Atividade prática 2 - Parte 1.

1) $U = \{Paris, NYC\}$

$V = \{Paris, Beijing, Otawa, London\}$

$W = \{Bruges, Stockholm, Moscow\}$

$Q(U,V)$ = "muito longe"

$R(V,W)$ = "muito perto"

$L(U,V)$ = "culturalmente afins"

$Q(U,V) = 0/(Paris, Paris) + 0.8/(Paris, Beijing) + 0.6/(Paris, Otawa) + 0.25 (Paris, London) + 0.7/(NYC, Paris) + 0.98/(NYC, Beijing) + 0.15/(NYC, Otawa) + 0.5/(NY, London)$

$R(V,W) = 1/(Paris, Bruges) + 0.4/(Paris, Estocolmo) + 0.2/(Paris, Moscow) + 0.9/(Beijing, Bruges) + 0.9/(Beijing, Stockolmn) + 0.7/(Beijing, Mosow) + 0.4/(Otawa, Bruges) + 0.15/(Otawa, Stockholm) + 0.05/(Otawa, Moscow) + 0.85/(London, Bruges) + 0.3/(London, Stockholm) + 0.1/(London, Moscow).$

$L(U,V) = 1/(Paris, Paris) + 0.2/(Paris, Beijing) + 0.6/(Paris, Otawa) + 0.8/(Paris, L) + 0.85/(NYC, P) + 0.3/(NYC, Beijing) + 0.8/(NY, Otawa) + 0.88/(NY, L)$

Determine:

$M(U,V)$ = ("muito longe" E "não culturalmente afins")

$P = (Q \circ R)$ = composição max-produto de $Q(U,V)$ e $R(V,W)$

$$\mu_{Q(U,V)} = \begin{array}{c|cccc} & Paris & Beijing & Otawa & London \\ \hline Paris & 0 & 0,8 & 0,6 & 0,25 \\ NYC & 0,7 & 0,98 & 0.15 & 0,5 \end{array}$$

$$\mu_{\overline{L(U,V)}} = \begin{array}{c|cccc} & Paris & Beijing & Otawa & London \\ \hline Paris & 0 & 0.8 & 0.4 & 0.2 \\ NYC & 0.15 & 0.7 & 0.2 & 0.12 \end{array}$$

$M(U,V) =$

$$\begin{pmatrix} 0 & 0.8 & 0.4 & 0.2 \\ 0.15 & 0.7 & 0.15 & 0.12 \end{pmatrix}$$

tilibra

$P = (Q \cdot R)$ max - produto $(Q(U,V)$ e $R(V,W))$

$$Q(U,V) = \begin{array}{c|cccc|c} & \text{Paris} & \text{Beij} & \text{Otawa} & \text{London} & \\ & 0 & 0.8 & 0.6 & 0.25 & \text{Paris} \\ & 0.7 & 0.98 & 0.15 & 0.5 & \text{NYC} \end{array}$$

$$R(V,W) = \begin{array}{c|ccc} & \text{Bruges} & \text{Stockholm} & \text{Moscow} \\ \text{Paris} & 1 & 0.4 & 0.2 \\ \text{Beij} & 0.1 & 0.4 & 0.7 \\ \text{Otawa} & 0.4 & 0.15 & 0.05 \\ \text{London} & 0.85 & 0.3 & 0.1 \end{array}$$

$P = (Q \cdot R)$ (max produto)

$$\max\begin{pmatrix} 0 \times 1 \\ 0.8 \times 0.1 \\ 0.6 \times 0.4 \\ 0.25 \times 0.85 \end{pmatrix} \overset{(1,1)}{=} 0.24 \quad \max\begin{pmatrix} 0 \cdot 0.4 \\ 0.8 \cdot 0.4 \\ 0.6 \cdot 0.15 \\ 0.25 \cdot 0.3 \end{pmatrix} \overset{(1,2)}{=} 0.32 \quad \max\begin{pmatrix} 0 \cdot 0.2 \\ 0.8 \cdot 0.7 \\ 0.6 \cdot 0.05 \\ 0.25 \cdot 0.1 \end{pmatrix} \overset{(1,3)}{=} 0.56$$

