Groverize Monotone Local Search. (Short Note)

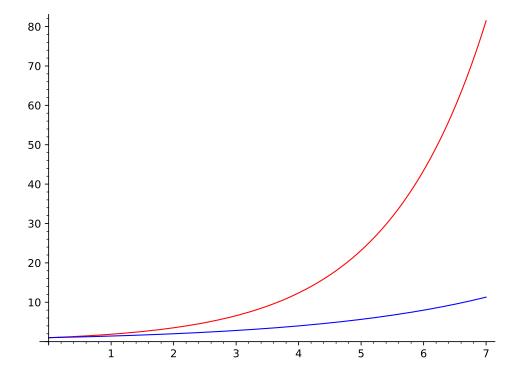
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1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the treewidth of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\sum_{k' \le k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \le \max_{k' \le k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} = \left(\max_{k' \le k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2(k'-t)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \le n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \le \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



Problem Name	Parameterized		New bound	Previous Bound
FEEDBACK VERTEX SET	3^k (r)	[cut-and-count]	1.6667^n (r)	
FEEDBACK VERTEX SET	3.592^{k}	[KociumakaP13]	1.7217^n	1.7347^n
Subset Feedback Vertex Set	4^k	[Wahlstrom14]	1.7500^n	1.8638^{n}
FEEDBACK VERTEX SET IN TOURNAMENTS	1.6181^{k}	[KumarL16]	1.3820^n	1.4656^{n}
Group Feedback Vertex Set	4^k	[Wahlstrom 14]	1.7500^n	NPR
Node Unique Label Cover	$ \Sigma ^{2k}$	[Wahlstrom14]	$(2 - \frac{1}{ \Sigma ^2})^n$	NPR
Vertex (r, ℓ) -Partization $(r, \ell \leq 2)$	3.3146^{k}	[BasteFKS15; KolayP15]	1.6984^{n}	NPR
Interval Vertex Deletion	8^k	[Cao8kinterval]	1.8750^{n}	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
Proper Interval Vertex Deletion	6^k	[HofV13; Cao15]	1.8334^{n}	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
BLOCK GRAPH VERTEX DELETION	4^k	[AgrawalLKS16]	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
Cluster Vertex Deletion	1.9102^{k}	[BoralCKP14]	1.4765^{n}	1.6181^n
Thread Graph Vertex Deletion	8^k	[Kante0KP15]	1.8750^{n}	NPR
Multicut on Trees	1.5538^{k}	[KanjLLTXXYZZZ14]	1.3565^{n}	NPR
3-HITTING SET	2.0755^{k}	[MagnusPhD07]	1.5182^{n}	1.6278^n
4-HITTING SET	3.0755^{k}	[FominGKLS10]	1.6750^{n}	1.8704^{n}
d -Hitting Set $(d \ge 3)$	$(d-0.9245)^k$	$[{\bf FominGKLS10}]$	$(2-\frac{1}{(d-0.9245)})^n$	[CochefertCGK16
Min-Ones 3-SAT	2.562^{k}	[abs-1007-1166]	1.6097^{n}	NPR
Min-Ones d-SAT $(d \ge 4)$	d^k		$(2-\frac{1}{d})^n$	NPR
Weighted d-SAT $(d \ge 3)$	d^k		$\frac{(2-\frac{1}{d})^n}{(2-\frac{1}{d})^n}$	NPR
Weighted Feedback Vertex Set	3.6181^k	[AgrawalLKS16]	1.7237^{n}	1.8638^n [F
Weighted 3-Hitting Set	2.168^{k}	ShachnaiZ15	1.5388^{n}	1.6755^n [C
Weighted d-Hitting Set $(d \ge 4)$	$(d-0.832)^k$ [For	ninGKLS10; ShachnaiZ15	$(2-\frac{1}{d-0.932})^n$	jo

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

References

- [Gro96] Lov K. Grover. A fast quantum mechanical algorithm for database search. 1996. arXiv: quant-ph/9605043 [quant-ph].
- [Fom+15] Fedor V. Fomin et al. Exact Algorithms via Monotone Local Search. 2015. arXiv: 1512. 01621 [cs.DS].