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To cite this article: Muhammad Iqbal Rahmadhan Putra *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **546** 052057

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Network Analysis of The Brazil Nut Effect Phenomenon with a Single Intruder

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Abstract. One phenomenon that can be observed in granular systems is the Brazil Nut Effect (BNE), that is, a phenomenon in which large-size particles lift up when vibrated vertically. In this experiment, structural changes in a pseudo-two-dimensional model of a granular system experiencing BNE were observed from the perspective of network analysis. The system consisted of 199 granular beds of 0.68 cm of diameter with a 2.5 cm diameter intruder placed in a 3mm wide double-window box that was slightly larger than the thickness of the bed and the intruder. The system was subjected to vibrations with a frequency of 13.33 Hz and an amplitude of 0.75 cm, so the BNE could be observed. For the purpose of the analysis, the granular beds were considered the nodes of a network and the relationships between adjacent beds (where contact force occurred) represented its edges. The analysis, consisting of image processing, network extraction, network parameters calculation and community detection, was performed using Wolfram Mathematica v. 11.3. The experiment was able to calculate the change in the network parameters including degrees, clustering coefficients, betweenness centrality, and modularity for the system with intruders and systems without intruders. The parameter values corresponding to each system were markedly different, clearly showing the influence of the intruder. The authors were also able to successfully map the evolution of the community structure in both types of granular systems one step at a time using a modularity optimization method.

1. Introduction

Network analysis offers methods for quantitatively probing and analyzing large, interacting systems whose associated networks have heterogeneous patterns that defy explanations attained by considering exclusively all-to-all, regular, or lattice-like interactions [1]. This kind of analysis has received significant attention from a sociological perspective and has recently emerged as a useful technique in anthropology, geography, psychology, communication studies, information science, organizational studies, economics, biology and neuroscience [2]. Especially in the era of Industry 4.0, network analysis could play a fundamental role in the optimization of manufacturing industries through the integration of all aspects of production and commerce across company boundaries for greater efficiency [3].



The most common representation of a network is as a graph. A graph consists of a set of nodes (to represent the entities) and a set of edges, each of which represents an interaction between a pair of entities (or between an entity and itself) [1].

If a mixture of two different types of granular materials is subjected to vertical vibrations, larger granules (or usually known as intruders) will rise to the surface. This phenomenon is known as the Brazil-Nut Effect (BNE) [4]. The simplest granular mixture to observe this effect is the one where a single intruder is mixed with an otherwise homogenous granular material. The factors that can influence BNE are the mass and diameter ratios of both types of granular material, the amplitude and frequency of the vibration, the arrangement of the bed particles and the initial configuration of the granular mixtures [5]. The influence of frequency and amplitude is expressed as a normalized acceleration factor, Γ [6].

The maximum compaction between 2-dimensional systems of the same size granular can forming the Hexagonal Close-Packed (HCP) structure as shown in Figure 1. The contact point between granules in a system is known as the contactopy. The potential energy value of such system can be calculated by counting the number of contactopies between individual particle. Contactopy has been used to predict directional movement of intruder by considering up to three layers of grains around the intruder [7]. It has been observed that even when simultaneously increasing the system contactopy and decreasing the potential energy, the BNE can still be inhibited as long as the contactopy has a certain value, which is system-dependent [8].

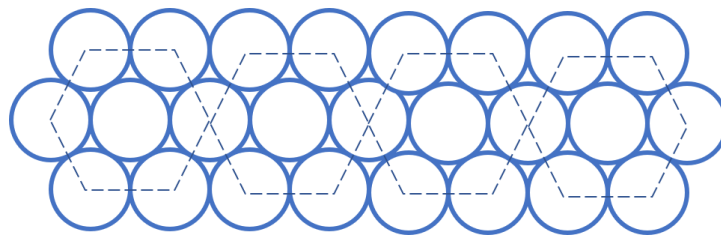


Figure 1. Hexagonal Close-Packed configuration.

