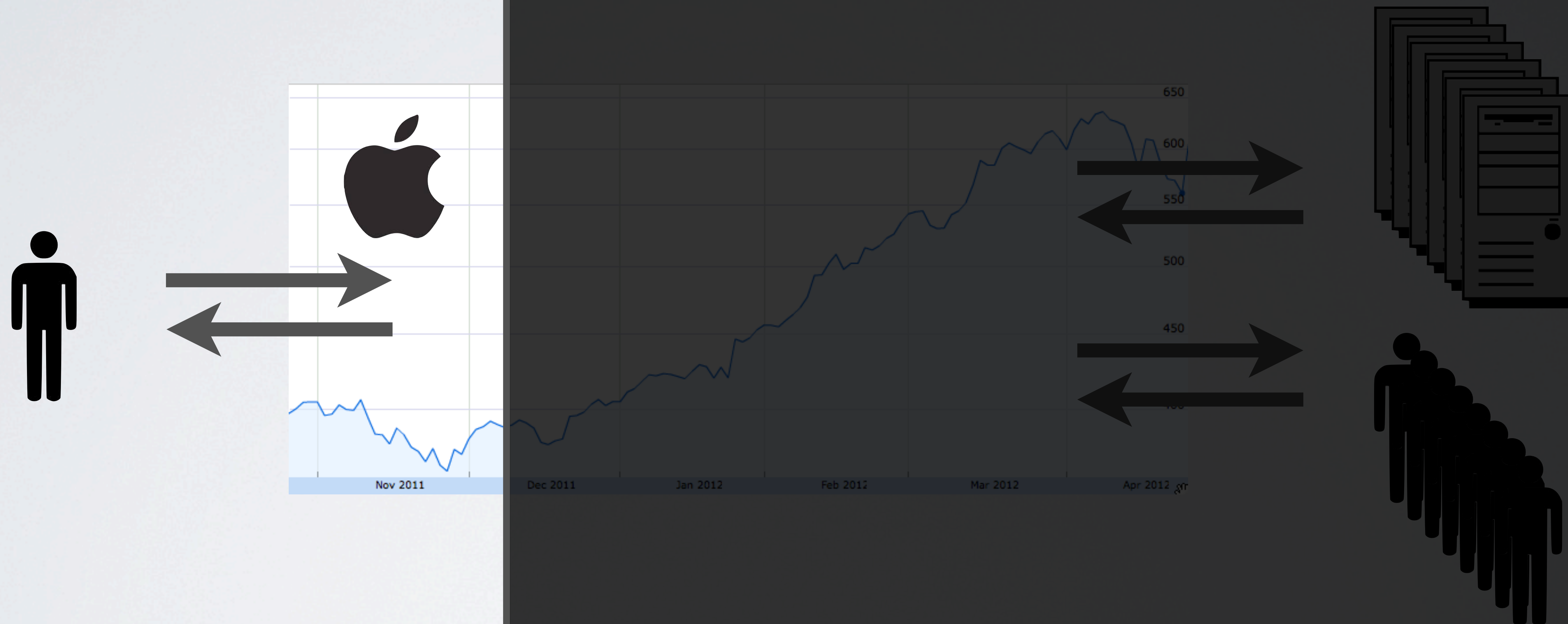
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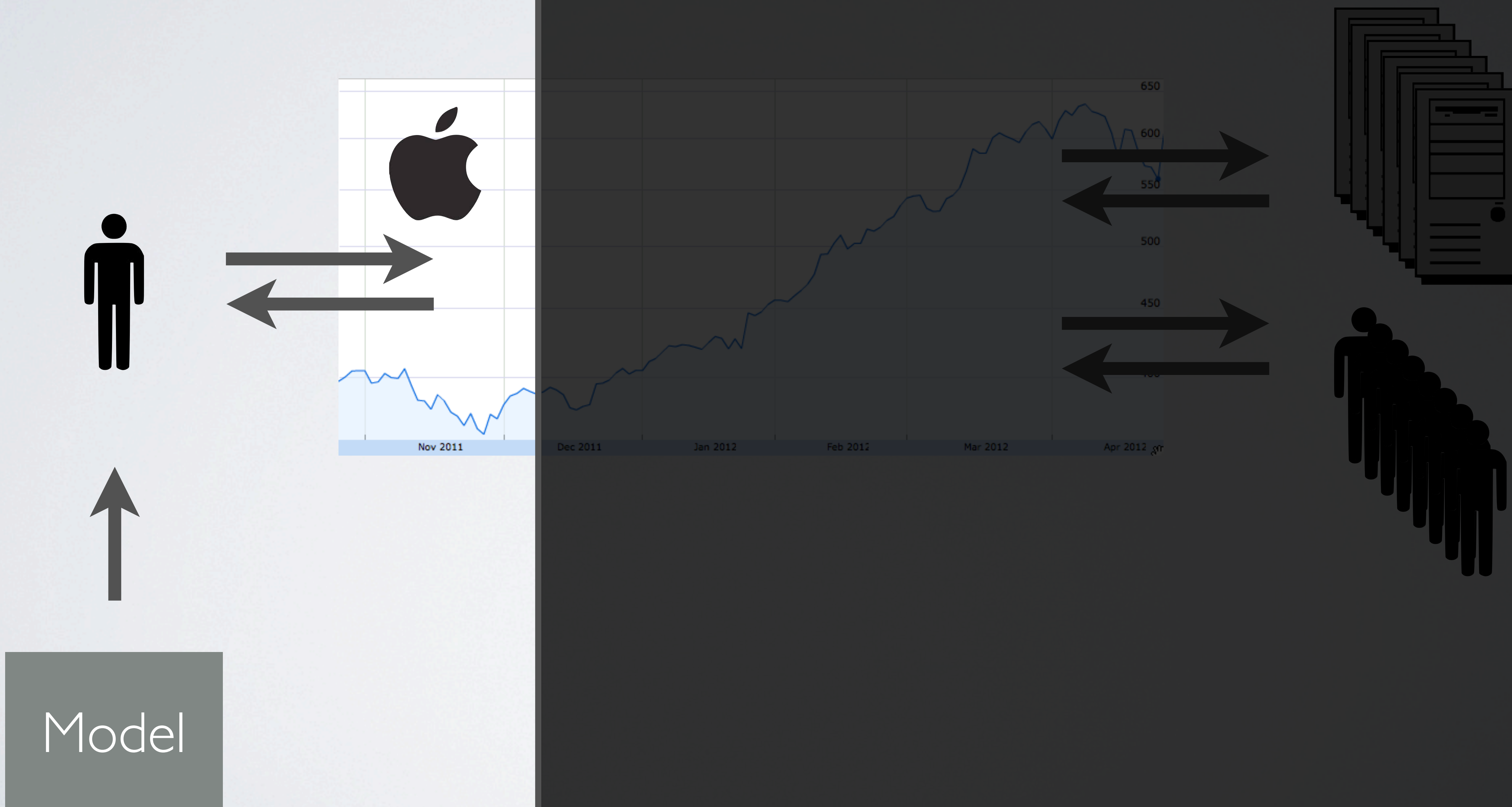
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Model

