Hardness of Computing Fault Tolerance.

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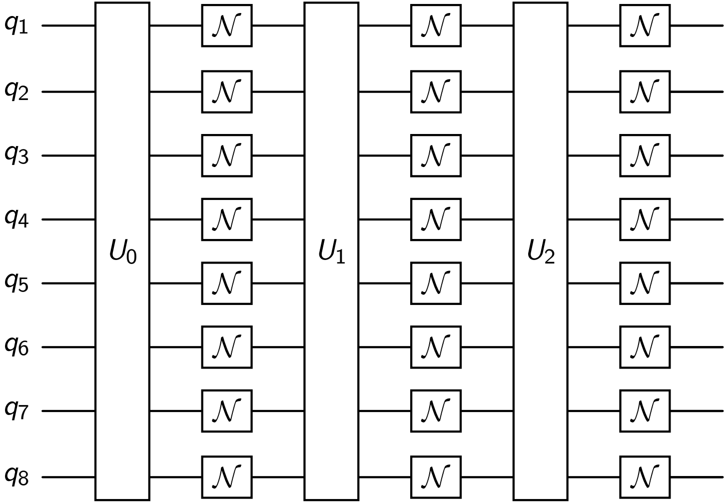
Introduction

- Brief overview of the topic
- Importance and relevance
- Objectives of the presentation

Key Points

- Main point 1
- Main point 2
- Main point 3

Nosiy Circuit.



Threshoold Theorem.

Pippenger's Construction.

Encode each bit with the repetition code $0 \mapsto 0^m$, $1 \mapsto 1^m$. Now observe that any logical operation, without decoding, can be made in $O(1)$ depth.
For example, $\mathrm{OR}(\bar{x}, \bar{y})$ can be computed by applying in parallel $\mathrm{OR}(x_i, y_i)$ for each i.

The 'Decoding' trick.

Instead of completely decoding, we would apply only a single step of partial decoding. We assume that in each code block the bits are partitioned into random disjoint triples, and we will apply a local correction to each of the triples by majority.

Claim

There are constants $\alpha, \eta \in (0,1)$ such that for any bit string x at distance $\leq \alpha n$ from the code (Repetition Code), A one cycle of local correction on x yield x' such:

$$d(x', C) \leq d(x, C)$$

The Franch's Construction.