$$\max\begin{pmatrix} 0.7 \cdot 1 \\ 0.98 \cdot 0.1 \\ 0.15 \cdot 0.4 \\ 0.5 \cdot 0.85 \end{pmatrix} \overset{(2,1)}{=} 0.7 \quad \max\begin{pmatrix} 0.7 \cdot 0.4 \\ 0.98 \cdot 0.4 \\ 0.15 \cdot 0.15 \\ 0.5 \cdot 0.3 \end{pmatrix} \overset{(2,2)}{=} 0.392 \quad \max\begin{pmatrix} 0.7 \cdot 0.2 \\ 0.98 \cdot 0.7 \\ 0.15 \cdot 0.05 \\ 0.5 \cdot 0.1 \end{pmatrix} \overset{(2,3)}{=} 0.686$$

$$P = Q \circ R = \begin{pmatrix} 0.24 & 0.32 & 0.56 \\ 0.7 & 0.392 & 0.686 \end{pmatrix}$$

2) $X = \{1,2,3,4\}$ $A = \{(1,1), (2,0.5), (3,0.4), (4,0.2)\}$

↳ small integers

Almost equal

	1	2	3	4
1	1	0.8	0	0
2	0.8	1	0.8	0
3	0	0.8	1	0.8
4	0	0	0.6	1

B = "other small integers".

$B = A \circ R$

comp. max-min

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 0.5 \\ 3 & 0.4 \\ 4 & 0.2 \end{pmatrix}$$

$$B = \begin{pmatrix} \max(\min(1,1), \min(0.5, 0.8), \min(0.4,0), \min(0.2,0)) \\ \max(\min(1, 0.8), \min(0.5, 1), \min(0.4, 0.8), \min(0.2, 0)) \\ \max(\min(1, 0), \min(0.5, 0.8), \min(0.4, 1), \min(0.2, 0.8)) \\ \max(\min(1, 0), \min(0.5, 0), \min(0.4, 0.8), \min(0.2, 1)) \end{pmatrix} = \begin{pmatrix} 1 \\ 0.8 \\ 0.5 \\ 0.4 \end{pmatrix}$$

Sistemas Nebulosos

Atividade Avaliativa

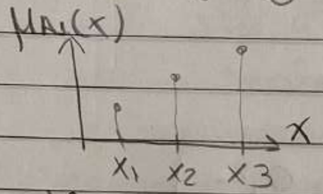
Parte 1.

Questão 5)

duas regras, cada regra representado por relação

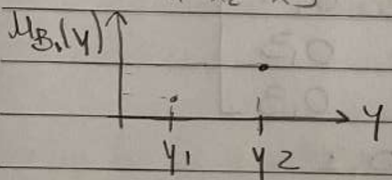
$R_1 : A_1 \to B_1$

temos função de pertinência discreta



$$\mu_{R_1} = \mu_{A_1}(x) \wedge \mu_{B_1}(y)$$

→ fazemos produto cartesiano e para cada tupla, aplicamos o t-norma (mínimo)



$x_1 = 0{,}2$; $x_2 = 0{,}4$; $x_3 = 0{,}5$; $y_1 = 0{,}1$; $y_2 = 0{,}3$

→ produto cartesiano

$\min(x_1, y_1) = 0{,}1$

$\min(x_1, y_2) = 0{,}2$

$\min(x_2, y_1) = 0{,}1$

$\min(x_2, y_2) = 0{,}3$

$\min(x_3, y_1) = 0{,}1$

$\min(x_3, y_2) = 0{,}3$

$R_2 : A_2 \to B_2$

$\mu_{R_2} = \mu_{A_2}(x) \wedge \mu_{B_2}(x)$

$x_1 = 1$, $x_2 = 1$, $x_3 = 0{,}3$; $y_1 = 0{,}6$; $y_2 = 0{,}2$

em formato de matriz (produto cartesiano + mínimo)

	y_1	y_2
x_1	0,6	0,2
x_2	0,6	0,2
x_3	0,3	0,2

temos agora as funções de pertinência das duas relações.
nosso fato é : x é A'
e nossa conclusão y é B'
a conclusão é a composição do fato com as regras, logo

$$\mu_{B'}(y) = (\mu_{A'}(x) \circ \mu_{R_1}) \vee (\mu_{A'}(x) \circ \mu_{R_2})$$