Empirical Evidence Equilibria

$$x^+ = f(x, a, s)$$

$$u(x, a, s), \delta$$

$$x^+ = f(x, a, s)$$

$$u(x, a, s), \delta$$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$x^+ = f(x, a, s)$$

$$u(x, a, s), \delta$$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

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$$u(x, a, s), \delta$$

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$$\Updownarrow$$

$$X^+ = F(X, a)$$

$$U(X, a), \delta$$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$
\boxed{
\begin{aligned}
&x^+ = f(x, a, s) \\
&u(x, a, s), \delta \\
&\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu
\end{aligned}
}
$$

$$
\begin{aligned}
y^+ &= \varphi(y, x, a) \\
s &= \omega(y)
\end{aligned}
$$

$$\Updownarrow$$

$$
\boxed{
\begin{aligned}
&X^+ = F(X, a) \\
&U(X, a), \delta
\end{aligned}
}
$$

$$
\begin{aligned}
Q(X, a) &= U(X, a) + \delta \mathbb{E}\left[V^*(X^+) \mid a\right] \\
V^*(x) &= \max_a Q(X, a)
\end{aligned}
$$

$$x^+ = f(x, a, s)$$
$$u(x, a, s), \delta$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$\longrightarrow$$

$$y^+ = \varphi(y, x, a)$$
$$s = \omega(y)$$

$$\sigma(X)[a] \sim \mathrm{e}^{-\frac{1}{\tau} Q(X,a)}$$

$$\Updownarrow$$

$$X^+ = F(X, a)$$
$$U(X, a), \delta$$

$$Q(X, a) = U(X, a) + \delta \mathbb{E}\left[V^*(X^+) \mid a\right]$$
$$V^*(x) = \max_a Q(X, a)$$

$$x^+ = f(x, a, s)$$

$$\sigma$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$\begin{aligned} y^+ &= \varphi(y, x, a) \\ s &= \omega(y) \end{aligned}$$

$$x^+ = f(x, a, s) \qquad\qquad y^+ = \varphi(y, x, a)$$

$$\sigma \qquad\qquad s = \omega(y)$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$x^+ = f(x, a, s) \qquad\qquad y^+ = \varphi(y, x, a)$$

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$$\sigma \qquad\qquad s = \omega(y)$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$\chi^k[d] = \lim_{t \to \infty} \mathbb{P}\left[(s^{t-k}, \ldots, s^{t-1}, s^t) = d\right]$$

$$x^+ = f(x, a, s) \qquad y^+ = \varphi(y, x, a)$$

$$\sigma \qquad s = \omega(y)$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$\chi^k[d] = \lim_{t \to \infty} \mathbb{P}\left[(s^{t-k}, \ldots, s^{t-1}, s^t) = d\right]$$

Consistency

$$\chi^k_\mu = \chi^k_\sigma$$

$$x^+ = f(x, a, s) \qquad\qquad y^+ = \varphi(y, x, a)$$

$$\sigma \qquad\qquad s = \omega(y)$$

$$\left.\begin{aligned} z^+ &= g(z, s) \\ s &= h(z) \end{aligned}\right\} \mu$$

$$z^t = (s^{t-k}, \ldots, s^{t-2}, s^{t-1})$$

$$\chi^k[d] = \lim_{t \to \infty} \mathbb{P}\left[(s^{t-k}, \ldots, s^{t-1}, s^t) = d\right]$$

Consistency

$$\chi^k_\mu = \chi^k_\sigma$$

$$x_i^+ = f_i(x_i, a_i, s_i)$$

$$\sigma_i$$

$$x_j^+ = f_j(x_j, a_j, s_j)$$

$$\sigma_j$$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$x_i^+ = f_i(x_i, a_i, s_i)$$

$$\sigma_i$$

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$$\sigma_i \qquad \sigma_j$$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$\Updownarrow$$

