Fourmlas Sheet.

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Probability.

Multiplicative Chernoff bound. Suppose $X_1, ..., X_n$ are independence random variables taking values in $\{0, 1\}$ Let X denote their sum and let $\mu = \mathbf{E}\left[\sum_i^n X_i\right]$ denote the sum's expected value. Then for any $\delta > 0$:

$$\begin{aligned} &\mathbf{Pr}\left[X \geq (1+\delta)\mu\right] \leq e^{-2\frac{\delta^2\mu^2}{n}} \\ &\mathbf{Pr}\left[|X-\mu| \geq \delta\mu\right] \leq 2e^{-\delta^2\mu/3}, \qquad 0 \leq \delta \leq 1 \end{aligned}$$

Jensen's inequality. If X is a random variable and ϕ is a convex function, then:

$$\begin{aligned} \phi\left(\mathbf{E}\left[X\right]\right) \leq \mathbf{E}\left[\phi\left(X\right)\right] \Rightarrow \mathbf{E}\left[X\right] &\leq \phi^{-1}\left(\mathbf{E}\left[\phi\left(X\right)\right]\right) \\ \mathbf{E}\left[X\right] &\leq \ln\left(\mathbf{E}\left[e^X\right]\right) \\ \mathbf{E}\left[X\right] &\geq e^{\mathbf{E}\left[\ln(X)\right]} \end{aligned}$$

Paley–Zygmund inequality. bounds the probability that a positive random variable is small, in terms of its first two moments. Could be thought as the lower bound Markov version. If a r.v X is always positive and has a finate variance, then for $0 \leq \tau \geq 1$:

$$\mathbf{Pr}\left[X > \tau\mathbf{E}\left[X\right]\right] \geq (1-\tau)^2 \frac{\mathbf{E}\left[X\right]^2}{\mathbf{E}\left[X^2\right]}$$