Recursion Code.

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Abstract

None

1 Construction.

Definition 1. *Let Δ be an integer greater than 2 and consider an algorithm $\mathcal{A}$ that for any n that is power of 3 construct a Δ-regular graph over n vertices. Now, let G be Δ-regular graph over n vertices generated by $\mathcal{A}$. Define the* ***third graph obtained by*** *G , labeled by $G^{\sim}$ to be the graph which $\mathcal{A}$ returns over $\frac{1}{3}n$ such that any of the edges could be associate by puncturing a $\frac{2}{3}$ fraction of the edges of each vertex.*

Definition 2 (Recursion Code)**.** *Let $C_0 = \Delta[1, \rho_0, \delta_0]$ be a binary linear code. We will define the recursion code in recursive manner. First for a sufficiently large integer n_0, which is also power of 3, $C(n_0)$ defined to be the Tanner code defined by the C_0 and graph $\mathcal{A}(n_0)$. Then let n be any power of 3, such that $n > n_0$, denote by G the graph that constructed by the running of $\mathcal{A}(n)$. Then let $C(n)$ be the code obtained by the joining the parity check matrix of the Tanner code $\mathcal{T}(G, C_0)$ and by the checks of the $C(n/3)$ over the bits associated with the $G^{\sim}$. We will call to that code family the* ***recursion code****.*

Lemma 1. *If $\rho_0 > \frac{2}{3}$, then the recursion code has a positive rate.*

Proof. By counting the restrictions we have that:

$$H(n) = \Delta n (1 - \rho_0) + H(n/3) \leq \frac{3}{2}\Delta(1 - \rho_0)\Delta n$$

So we dimension of the code is at least $\frac{1}{2}\Delta n - H(n)$ which is

$$\frac{1}{2}n\Delta - \frac{3}{2}\Delta(1 - \rho_0)\Delta n = \frac{1}{2}\Delta n(3\rho_0 - 2)$$

So for any $\rho_0 > \frac{2}{3}$ we have that the rate of the C_n is grater than constant. $\square$

Recursion Decoder. blablabla