$$Y^+ = \Phi(Y)$$

$$s = \Omega(Y)$$

$$x_i^+ = f_i(x_i, a_i, s_i) \qquad\qquad x_j^+ = f_j(x_j, a_j, s_j)$$

$\sigma_i \qquad\qquad \sigma_j$

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$\Updownarrow$$

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$$s = \Omega(Y)$$

π_Φ

$x_i^+ = f_i(x_i, a_i, s_i)$

σ_i

$x_j^+ = f_j(x_j, a_j, s_j)$

σ_j

$$y^+ = \varphi(y, x, a)$$

$$s = \omega(y)$$

$$Y^+ = \Phi(Y)$$

$$s = \Omega(Y)$$

π_Φ

μ_i

Models → Strategies → Models

Models → Strategies → Models

μ_i σ_i

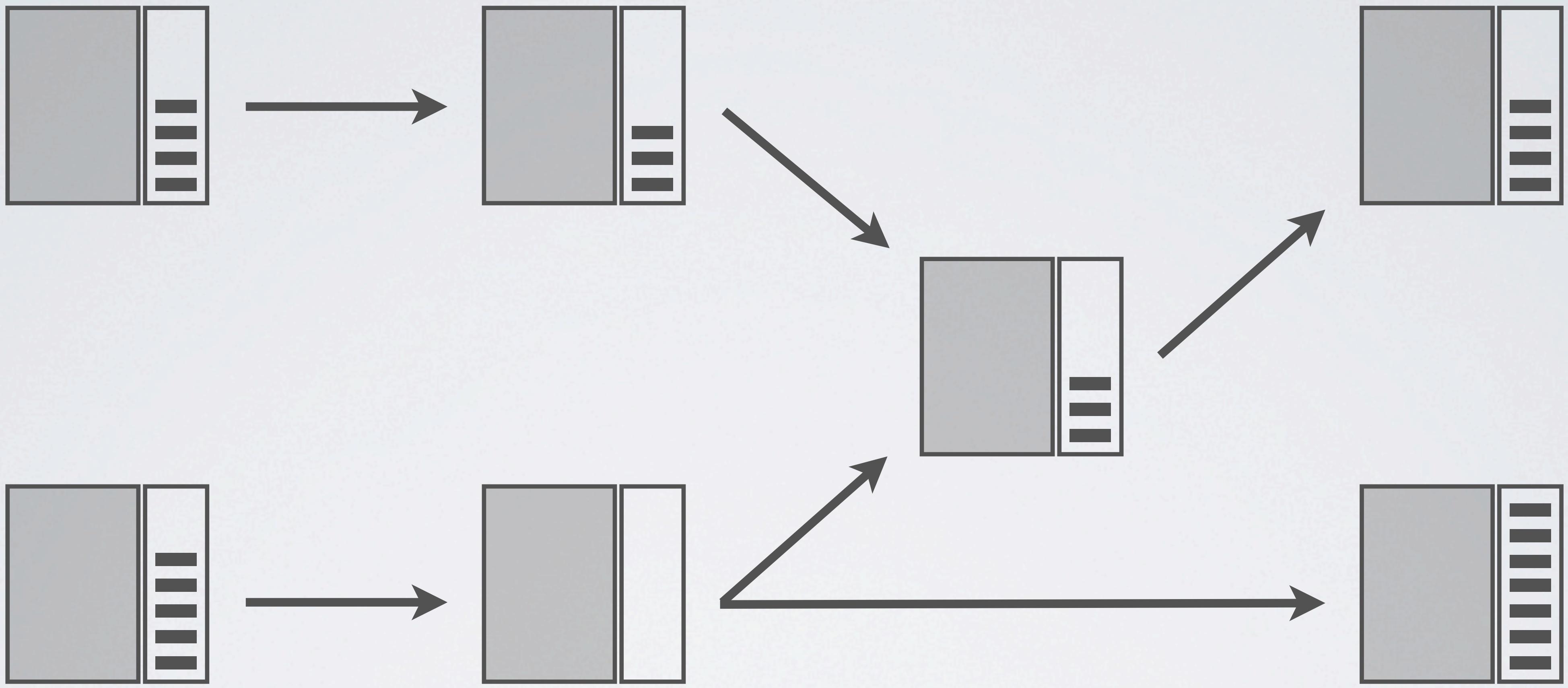
Models $\rightarrow$ Strategies $\rightarrow$ Models