$A' = x_1 = 0,\ x_2 = 1,\ x_3 = 0$

$\rightarrow \mu_{A'}(x) \circ \mu_{R_1}(x,y)$

$$\mu_{A'}(x) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad \mu_{R_1} = \begin{bmatrix} 0,1 & 0,2 \\ 0,1 & 0,3 \\ 0,1 & 0,3 \end{bmatrix}$$

aplicando máx-min

$\mu_{A'}(x) \circ \mu_{R_1}(x,y)(1,1) = \text{máx}(0 \ \ 0,1 \ \ 0) = 0,1$

$\quad '' \quad (1,2) = \text{máx}(0 \ \ 0,3 \ \ 0) = 0,3$

$\mu_{A'}(x) \circ \mu_{R_1}(x,y) = [0,1 \ \ 0,3] = \mu_{C_1}$

$\rightarrow \mu_{A'}(x) \circ \mu_{R_2}(x,y)$

$$\mu_{A'}(x) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad \mu_{R_2} \begin{bmatrix} 0,6 & 0,2 \\ 0,6 & 0,2 \\ 0,3 & 0,2 \end{bmatrix}$$

$\mu_{A'}(x) \circ \mu_{R_2}(x,y)(1,1) = \text{máx}(0 \ \ 0,6 \ \ 0) = 0,6$

$\quad '' \quad (1,2) = \text{máx}(0 \ \ 0,2 \ \ 0) = 0,2$

$\mu_{A'}(x) \circ \mu_{R_2}(x,y) = [0,6 \ \ 0,2] = \mu_{C_2}$

$\mu_{B'}(y) = \mu_{C_1} \vee \mu_{C_2}$ $\Rightarrow$ usando operador máx

$$\mu_{B'}(y) = [0,6 \ \ 0,3]$$

Questão 6)

deve ser executado graficamente

$R_1 : A_1 \rightarrow C_1$

$R_2 : A_2 \rightarrow C_2$

temos μ_{A1}, μ_{A2}, μ_{C1}, μ_{C2}, $\mu_{A'}$

precisamos encontrar $\mu_{C'}$

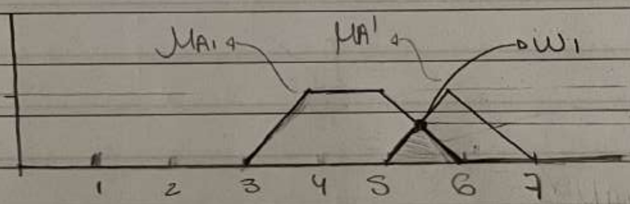
$w_1 = \mu_{A'} \circ \mu_{A1}$

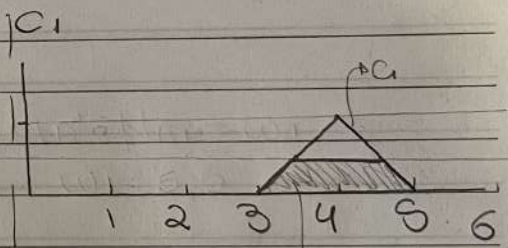
$w_2 = \mu_{A'} \circ \mu_{A2}$

$\mu_{C'} = \max(\min(w_1, C_1), \min(w_2, C_2))$

(em resumo as relações e regras funcionam da mesma maneira que a questão anterior)