In granular systems, the network represents a chain force, caused by the contact between particles, that is formed at both, the mesoscale and macroscale levels. This experiment defines granular particles as nodes whose coordinates are located at the center of the particle. These particles form a network whose edges between two vertices occur when two particles contact each other. The limitation of this experiment is that the relationship between particles is equal and reciprocal so that the type of network used is an undirected and unweighted network.

Once the granular system with one intruder is represented as a network, parameters such as degrees, clustering coefficients, betweenness centrality, and modularity can be calculated [1] [2]. The community structure of the network is determined by optimizing the value of the modularity parameter.

The degree of a node is the number of edges associated with that node. In the context of granular systems, degrees, commonly known also as coordination numbers can be interpreted as the number of particles in contact with other particles [9]. The clustering coefficient is a measurement of the propensity of a node in a network to cluster. One way to define the clustering coefficient of a given node i is to calculate the number of triangles (3-cycles) which include node i and divide it by the number of triples centered on the node. A triple is a set of three nodes connected by either three edges (forming triangles) or two edges. The betweenness centrality of node i is a measure of how important the node is in a network and can be computed by comparing the number of shortest paths that pass through node i with the overall shortest path. Modularity is defined as the number of edges in the network reduced by the expected number of edges in the same network if they were placed randomly. Modularity can be used as a parameter to form community structures in a network. A community

structure is generally defined as an arrangement of nodes into groups so that the edge density within nodes in one group will be greater than the one between the nodes in the group and other nodes [10].

2. Experiments

199 bed particles with a thickness of 2mm and a diameter of 0.68 cm were placed in a 19 cm x 10 cm acrylic container along with a ring-shaped intruder particle with a 1cm inner diameter and a 2.4 cm outer diameter. The system normalized acceleration was specified as

$$\Gamma = \frac{A(2\pi f)^2}{g} \quad (1)$$

Where g is the gravitational acceleration constant, 9.8 m/s^2 . To ensure the occurrence of the BNE we used a constant vibrational frequency f of 13.33 Hz and an amplitude A of 0.75 cm resulting in a normalized acceleration (dimensionless) Γ of 5.37.

The arrangement of tools and materials of experiment is shown in Figure 2 below.

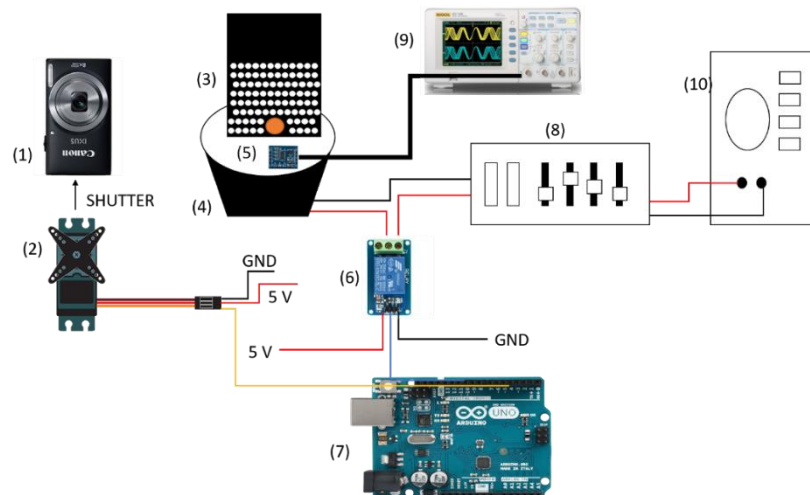


Figure 2. The arrangement of tools and materials of experiment (1) Camera, (2) Servo, (3) Acrylic container containing bed particles and intruder, (4) Speaker, (5) Accelerometer, (6) Relay, (7) Arduino Uno, (8) Amplifier, (9) Oscilloscope, and (10) Audio generator.

The acrylic container was placed on top of the speaker, a 12-inch Curve Woofer model 30H12SRW3, that was our chosen vibration tool. Sine waves were generated by a GW Instek Audio Generator GAG-809 and the output from the signal generator was amplified using an EKO Power Amplifier model AB_22. The vibration frequency was read by a RIGOL Oscilloscope DS1052E 50Mhz 2 Channel.