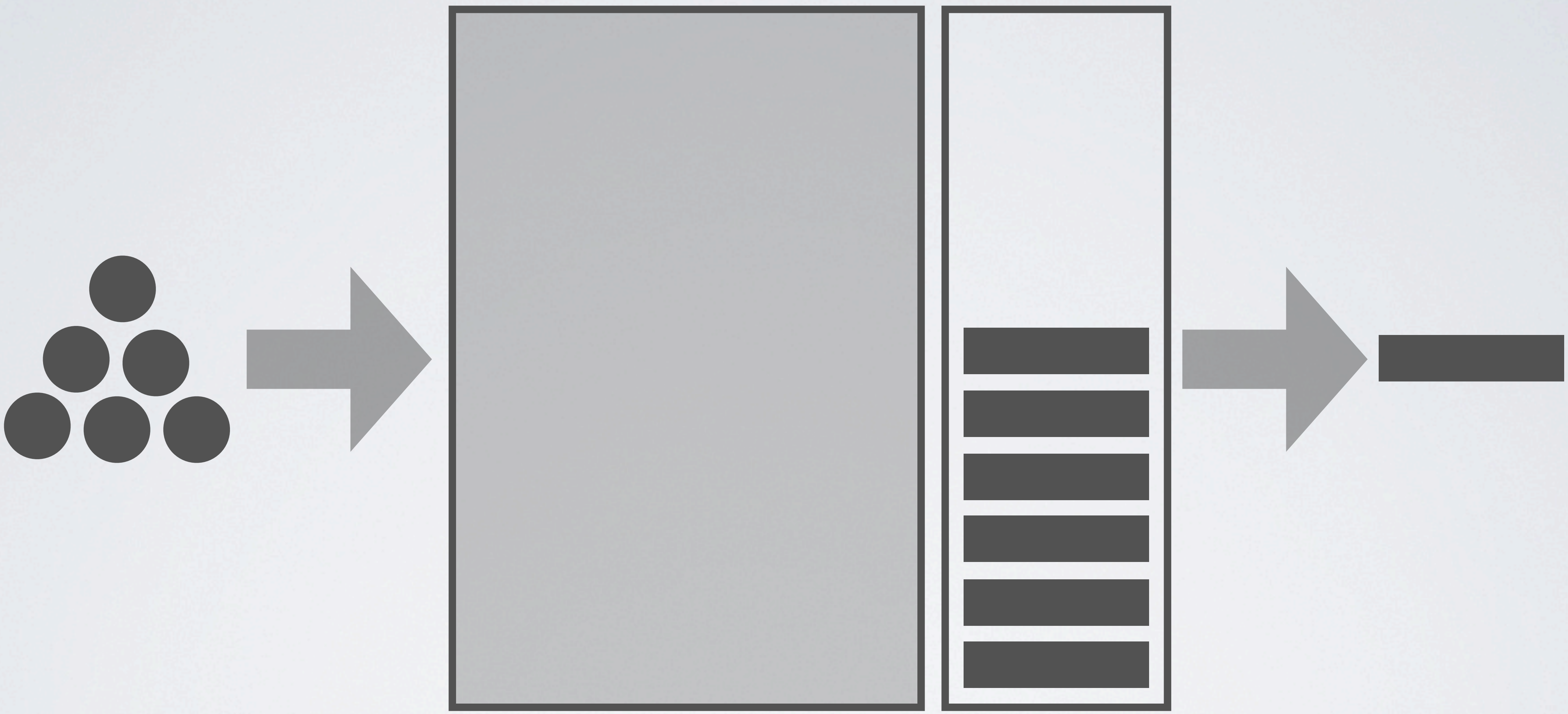
σ $\qquad$ μ_i

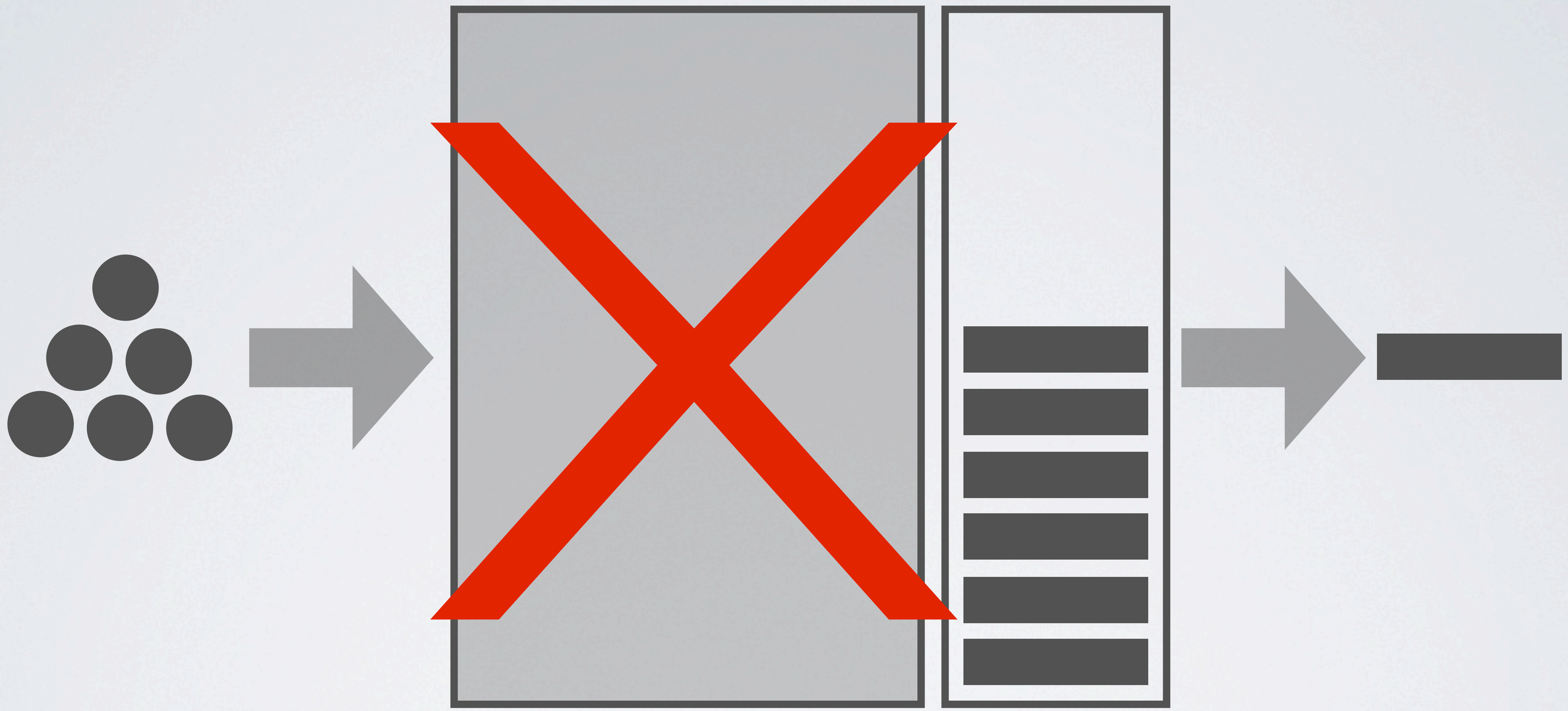
Models $\rightarrow$ Strategies $\rightarrow$ Models

