

Quantum LTC With Positive Rate

David Ponnarovsky

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preamble. preamble.

Claim for any $[[n, k, d]]$ CSS code property 1 holds

. **Proof.** let $y \in \{0, 1\}^n$ be a vector such $y \in G_z^\delta$, let assume that $|y|_{c^{\perp}} \leq C_2 d$ then for any $c \in C_x^\perp$:

$$\delta r_z \geq |H_z y| = |H_z (y + c)|$$

Robusstness Let $\omega \leq \Delta^2$. Let C_A and C_B be codes of length Δ with minimum distance d_A and d_B . We shall say that the dual tensor code $C = C_A \otimes \mathbb{F}_2^B + \mathbb{F}_2^A \otimes C_B$ is ω -robust, if for any codeword $c \in C$ of Hamming weight $|c| \leq \omega$, there exist $A' \subset A, B' \subset B, |A'| \leq |c|/d_B, |B'| \leq |c|/d_A$, such that $c_{ab} = 0$ whenever $a \notin A', b \notin B'$.

Definition. Sub-Tensor Pair We will say that C'_A, C'_B are sub-tensor pair of C_A, C_B if each of the code is subspace of C_A, C_B respectively and in addition one of the minimal codeword in C_A is also contained in C'_A (and similar to C'_B).

Note that the distance of each subcode is equal to the one from which it's derived. And also such code can be generated efficiently by choosing Δ non trivial coordinate of one of the minimal codewords and sets a check nodes over them.

Claim. Subcode Robusstness. Consider the subspaces $C'_A \subset C_A, C'_B \subset C_B$, such that the dual tensor of C_A, C_B is ω -robust then the dual tensor of C'_A, C'_B is also ω -robust.

Proof. Let c be a codeword in the dual tensor of C'_A, C'_B then it's clear that c is also in the dual tensor of C_A, C_B and therefore there exists V, U subsets of A, B respectively such that c supported only on them, and their size is less than $|c|/d_B, |c|/d_A$. As the length's space of each of the subcode is identical to its container, and by the fact that the minimal weight of codeword is greater than the minimal weight in the containing code it's follow that (1) $U \subset A' = A$ and $|c|/d_A$.