The experiment also used an Arduino Uno R3 board with the ATmega 328 microcontroller, a 5V-1 Channel Relay interface board as an electronic switch, an MMA7361 accelerometer and a SG90 servo motor. The vibration procedure was done discontinuously. The image of granular systems condition was taken when the system was not vibrating, from initial configuration until the intruder is on the surface of bed particles.

After the experimental images were obtained, image processing using transformations such as binary and hit-miss was performed using Wolfram Mathematica [12] to extract the node coordinates of each particle and determine whether any given two vertices were interconnected or not by comparing their Euclidian distance with the sum of the particle radiuses (Eq. 3).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq r_i + r_j \quad (2)$$

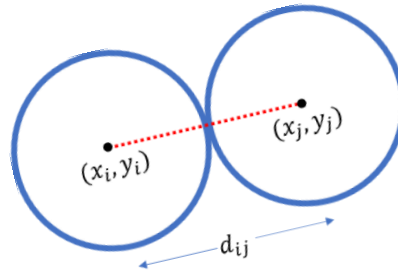


Figure 3. Illustration to detect the presence or absence of contact by comparing the Euclidean distance with the sum of the radius of each particle.

Suppose the network formed is a weightless network G consisting of a set of vertices V and L sides where $V = v_1, v_2, v_3, \dots, v_N$ and $L = l_1, l_2, l_3, \dots, l_M$ with N as the number of vertices and M as the number of edges. This network can be represented as an adjacency matrix A consisting of a_{ij} elements whose values are given by the following rule:

$$a_{ij} = \begin{cases} 1, & \text{if node } i \text{ and } j \text{ are connected} \\ 0, & \text{if node } i \text{ and } j \text{ are not connected} \end{cases}$$

The local degree k_i values are determined by the following expression:

$$k_i = \sum_{j=1}^N A_{ij} \quad (3)$$

The local clustering coefficient C_i value is computed using the following formula:

$$C_i = \frac{\sum_{h,j} A_{hj} A_{ih} A_{ij}}{k_i(k_i - 1)} \quad (4)$$

The expression below returns the local betweenness centrality B_i values are found with the expression below where the number of shortest paths from node g to node h is ψ_{gh} and the number of the shortest paths that pass through node i is $\psi_{gh}(i)$.

$$B_i = \sum_{g,h; g \neq h} \frac{\psi_{gh}(i)}{\psi_{gh}}, \quad i \in \{g, h\} \quad (5)$$

The modularity values in general are determined by Eq. 6 where c_i and c_j are formed communities with vertices i and j inside them.

$$Q = \frac{1}{2m} \sum_{i,j} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j) \quad (6)$$

In this study, the method of forming community structures from the network used the greedy optimization of modularity method proposed by A. Clauset et al [11] because the number of nodes is large and is done for many networks so it requires a small computational cost. Calculation of these parameters using the Wolfram Mathematica program with additional IGraph packages

In this study, the greedy optimization algorithm proposed by A. Clauset et al. [11] was used for forming community structures within the network. This method was chosen given the large number of nodes and is done for many networks, so the computational cost is small. All the calculations were done with IGraph/M, the Mathematica interface for graph, a collection of open source network analysis tools [13].

3. Result and Discussion

From the experiment, six data sets were obtained, three from a granular system without intruders and the rest of them from a system with one intruder. Each data set consisted of images obtained at 11 different times. After representing each data set as a network, the relevant parameters were computed. The following section will discuss the results of those calculations.

3.1. Degree

The global degree value was obtained by averaging the degree value of all the nodes. This value represents the number of contact forces that occurred between particles over time.

Figure 4 shows the influence of intruders when determining the global degree of a system. In this case, the system with one intruder experienced a decrease in the number of contact forces over time reaching a minimum between steps 7 and 8. Afterwards, the value of the degree rose back again.