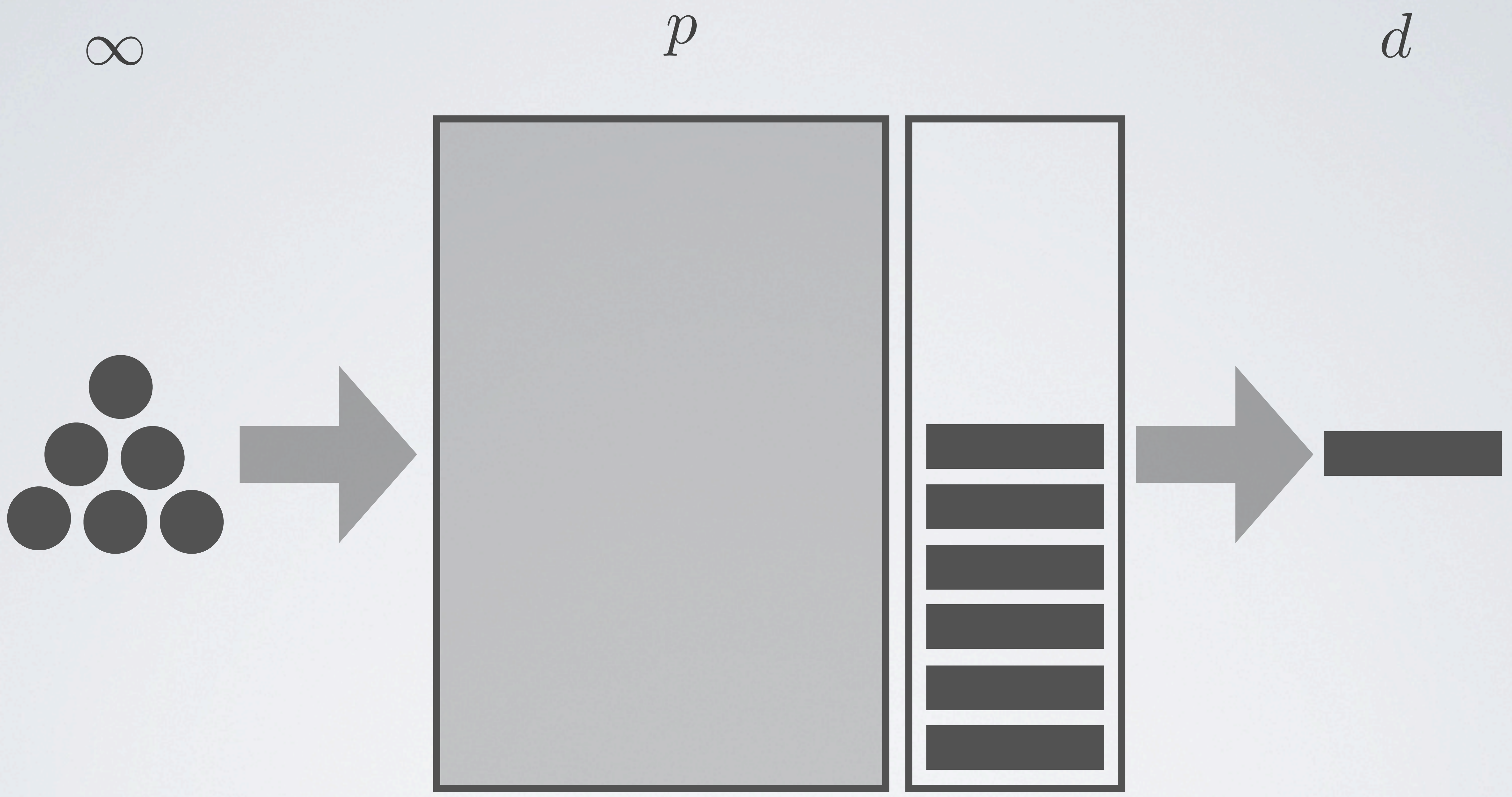
Brouwer's fixed point theorem

Decentralized Control





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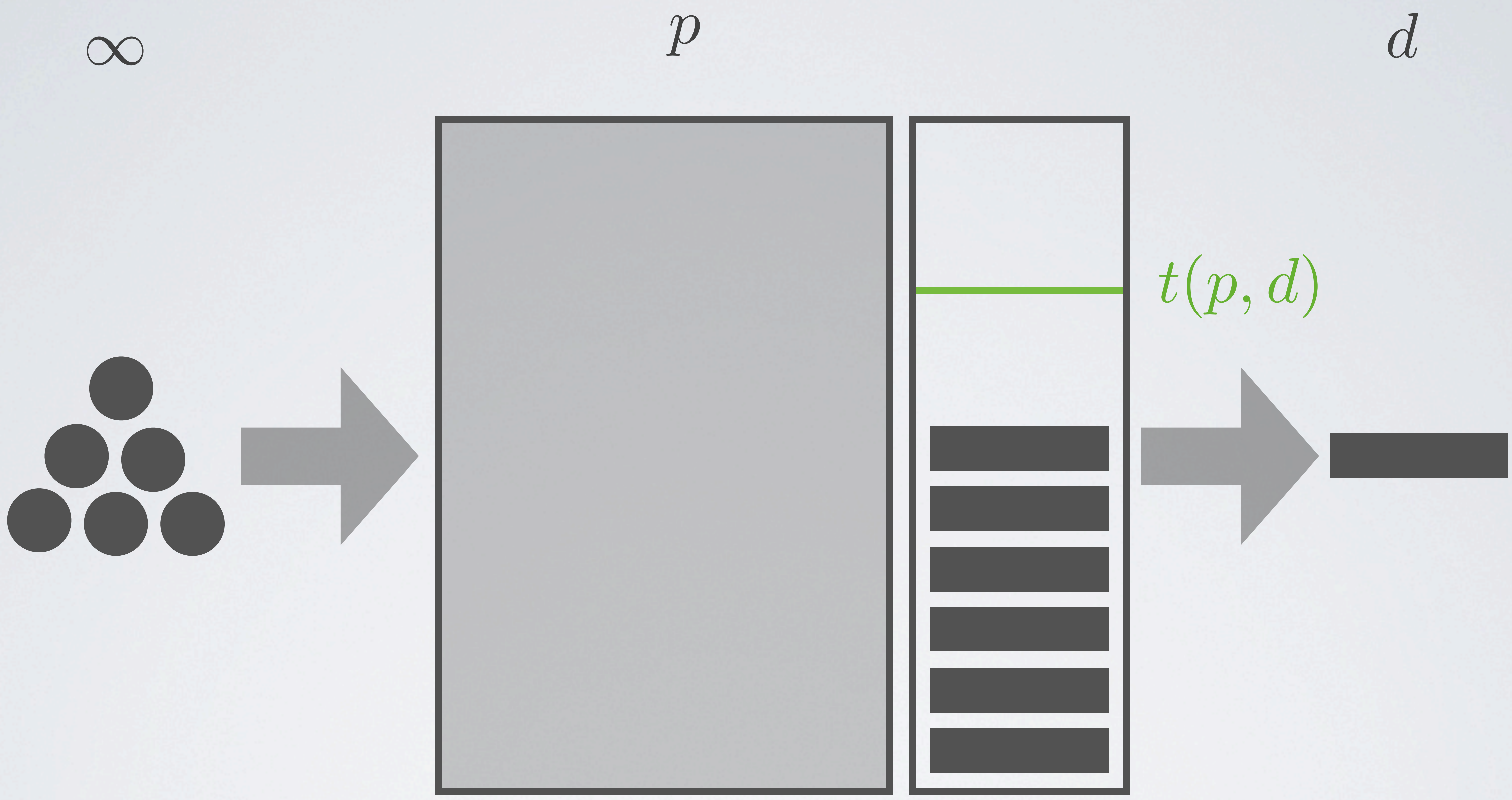

