Groverize Monotone Local Search. (Short Note)

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1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

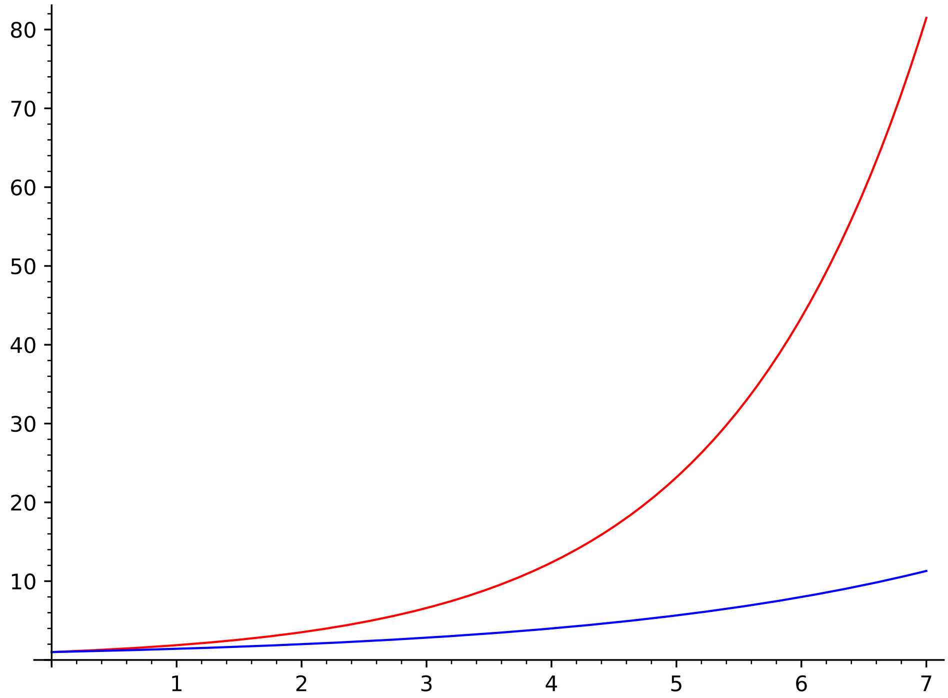
$$\sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\left(k'-t\right)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq$$

$$\Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$

Problem Name	Parameterized	Groverize	New bound	Previous Bound
Feedback Vertex Set	3^k (r) [Cyg+11]	`1.3744^{k}`	1.6667^n (r)	
Feedback Vertex Set	3.592^k [KP14]	`1.3865^{k}`	1.7217^n	1.7347^n [FTV15]
Subset Feedback Vertex Set	4^k [**Wahlstrom14**]	`1.3919^{k}`	1.7500^n	1.8638^n [Fom+14]

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).



References

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[Fom+15] Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: 1512.01621 [cs.DS].