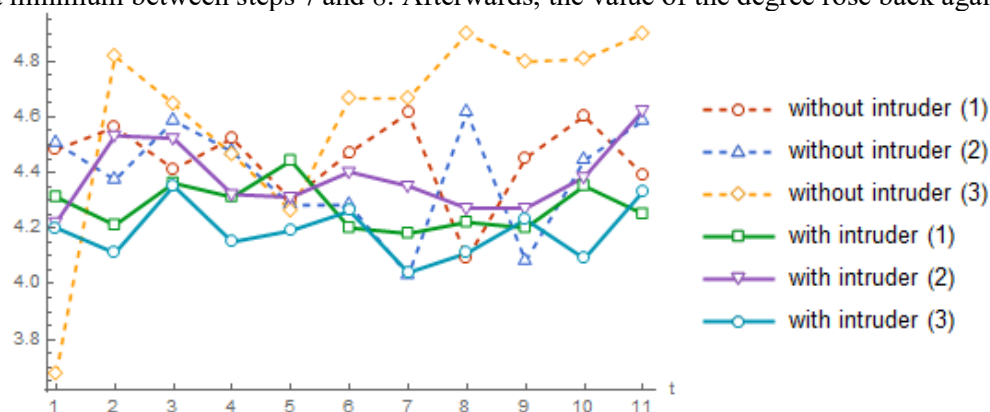


Figure 4. The average degree at each time step for the six data sets.

However, the granular system without intruders had a higher global degree value overall (above 4.3) compared to the other systems except in the first and fifth time step. The reason behind the low value in the first step was the randomness of the initial state of the system. After subjecting it to vertical vibrations, it became more stable, exhibiting a high average degree.

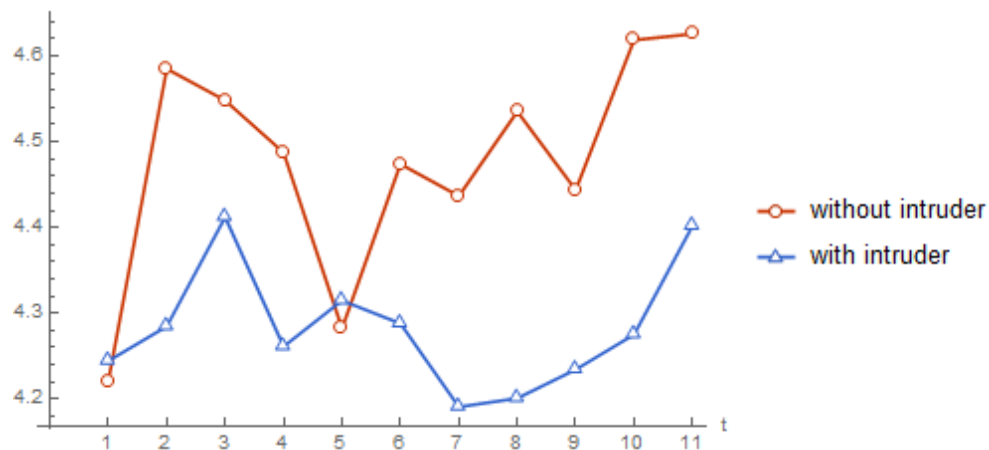


Figure 5. The mean of average degree at each time step.

3.2. Clustering coefficient

The clustering coefficient indicates the propensity of a node to cluster, the larger its number, the more likely it is that the node will be part of a group. Those coefficients are related to the number of triangles, dense formations between three granules, that occur.

Figure 6 shows that the clustering coefficient values for the system with an intruder oscillated between 0.24 and 0.34, while the values for the system without intruders ranged between 0.20 and 0.35. This indicates that the clustering coefficient value was not significantly affected during the Brazil Nut Effect phenomenon.