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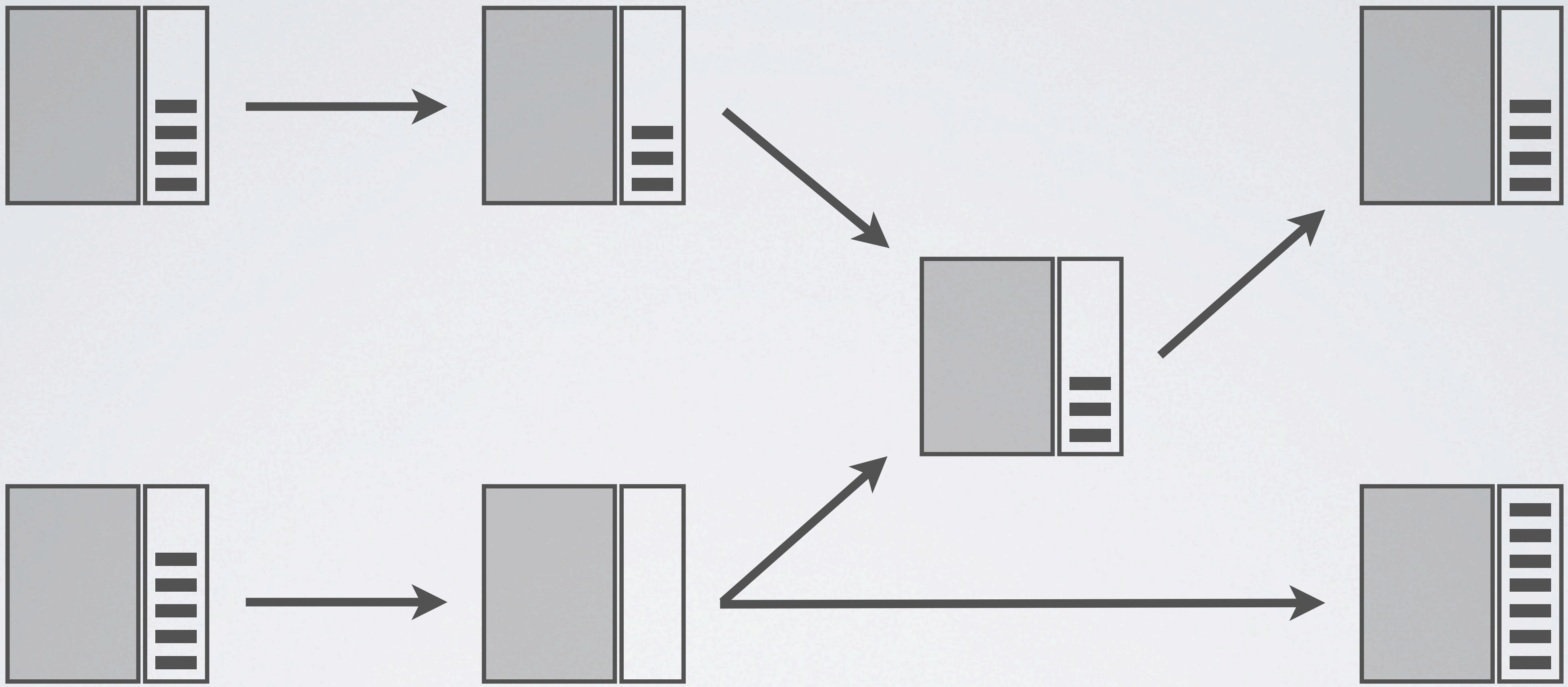