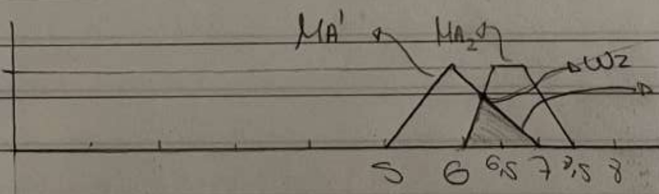
$\rightarrow w_1$

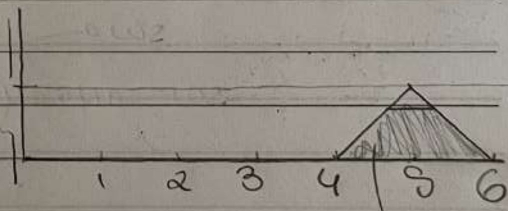




$\min(w_1, C_1)$

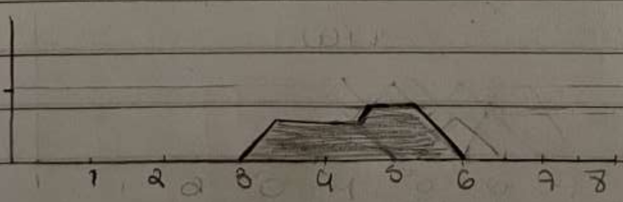
$\rightarrow w_2$





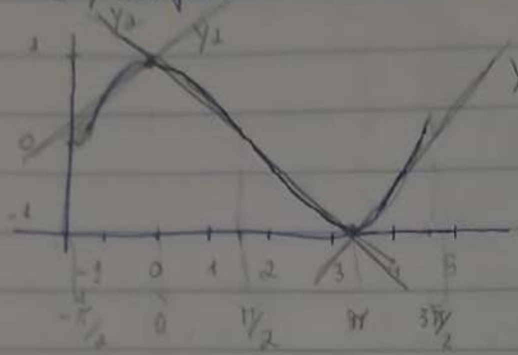
$\min(w_2, C_2)$

$\rightarrow C'$



(algumas linhas podem ser vistas pois foram usadas como esboço).

2) Seja a função $y = \text{cosseno}(x)$, para x definido no intervalo de $[-\pi/2, 3\pi/2]$, conforme abaixo



Pontos p/ formar a equação:

y_1	(x,y)
P_1	$(-\frac{\pi}{2}, 0)$
P_2	$(0, 1)$

y_2	(x,y)
P_1	$(0,1)$
P_2	$(\pi, -1)$

y_3	(x,y)
P_1	$(\pi, -1)$
P_2	$(\frac{3\pi}{2}, 0)$

(a) Empregue o mecanismo de inferência de Sugeno com consequentes de ordem 1 (linear) e obtenha uma expressão analítica para aproximar esta função.

Dica: use funções de pertinência do tipo triangular p/ "fuzzificação" da variável x.

- se x é A_1 então $y_1(x) = p_1 x + q_1$ | $\mu_{A_1}(x) = ?$
- se x é A_2 então $y_2(x) = p_2 x + q_2$ | $\mu_{A_2}(x) = ?$
- se x é A_3 então $y_3(x) = p_3 x + q_3$ | $\mu_{A_3}(x) = ?$

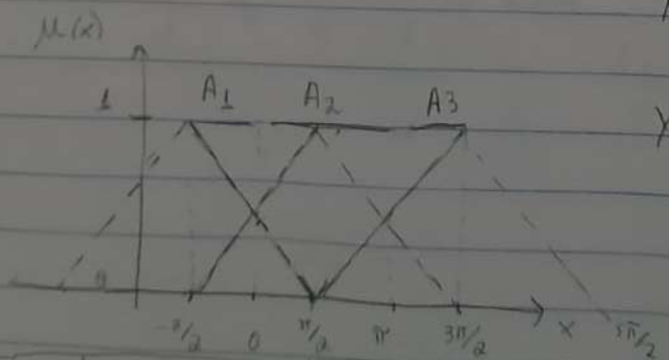
$$y_s = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3}{w_1 + w_2 + w_3}$$

$$y_1 = \frac{2}{\pi}x + 1$$

$$y_2 = -\frac{2}{\pi}x + 1$$

$$y_3 = \frac{2}{\pi}x - 3$$

Obtido através dos pontos das retas.



μ_{A_1}	(x,y)
P_1	$(-\pi/2, 1)$
P_2	$(\pi/2, 0)$

μ_{A_2}	(x,y)
P_1	$(-\pi/2, 0)$
P_2	$(\pi/2, 1)$

μ_{A_3}	(x,y)
P_1	$(\pi/2, 0)$
P_2	$(3\pi/2, 1)$

$$y_s = \frac{w_1 \cdot \left(\frac{2}{\pi}x+1\right) + w_2\left(\frac{-2}{\pi}x+1\right) + w_3\left(\frac{2}{\pi}x-3\right)}{w_1+w_2+w_3}$$

