

$\sqrt{n} \mapsto \Theta(n)$ Magic States 'Distillation' Using Quantum LDPC Codes.

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1 The Construction.

Let x_0 be a codeword of C_X/C_Z^\perp , Denote by $w \in \mathbb{F}_2^n$ the binary string presents Z -generator that anti commute with the X -generator corresponds to x_0 . Let $\mathcal{X} = \{x_0, x_1, \dots, x_{k'}\} \in \mathbb{F}_2^n$ be a subset of a base for the code C_X/C_Z^\perp . Such $(\text{span } \mathcal{X}/x_0)_w$ is Triorthogonalcode. Let us denote by \mathcal{X}' the base $\{y_1, y_2, \dots, y_{k'}\} \in \mathbb{F}_2^n$ defined such: $y_i = x_j + x_0$.

Denote by E the circuit that encodes the logical i th bit to y_i , by $T^{(w)}$ the application of T gates on the qubits for which w act non trivial, means $T^{(w)}$ is a tensor product of T 's and identity where on the i th qubit $T^{(w)}$ apply T if w_i is 1 and identity otherwise. And finally by D denote the gate that decode binary strings in \mathbb{F}_2^n back into the logical space.

2 Proof of Theorem 1.

Claim 2.1. *There exists family of non-trivial distance quantum LDPC codes Q such the codes span \mathcal{X}' chosen respect to them has a positive rate. Furthermore, the rate of span \mathcal{X}' is a asymptotically converge to Q rate:*

$$|\rho(Q) - \rho(\text{span } \mathcal{X}')| = o(1)$$

Proof.

□

Claim 2.2. *Let $|\mathcal{X}'\rangle \propto \sum_{x \in \text{span } \mathcal{X}'} |x\rangle$. Then $T^{(w)} |\mathcal{X}'\rangle \propto \sum_{x \in \text{span } i} x$*