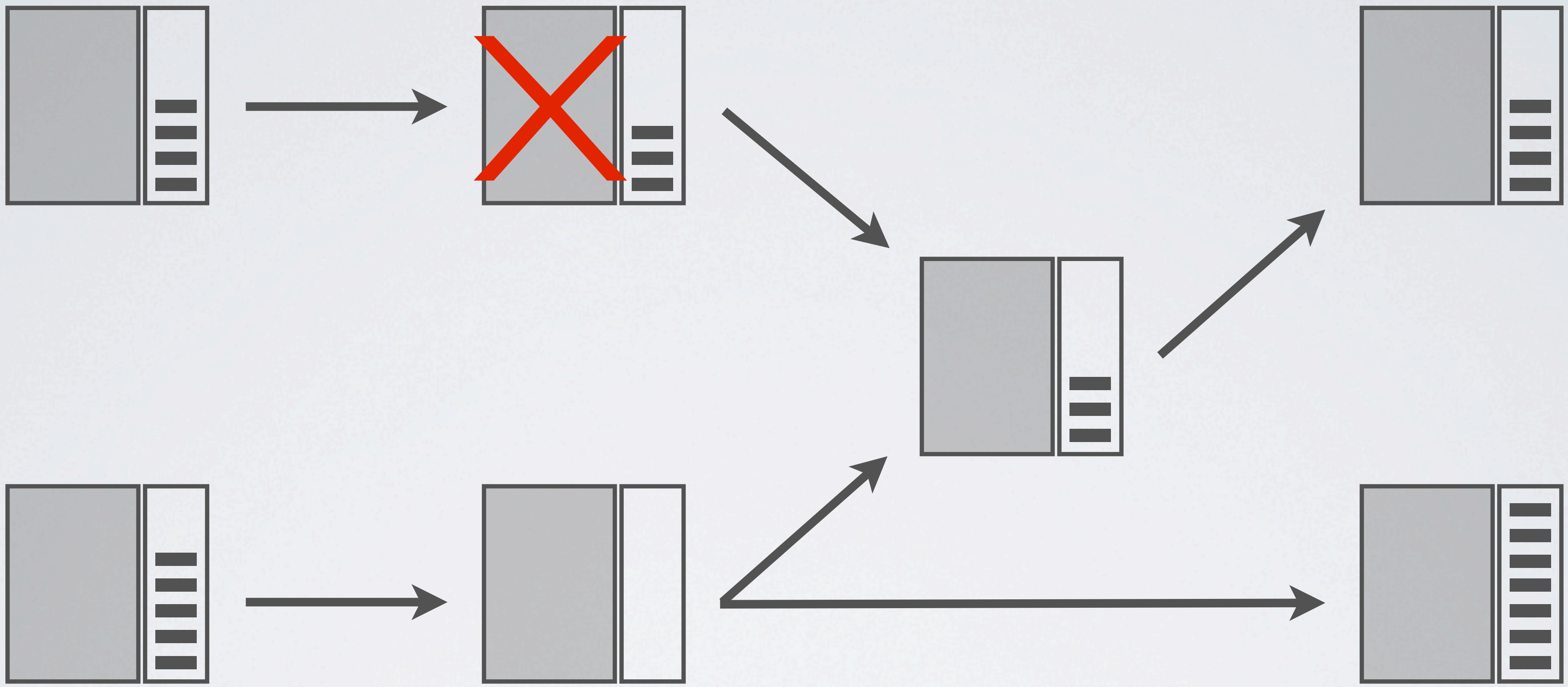

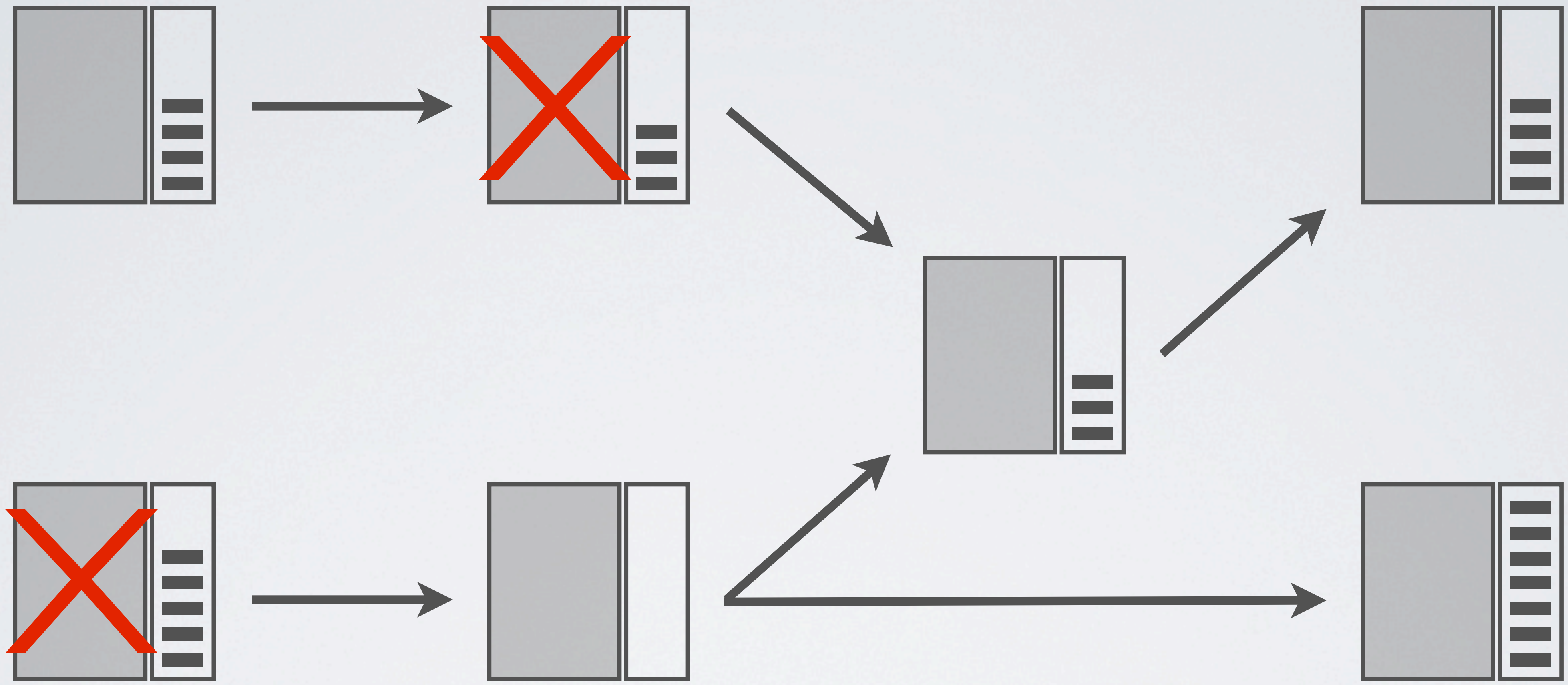

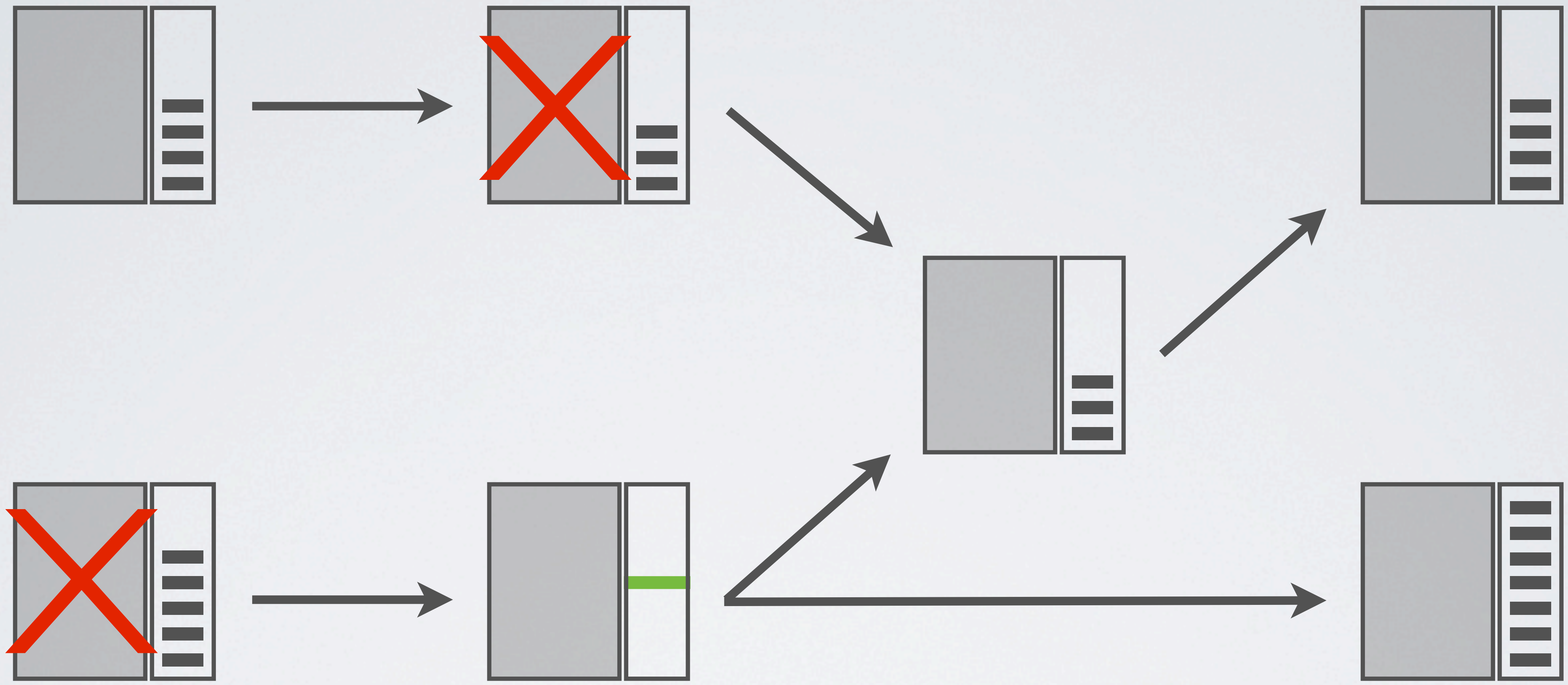

