Probability
Assignment
November 18, 2022

Instructions:

- *The total number of marks is 50. 5 marks for each question.*

- *You can return your handwritten assignment after the class on Tuesday.*

- *You need not submit the coding assignments (and hence no marks).*

Problem 1. Show $P(A \cap B) \geq P(A) + P(B) - 1$

Problem 2. A box contains 1000 light bulbs. The probability that there is atleast one defective bulb in the box is 0.1 and the probability that there are atleast 2 defective bulbs is 0.05. Find the probability that

- There are no defective bulbs

- There is exactly one defective bulb

- There is atmost one defective bulb

Problem 3. A fair dice is rolled multiple times till we get at-least one 5 and one 6. What is the probability that we need 5 tosses.

Problem 4. A fair coin is tossed n times. What is the probability that the difference between number of heads and number of tails is $n - 3$.

Problem 5. Let E and F be mutually exclusive events of a random experiment. A random experiment is performed repeatedly (and independently) till either E or F occurs. Show that the probability of E occurring before F if $P(E)/(P(E) + P(F))$.

Problem 6. Suppose 5 percent men and 15 percent women are left handed. If a left handed person is chosen at random, what is the probability of this person being male?

Problem 7. Let X be a random variable uniformly distributed over $\{0, 1, 2, \cdots, N\}$, find $\mathbb{E}[X]$.

Problem 8. Let X be a normal distribution with mean μ and variance σ^2. What is the mean and varaince of random variable $Y := X^2$.

Problem 9. Let X be exponential random variable. Calculate the probabilities of

1. $|X| \leq 3$

2. $X \leq 4$ or $X \geq 10$

Problem 10. Consider the game of coin tosses between three persons as follows. Each person tosses a coin once. If the result of coin toss of any one person is different that other two, the game ends. Otherwise they go for the next round. What is the probability that the game ends in the first round of tosses?

Problem 11 (coding assignment). Let U be a uniform random variable between 0 and 1. Generate 10^5 samples of U (using a C/Python program) and save into a file. From the saved data

- plot empirical CDF $F_U(x) = \frac{\#\text{points with value} \leq x}{N}$ for different values of x

- find empirical mean and variance.