$$\frac{d^2\phi}{dx^2} = 4\pi G \rho(x)$$

$\phi(0) = 5$

$\phi(3) = 4$

$\rho(x) \to x \in [0,3]$ $\qquad \underline{\underline{\Omega = [0,3]}}$

$$\rho(x) = \begin{cases} 0, & x \in [0,1] \\ 1, & x \in (1,2] \\ 0, & x \in (2,3] \end{cases}$$

$v \in \mathcal{U}$

$$\frac{d^2\phi}{dx^2} v = 4\pi G \rho(x) v \quad / \int_\Omega$$

$$\int_0^3 \frac{d^2\phi}{dx^2} v\,dx = \int_0^3 4\pi G \rho(x) \cdot v\,dx$$

$$\underbrace{\phi' v \Big|_0^3}_{\phi'(3)\underset{0}{v(3)} - \phi'(0)\underset{0}{v(0)}} - \int_0^3 \phi' v'\,dx = 4\pi G \int_1^2 v\,dx$$

$$\underbrace{-\int_0^3 \phi' v'\,dx}_{B(\phi, v)} = \underbrace{4\pi G \int_1^2 v\,dx}_{L(v)}$$

$\phi = w + u \qquad w \in \mathcal{U} \subset H^1(\Omega)$

$w(0) = 0$

$w(3) = 0$

$\mathcal{U} = \{f : f(0) = 0 \wedge f(3) = 0\}$

$u(x) = 5 - \frac{x}{3} \qquad \phi' = w' - \frac{1}{3}$

$$B(\phi, v) = B(w + u, v) = B(w, v) + B(u, v)$$

$$B(w, v) = \tilde{L}(v) \rightarrow \tilde{L}(v) = L(v) - B(u, v)$$

$$-\int_0^3 \left(w' - \frac{1}{3}\right) v' dx = 4\pi G \int_1^2 v dx$$

$$\underbrace{-\int_0^3 w' v' dx}_{B(w,v)} = \underbrace{4\pi G \int_1^2 v dx + \frac{1}{3}\int_0^3 v' dx}_{\tilde{L}(v)}$$

$$h = \frac{3}{n-1}$$

$$n \in \{1, 2, \dots, n-1\}$$

$$e_n(x) = \begin{cases} \frac{x}{h} - n + 1, & x \in [h(n-1); h \cdot n] \\ n - \frac{x}{h} + 1, & x \in [hn; h(n+1)] \end{cases}$$

$$B \cdot X = L$$

$$w \approx w_h = \alpha_1 e_1 + \dots + \alpha_{n-1} e_{n-1}$$

$$\phi \approx \phi_h - \bar{u} + w$$

Bartosz Dudek