$\mu_{A1}(x) = trimf\left(-\frac{3\pi}{2}, -\pi/2, \pi/2\right)$
$\mu_{A2}(x) = trimf\left(-\frac{\pi}{2}, \pi/2, 3\pi/2\right)$
$\mu_{A3}(x) = trimf\left(\pi/2, 3\pi/2, 5\pi/2\right)$

$$x \in \left[-\pi/2, 3\pi/2\right] \rightarrow \begin{cases} w_1 = \mu_{A1}(x) = -\frac{1}{\pi}x + \frac{1}{2} \\ w_2 = \mu_{A2}(x) = \\ w_3 = \mu_{A3}(x) = \frac{1}{\pi}x - \frac{1}{2} \end{cases}$$

$\mu_{A1}(x) =$
$(-\pi/2, 1), (\pi/2, 0)$
$\mu_{A1}(x) \Rightarrow \boxed{-\frac{1}{\pi}x + \frac{1}{2} = y}$

$\mu_{A3}(x) =$
$(\pi/2, 0)\ (3\pi/2, 1)$
$\mu_{A3}(x) \Rightarrow \boxed{\frac{1}{\pi}x - \frac{1}{2} = y}$

$\mu_{A2}(x)\ [-\pi/2, \pi/2]$
$(-\pi/2, 0)\ (\pi/2, 1)$
$\mu_{A2\text{-}1} \Rightarrow \boxed{\frac{1}{\pi}x + \frac{1}{2} = y}$

$\mu_{A2}(x)\ [\pi/2, 3\pi/2]$
$(\pi/2, 1)\ (3\pi/2, 0)$
$\mu_{A2\text{-}2} \Rightarrow \boxed{-\frac{1}{\pi}x + \frac{3}{2} = y}$

$$Y_s = Y_{s1}\Big|_{-\pi/2}^{\pi/2} + Y_{s2}\Big|_{\pi/2}^{3\pi/2} \longrightarrow$$

tilibra

$$Y_{S1} = \frac{w_1 y_1 + w_2 y_{2-1}}{w_1 + w_2} = \frac{\left(-\frac{x}{\pi} + \frac{1}{2}\right)\left(\frac{2x}{\pi} + 1\right) + \left(\frac{x}{\pi} + \frac{1}{2}\right)\left(\frac{-2x}{\pi} + 1\right)}{\left(-\frac{1}{\pi}x + \frac{1}{2}\right) + \left(\frac{x}{\pi} + \frac{1}{2}\right)}$$

$$Y_{S1}\Big|_{-\pi/2}^{\pi/2} = \frac{\left(-\frac{2x^2}{\pi^2} - \frac{x}{\pi} + \frac{x}{\pi} + \frac{1}{2}\right) + \left(\frac{-2x^2}{\pi^2} + \frac{x}{\pi} - \frac{x}{\pi} + \frac{1}{2}\right)}{1} = -\frac{4x^2}{\pi^2} + 1$$

$$Y_{S2} = \frac{w_2 y_{2-2} + w_3 y_3}{w_2 + w_3} = \frac{\left(\frac{-2x}{\pi} + 1\right)\left(\frac{-x}{\pi} + \frac{3}{2}\right) + \left(\frac{2x}{\pi} - 3\right)\left(\frac{x}{\pi} - \frac{1}{2}\right)}{\left(\frac{-x}{\pi} + \frac{3}{2}\right) + \left(\frac{x}{\pi} - \frac{1}{2}\right)}$$

$$Y_{S2}\Big|_{\pi/2}^{3\pi/2} = \frac{\left(\frac{2x^2}{\pi^2} - \frac{3x}{\pi} - \frac{x}{\pi} + \frac{3}{2}\right) + \left(\frac{2x^2}{\pi^2} - \frac{x}{\pi} - \frac{3x}{\pi} + \frac{3}{2}\right)}{1} =$$

$$Y_{S2}\Big|_{\pi/2}^{3\pi/2} = \frac{4x^2}{\pi^2} - \frac{6x}{\pi} - \frac{2x}{\pi} + 3$$

$$Y_S = \begin{cases} -\frac{4x^2}{\pi^2} + 1, & x \in [-\pi/2, \pi/2] \\ \frac{4x^2}{\pi^2} - \frac{8x}{\pi} + 3, & x \in [\pi/2, 3\pi/2] \end{cases}$$