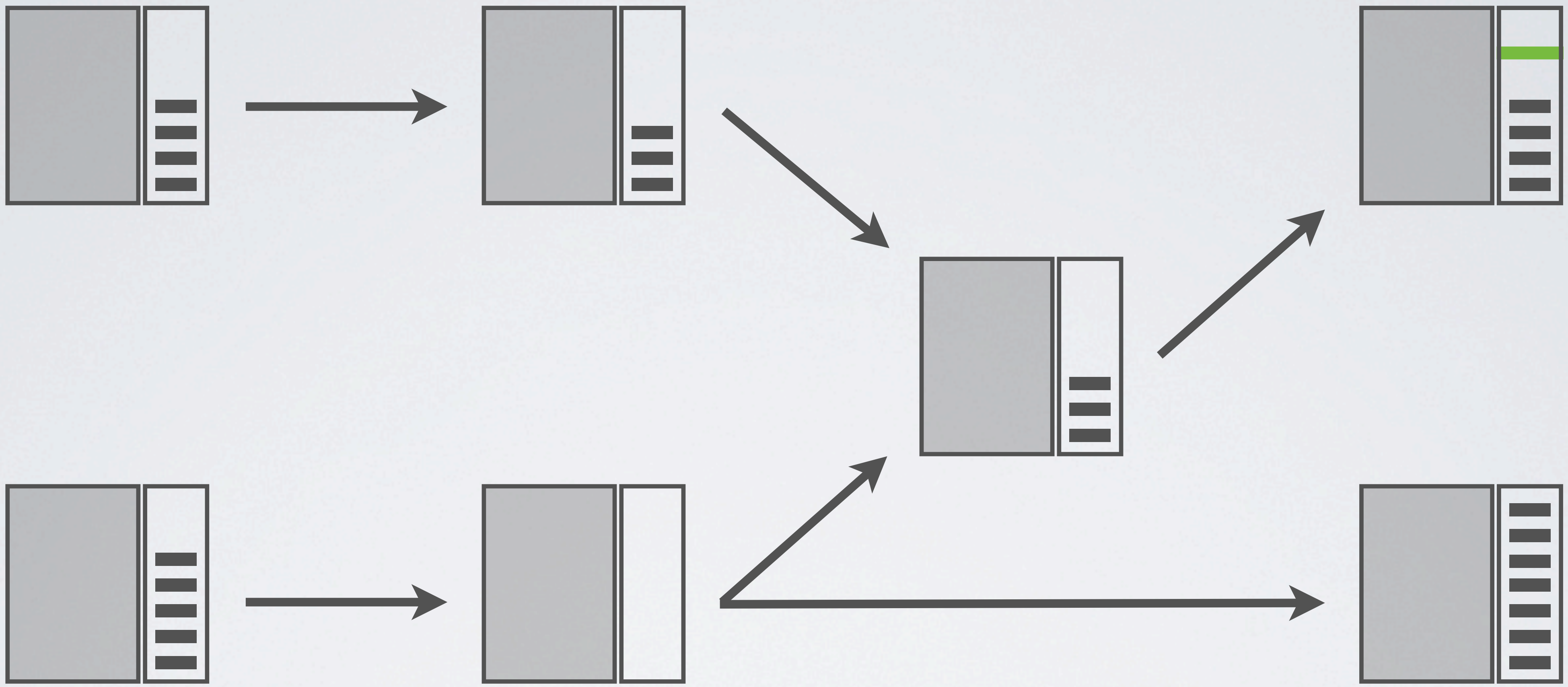

N. Dudebout and J.S. Shamma, "*Empirical Evidence Equilibria in Stochastic Games*", submitted to CDC 2012.