# Game Theory

## Nicolas Dudebout

# What is game theory?

**What** is game theory?

**How** do we study it?

**What** is game theory?

**How** do we study it?

**Where** is research headed?

# What?

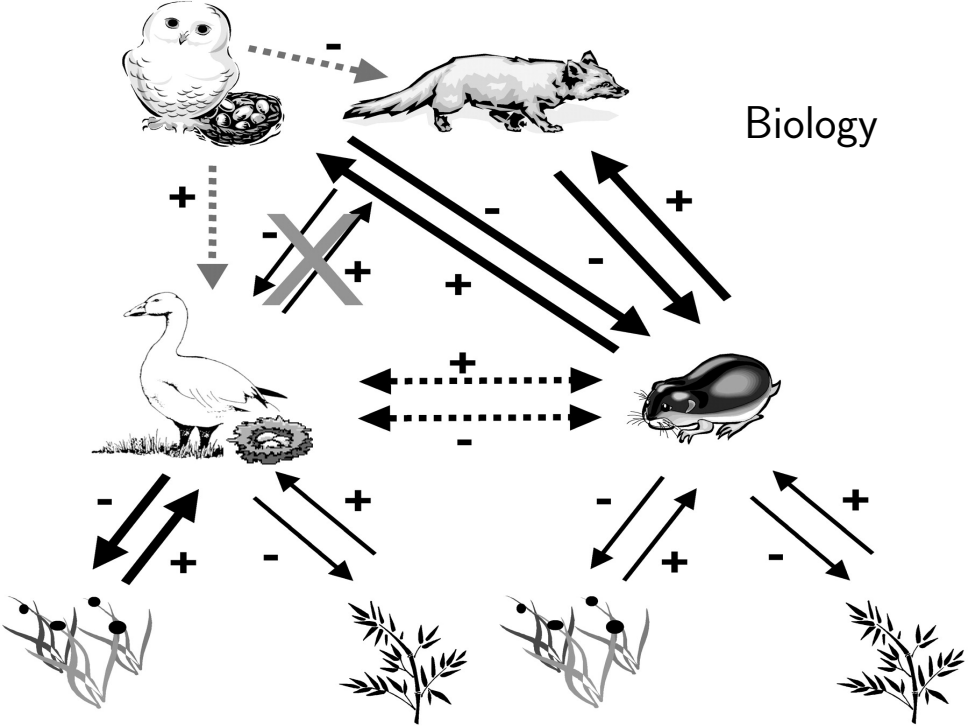The study of interacting decision makers

Economy

Biology

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**facebook**

Email

Password

☐ Keep me logged in    Forgot your password?

Login

## Facebook helps you connect and share with the people in your life.

Sociology

## Sign Up

It's free and always will be.

First Name:

Last Name:

Your Email:

Re-enter Email:

New Password:

I am:  Select Sex:

Birthday:  Month:  Day:  Year:

Why do I need to provide this?

**Sign Up**

**Create a Page** for a celebrity, band or business.

English (US)   Español   Português (Brasil)   Français (France)   Deutsch   Italiano   العربية   हिन्दी   中文(简体)   日本語   »

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Google

google ads                                                          Search        Instant is off ▼

About 53,100,000 results (0.07 seconds)                                          Advanced search

■ Everything

🖼 Images

▶ Videos

**Google AdSense** 🔍
Offers a contextual advertising solution to web publishers Delivers text-based **Google
AdWords** as that are relevant to site content pages.
https://www.**google**.com/**adsense**/ - Cached - Similar

News

⛅ Shopping

▼ More

  **Google** ads                      AdSense Help
  Submit a new application             How do I add more sites to my account?
  AdSense Tour                         **Google** AdSense Case Study
  Adsense reports                      How do I change my email address ...

More results from google.com »

Atlanta, GA
Change location

**Welcome to Google Advertising Programs** 🔍
Allows you to buy advertising on the **Google** search engine, or on other sites through its
**AdSense** program. Includes a tour and FAQ.
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Any time
  Latest
  Past 24 hours
  Past 2 days
  Past week
  Past month
  Past year
  Custom range...