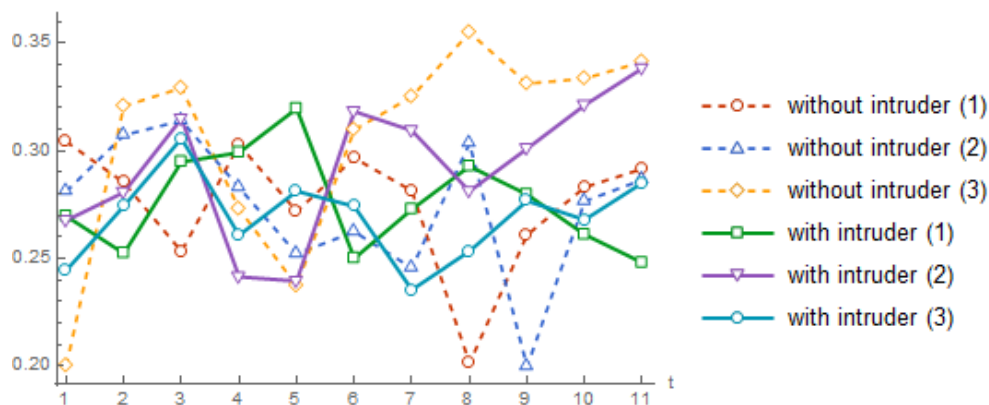


Figure 6. The clustering coefficient at each time step for the six data sets.

3.3. Betweenness centrality

The global value of the betweenness centrality parameter is obtained by finding the average value of the parameter for all the nodes. Its number indicates how important the node is for the network system which is seen from how often the node is traversed by the force chain shortest path from the other two nodes. Based on the results, there's a significant difference between the average value of the granular system with intruder and the one without intruder. In the first case the value ranged from 0.036 to 0.0412, while the parameter for the system without intruder hovered around 0.0355 - 0.038, a 57% smaller range.

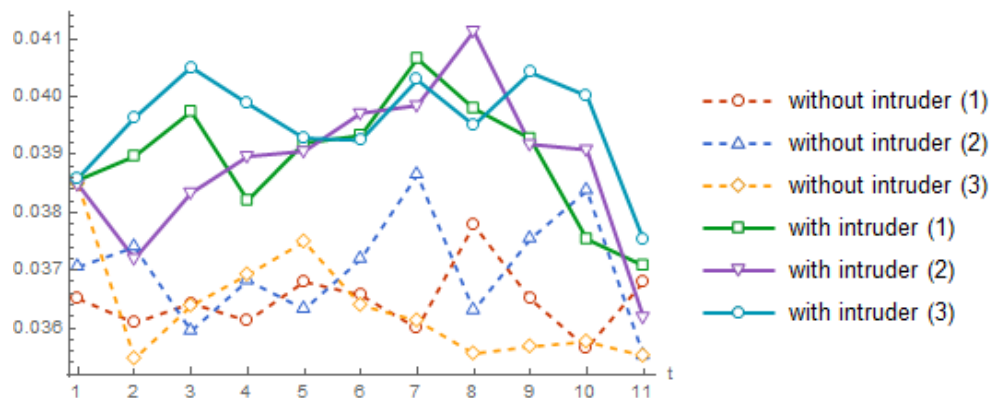


Figure 7. The average betweenness centrality at each time step for the six data sets.

The first, second and third data sets related to the system with an intruder displayed a similar behavior. After the granular system had been subjected to vibrations, the betweenness centrality initially rose remaining relatively constant afterwards until the last few time steps where its value decreased.

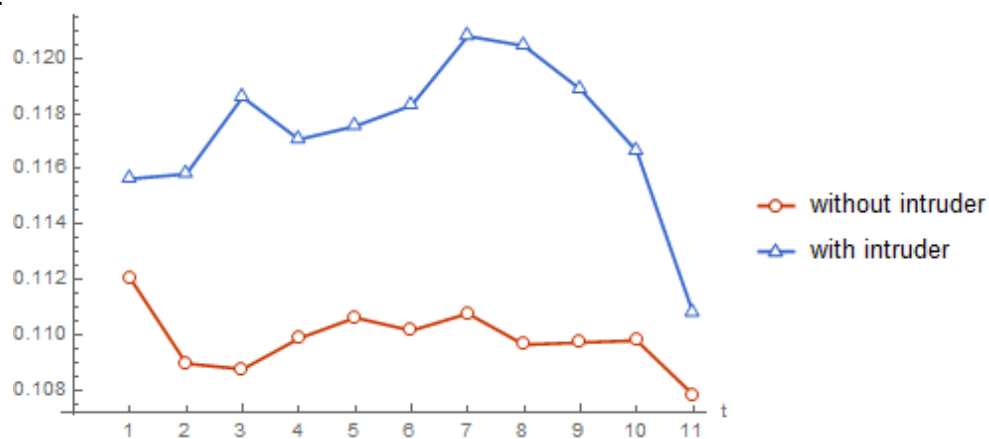


Figure 8. Comparison of the mean of average betweenness at each time step for systems without and with intruder.

