BASIC OPTIMIZATION

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IITH Future Wireless Communication (FWC)

FWC22098 Assignment-optimization

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Contents

1 Problem

If $\mathbf{p(x)}$ be a polynomial of degree 3 satisfying $\mathbf{p(-1) = 10}$, $\mathbf{p(1) = -6}$ and $\mathbf{p(x)}$ has maximum at $\mathbf{x = -1}$ and $\mathbf{p(x)}$ has minima at $\mathbf{x = 1}$. Find the distance between the local maximum and local minimum of the curve.

2 Solution

Let the polynomial be

$$p(x) = ax^3 + bx^2 + cx + d \tag{1}$$
$$p(-1) = -a + b - c + d = 10 \tag{2}$$
$$p(1) = a + b + c + d = -6 \tag{3}$$
$$\frac{dp(-1)}{dx} = 3a - 2b + c = 0 \tag{4}$$
$$\frac{dp'(1)}{dx} = 6a + 2b = 0 \tag{5}$$

From equations (2), (3), (4), (5)
We have 4 equations and 4 unknowns
The general equation of a line is,

$$\mathbf{n}^\top \mathbf{x} = \mathbf{c} \tag{6}$$
$$\mathbf{n} = \begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} & \mathbf{n_3} & \mathbf{n_4} \end{pmatrix} \tag{7}$$

where

$$\mathbf{n_1} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{n_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{n_3} = \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{n_4} = \begin{pmatrix} 6 \\ 2 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{c} = \begin{pmatrix} 10 \\ -6 \\ 0 \\ 0 \end{pmatrix}, \tag{9}$$

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & -2 & 1 & 0 \\ 6 & 2 & 0 & 0 \end{bmatrix}^\top \mathbf{x} = \begin{pmatrix} 10 \\ -6 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{n}^{-\top} \mathbf{c} \tag{10}$$

$$\mathbf{x} = \begin{bmatrix} 0.0625 & -0.0625 & 0.125 & 0.125 \\ -0.1875 & 0.1875 & -0.375 & 0.125 \\ -0.5625 & 0.5625 & -0.125 & -0.125 \\ 0.625 & 0.3125 & 0.375 & 0.125 \end{bmatrix} \begin{pmatrix} 10 \\ -6 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -3 \\ -9 \\ 5 \end{pmatrix} \tag{11}$$

So the values of a,b,c and d are 1,-3,-9 and 5 respectively.
Finally the cubic polynomial is

$$p(x) = x^3 - 3x^2 - 9x + 5 \tag{12}$$

we can find the maxima of eq(12) by using gradient ascent method $\implies x_{n+1} = x_n + \alpha \nabla f(x_n)$

$$\implies x_{n+1} = x_n + \alpha \left(3x^2 - 6x - 9\right) \tag{13}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Maxima} = 9.999999999995849} \tag{14}$$
$$\boxed{\text{Maxima Point} = -0.9999991682168597} \tag{15}$$

we can find the minima of eq(12) by using gradient descent method $\implies x_{n+1} = x_n - \alpha \nabla f(x_n)$

$$\implies x_{n+1} = x_n - \alpha \left(3x^2 - 6x - 6\right) \tag{16}$$

Taking $x_0 = 1.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Minima} = -21.999999999995847} \tag{17}$$
$$\boxed{\text{Minima Point} = 2.999999168216859} \tag{18}$$

The distance between the local maximum and local minimum of the curve

$$A = (-0.9999991682168597, 9.999999999995849) \tag{19}$$
$$B = (2.999999168216859, -21.999999999995847) \tag{20}$$

$$||A - B|| = 4\sqrt{65} \tag{21}$$

3 Construction

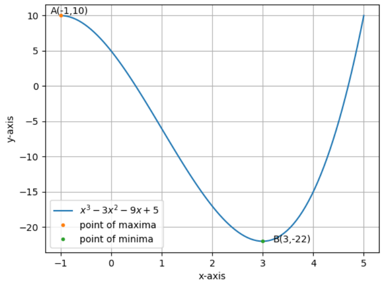

Figure of construction

4 Software

Below python code realizes the above construction :
https://github.com/dudekulauseni123/FWC0982022