**Google AdWords** - Online Advertising by **Google** 🔍
Advertise with **Google AdWords** ads in the Sponsored Links section next to search results to
boost website traffic and sales. With **Google AdWords** ...
**AdWords** Beginners Guide - What is **Google AdWords**? - **AdWords** ...
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More search tools

Something different
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  affiliate links
  banner ads
  text ads
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**How to Make Money With Google AdSense** 🔍
Now you can earn a share of the revenue that **Google** earns from **AdWords** by displaying
these same text ads on your site. In other words, you're helping **Google** ...
www.2createawebsite.com › Making Money - Cached - Similar

**AdSense** - Wikipedia, the free encyclopedia 🔍
**AdSense** is an ad serving application run by **Google** Inc. Website owners can enroll in this
program to enable text, image, and video advertisements on their ...
en.wikipedia.org/wiki/**AdSense** - Cached - Similar

Computer
Science

Engineering

Different agendas

# What?

# What?

- study of interacting decision makers

# What?

- study of interacting decision makers
- interdisciplinary field

# What?

- study of interacting decision makers
- interdisciplinary field
- different agendas

# How?

# Decision maker

# Decision maker

- choices, $C$

# Decision maker

- choices, $C$
- preferences, $\succeq$

# Decision maker

- choices, $C$
- preferences, $\succeq$
  utility function, $u : C \to \mathbb{R}$

$$c_1 \succeq c_2 \iff u(c_1) \geq u(c_2)$$

$C = \{L, R\}$

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$u : C \to \mathbb{R}$
$L \mapsto 0$
$R \mapsto 1$
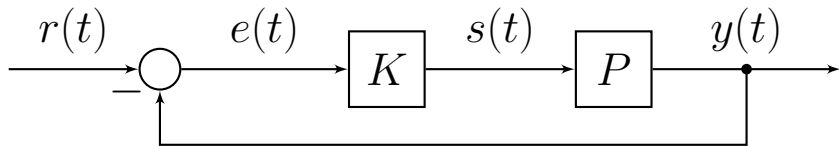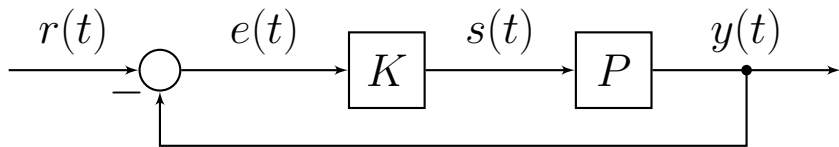
- $C = \{\text{stabilizing controller } K\}$
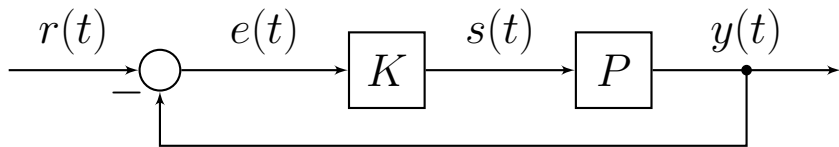
- $C = \{\text{stabilizing controller } K\}$
- $u(K) = -\tau_r(K)$

# Optimality

Decision maker:

- choices, $C$
- utility function, $u$

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Decision maker:

- choices, $C$
- utility function, $u$

Goal of decision maker:

$$\max_{c \in C} u(c)$$

# Game Theory

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- players, $\{i\}$

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- choices for player $i$, $C_i$

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  $c \in C = (c_i, c_{-i})$

# Game Theory

- players, $\{i\}$
- choices for player $i$, $C_i$
- joint choices, $C = \prod_i C_i$
  $c \in C = (c_i, c_{-i})$
- utility function for player $i$, $u_i : C \to \mathbb{R}$

# Optimality?

Goal of decision maker $i$:

$$\max_{c \in C} u_i(c_i, c_{-i}) \left( \neq \max_{c \in C} u_i(c_i) \right)$$

# Example: Prisoner's dilemma

|     | $C$       | $D$        |
| --- | --------- | ---------- |
| $C$ | $2, 2$    | $-1, 3$    |
| $D$ | $3, -1$   | $0, 0$     |