To further investigate the effect of intruders on the value of the global betweenness centrality of the system, a mapping of the betweenness centrality values has been done locally on each node. In figure 9 there are 4 samples from the second experiment, namely at the first, fourth, eighth, and eleventh times. It can be seen that at the first and eleventh time betweenness centrality values are spread evenly at low values, because the system seems to be a system without intruders. Whereas at time 4 and time 8 (which is the peak value betweenness) there is a high betweenness value around the intruder.

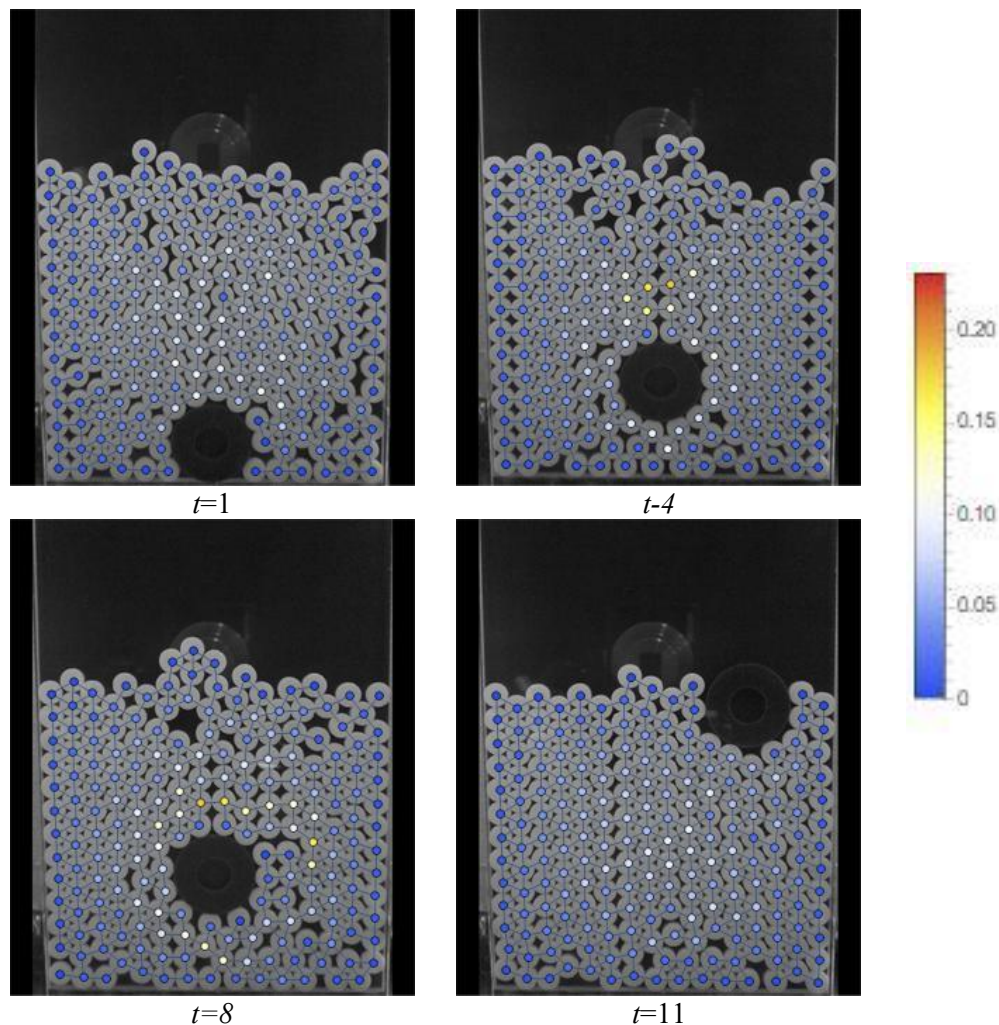


Figure 9. Local betweenness centrality mapping on the second experiment system with intruder.

