a)

$$\operatorname{sh} x = \begin{cases} S_1 = x,\ a_1 = x,\ f_1 = 1 \\ S_n = S_{n-1} + a_n,\ n \geq 2 \\ a_n = \dfrac{x^2}{f_n} \cdot a_{n-1},\ n \geq 2 \\ f_n = 2n \cdot (2n+1) \cdot f_{n-1},\ n \geq 2 \end{cases}$$

b)

$$\ln(1+x) = \begin{cases} a_1 = x,\ S_1 = x \\ S_n = S_{n-1} + \dfrac{a_n}{n},\ n \geq 2 \\ a_n = (-1)^{n-1} \cdot a_{n-1},\ n \geq 2 \end{cases}$$

c)

$$\frac{1}{1+x} = \begin{cases} a_0 = 1,\ S_0 = 1 \\ S_n = S_{n-1} + a_n,\ n \geq 1 \\ a_n = (-1)^n \cdot x,\ n \geq 1 \end{cases}$$

d)

$$\ln\left(\frac{1+x}{1-x}\right) = \begin{cases} a_1 = x,\ S_1 = x \\ a_n = a_{n-1} \cdot x^2,\ n \geq 2 \\ S_n = S_{n-1} + \dfrac{a_n}{n} \\ [\text{return } 2 \cdot S_n] \end{cases}$$

e) $\frac{1}{(1+x)^2} = \begin{cases} S_1 = 1,\ a_1 = 1 \\ a_n = x \cdot a_{n-1},\ n \geq 2 \\ S_n = S_{n-1} + (-1)^{n-1} \cdot n \cdot a_n,\ n \geq 2 \end{cases}$

f) $\frac{1}{1+x^2} = \begin{cases} S_0 = 1,\ a_0 = 1 \\ a_n = x^2 \cdot a_{n-1},\ n \geq 1 \\ S_n = S_{n-1} + (-1)^n \cdot a_n,\ n \geq 1 \end{cases}$

g) $\sqrt{1+x} = \begin{cases} a_1 = \frac{x}{2},\ S_1 = \frac{x}{2} \\ a_n = (-1)^{n+1} \cdot \frac{(2k-1)}{2k} \cdot x \cdot a_{n-1},\ n \geq 2 \\ S_n = S_{n-1} + a_n,\ n \geq 2 \end{cases}$

[return $(1+S_n)$]

0.3

a) $\sum_{k=0}^{n} \frac{x^{2k}}{2^k \cdot k!} = \begin{cases} S_0 = 1,\ a_0 = 1 \\ S_n = S_{n-1} + a_n,\ n \geq 1 \\ a_n = \frac{x^2}{2n} \cdot a_{n-1},\ n \geq 1 \end{cases}$

b) $\sum_{k=0}^{n} \frac{(-1)^k x^{(2k+1)}}{k!(2k+1)} = \begin{cases} S_0 = x,\ a_0 = x \\ S_n = S_{n-1} + \frac{a_n}{(2n+1)},\ n \geq 1 \\ a_n = -\frac{x^2}{n} \cdot a_{n-1},\ n \geq 1 \end{cases}$

$\frac{a_k}{a_{k-1}} = -\frac{x^2 \cdot (2k-1)}{k\ (2k+1)}$

$a_k = \frac{-x^2}{k} \cdot a_{k-1},\ k \geq 1$