Groverize Monotone Local Search. (Short Note)

David Ponarovsky

April 24, 2023

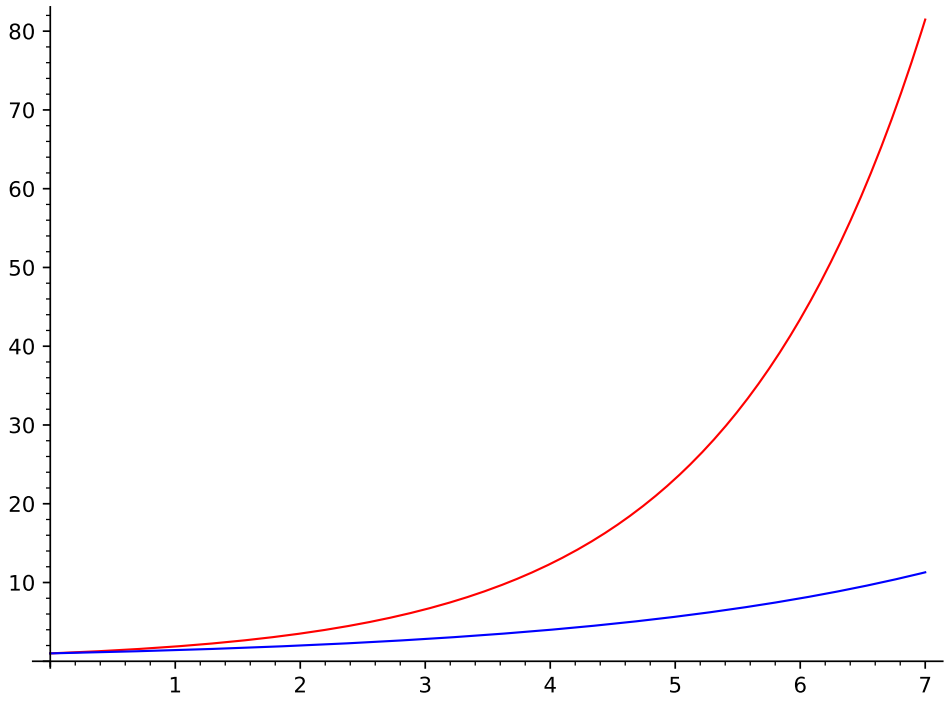
1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2(k'-t)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq$$

$$\Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



Problem Name	Parameterized	Groverize	New bound	Previous Bound
Feedback Vertex Set	3^k (r) [Cyg+11]	1	1.6667^n (r)	
Feedback Vertex Set	3.592^k [KP14]	1	1.7217^n	1.7347^n [FTV15]
Subset Feedback Vertex Set	4^k [**Wahlstrom14**]	1	1.7500^n	1.8638^n [Fom+14]

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

References

[Gro96] Lov K. Grover. *A fast quantum mechanical algorithm for database search*. 1996. arXiv: quant-ph/9605043 [quant-ph].

[Cyg+11] Marek Cygan et al. "Solving connectivity problems parameterized by treewidth in single exponential time (extended abstract)". In: *2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011*. IEEE Computer Soc., Los Alamitos, CA, 2011, pp. 150–159. DOI: 10.1109/FOCS.2011.23. URL: https://doi.org/10.1109/FOCS.2011.23.

[Fom+14] Fedor V. Fomin et al. "Enumerating minimal subset feedback vertex sets". In: *Algorithmica* 69.1 (2014), pp. 216–231. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9731-6. URL: https://doi.org/10.1007/s00453-012-9731-6.

[KP14] Tomasz Kociumaka and Marcin Pilipczuk. "Faster deterministic Feedback Vertex Set". In: *Inform. Process. Lett.* 114.10 (2014), pp. 556–560. ISSN: 0020-0190. DOI: 10.1016/j.ipl.2014.05.001. URL: https://doi.org/10.1016/j.ipl.2014.05.001.

[FTV15] Fedor V. Fomin, Ioan Todinca, and Yngve Villanger. "Large induced subgraphs via triangulations and CMSO". In: *SIAM J. Comput.* 44.1 (2015), pp. 54–87. ISSN: 0097-5397. DOI: 10.1137/140964801. URL: https://doi.org/10.1137/140964801.

[Fom+15] Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: 1512.01621 [cs.DS].