The influence of the intruder is to increase the chances of nodes around it to be on the shortest path of the other two nodes. When the intruder is not in the middle of the system, all nodes get almost the same opportunity to be on the shortest path of the other two nodes so that the betweenness value is spread evenly on a small number. This is the explanation how the change in value betweenness in the system with intruders while the system without intruder betweenness values tend to be constant.

3.4. Modularity

Modularity measures the strength of the partition of a network into modules. Networks with high modularity have dense connections between the nodes within modules but sparse connections between nodes in different modules. The modularity values shown below were calculated with a greedy optimization algorithm and don't show any particular pattern. These modularity values were used as a parameter to form a community structure for the networks representing the granular systems.

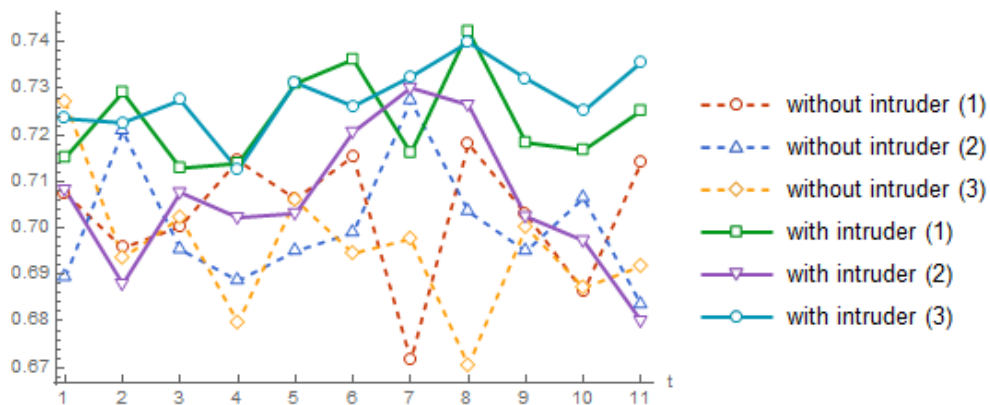


Figure 10. Modularity value for each time step for the six data sets.

The following figure displays the evolution of the community structure at the first, fourth, ninth and eleventh time steps of the second data set of the experiment, corresponding to a granular system with an intruder.