# Example: Prisoner's dilemma

|       | $C$ | $D$  |
|-------|-----|------|
| $C$   | 2   | $-1$ |
| $D$   | 3   | 0    |

# Example: Prisoner's dilemma

|       | $C$ |   |
| :---: | :---: | :---: |
| $C$ | 2 |   |
| $D$ | 3 |   |

# Example: Prisoner's dilemma

$$C$$

| | $C$ | |
|---|---|---|
| $C$ | 2 | |
| $D$ | 3 | |

Best response, $\text{BR}_i : C_{-i} \rightrightarrows C_i$

# Example: Prisoner's dilemma

|     | $C$ |     |
| --- | --- | --- |
| $C$ | 2   |     |
| $D$ | 3   |     |

Best response, $\text{BR}_i : C_{-i} \rightrightarrows C_i$

- $\text{BR}_1(C) = \{D\}$

# Example: Prisoner's dilemma

|       | $D$    |
| :---- | :----- |
| $C$   | $-1$   |
| $D$   | $0$    |

Best response, $\text{BR}_i : C_{-i} \rightrightarrows C_i$

- $\text{BR}_1(C) = \{D\}$

# Example: Prisoner's dilemma

|     | $D$  |
| --- | ---- |
| $C$ | $-1$ |
| $D$ | $0$  |

Best response, $BR_i : C_{-i} \rightrightarrows C_i$

- $BR_1(C) = \{D\}, BR_1(D) = \{D\}$

# Example: Prisoner's dilemma

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Best response, $\mathsf{BR}_i : C_{-i} \rightrightarrows C_i$

- $\mathsf{BR}_1(C) = \{D\}, \mathsf{BR}_1(D) = \{D\}$
- $\mathsf{BR}_2(C) = \{D\}, \mathsf{BR}_2(D) = \{D\}$

# Nash equilibrium

$a^* = (a_i^*, a_{-i}^*)$ is a Nash equilibrium:

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- ► $\forall i$, $a_i^*$ is a best response to $a_{-i}^*$

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- no unilateral deviation is profitable

# Nash equilibrium

$a^* = (a_i^*, a_{-i}^*)$ is a Nash equilibrium:

- $\forall i$, $a_i^*$ is a best response to $a_{-i}^*$
- no unilateral deviation is profitable
- $\forall i$, $\forall a_i \in A_i$,

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$$

# Existence of Nash equilibria

Every $n$-player game has a Nash equilibrium.

# Extensions

# Extensions

- history-dependent strategy

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- history-dependent strategy
- imperfect information

# Extensions

- history-dependent strategy
- imperfect information
- cooperation

# Extensions

- history-dependent strategy
- imperfect information
- cooperation
- large populations

# Back to the agendas

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- descriptive

# Back to the agendas

- descriptive
- predictive

# Back to the agendas

- descriptive
- predictive
- manipulative

# How?

# How?

- interacting decision maker

# How?

- interacting decision maker
- best response

# How?

- interacting decision maker
- best response
- Nash equilibrium

# Where?

# Learning

Controls $\Rightarrow$ Game Theory:
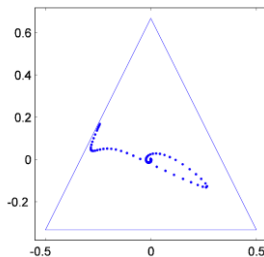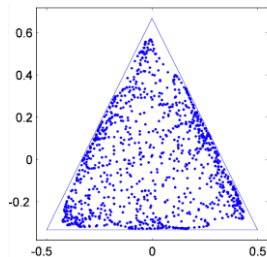
# Learning

Controls $\Rightarrow$ Game Theory:

- ▶ stability and robustness

# Learning

Controls $\Rightarrow$ Game Theory:

- ▶ stability and robustness
- ▶ derivative control

# Decentralized control

Game Theory $\Rightarrow$ Controls:

# Decentralized control

Game Theory $\Rightarrow$ Controls:

- network formation

# Decentralized control

Game Theory $\Rightarrow$ Controls:

- ▶ network formation
- ▶ communication limitations

# Dynamic Games

# Dynamic Games

- network security

# Dynamic Games

- network security
- learning in repeated games

# Where?

# Where?

- learning

# Where?

- learning
- decentralized control

# Where?

- learning
- decentralized control
- dynamic games

Questions?
Comments?