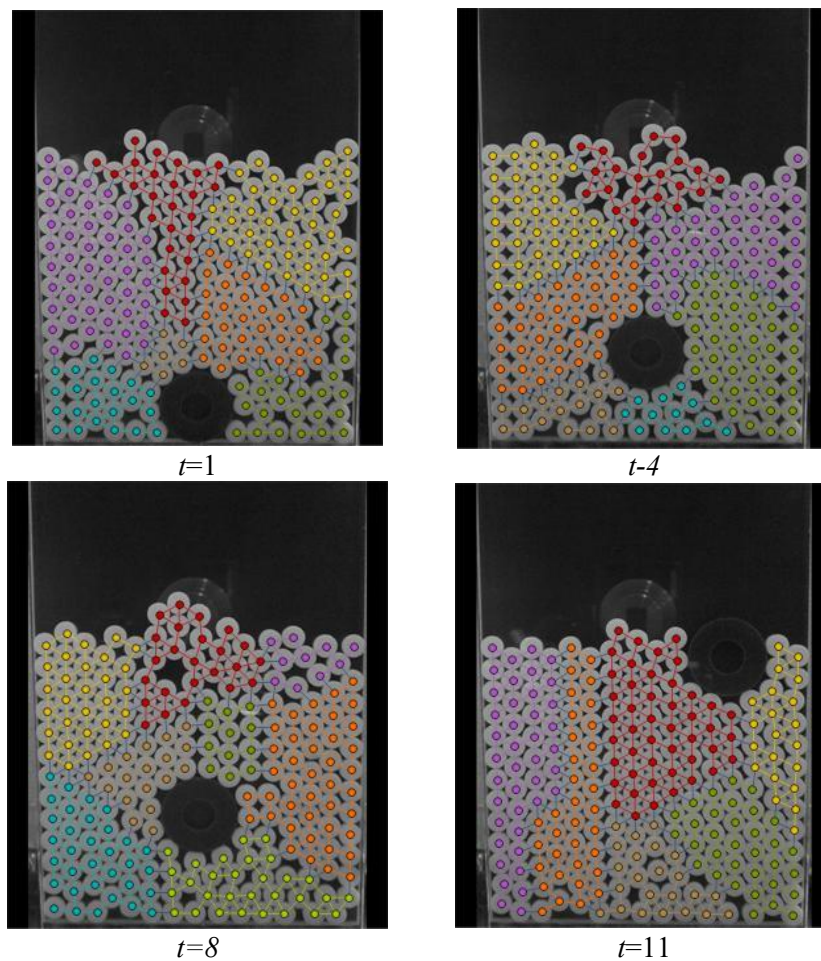


Figure 11. The evolution of the community structure for the second data set (granular system with intruder) at four different time steps.

In general, the difference between both types of systems can be seen by comparing the average modularity value for each one of them. The parameter value of the one with an intruder is consistently higher than the number obtained from the homogeneous system. Figure 12 shows the difference in mean modularity between both types of systems, strongly indicating that intruders have an influence in forming the community of a system. An intruder will make the system more tightly connected when it is in the middle so that the value of the modularity becomes higher. In contrast, community formation in systems without intruders is only influenced by vibrations and therefore, the value tends to be similar at each time step.

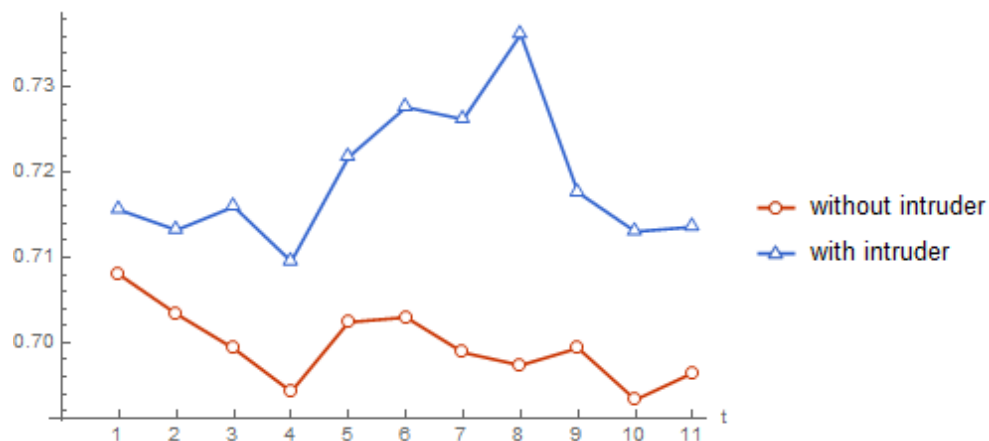


Figure 12. Comparison of the mean of modularity at each time step for systems without and with intruder.

4. Conclusion and Future Work

Based on the results obtained, the authors consider that network analysis can be used as a tool to characterize granular systems experiencing the Brazil Nut Effect. The three data sets corresponding to the system with one intruder displayed the same tendency in degrees, betweenness centrality, and modularity. However, the data set corresponding to the granular system without intruders showed different results in terms of global degree, global betweenness centrality values, and modularity. An explanation of the effects of intruders has been explained in the previous section in detail, which is consistent for all data. Changes in community structure due to the movement of an intruder were also detected.

Based on these results, the authors are confident in the possibility of extending the use of network analysis to other BNE phenomena, including the segregation of granular binary mixture systems. Finally, the authors also believe that the observations made by network analysis should be compared with the results of previous studies to get a better understanding of the underlying physical phenomena.

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