

# Magic States Distillation Using $\Delta$ -Toric (good qLDPC?).

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Let  $|f\rangle$  be a codeword in  $C_X$ , and let  $X_g$  be the indicator that equals 1 if  $f$  has support on  $X_g$ , and 0 otherwise. Observe that applying  $T^{\otimes}$  on  $|f\rangle$  yields the state:

$$\begin{aligned} T^{\otimes n} |f\rangle &= T^{\otimes n} \left| \sum_g X_g g \right\rangle = \exp \left( i\pi/4 \sum_g X_g |g| - 2 \cdot i\pi/4 \sum_{g,h} X_g X_h |g \cdot h| \right. \\ &\quad \left. + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| - 8 \cdot i\pi/4 \cdot \text{integers} \right) |f\rangle \\ &= \exp \left( i\pi/4 \sum_g X_g |g| - 2 \cdot \pi/4 \sum_{g,h} X_g X_h |g \cdot h| + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| \right) |f\rangle \end{aligned}$$

Now assume that the code  $C_X$  is the quantum Tanner code, denote by  $G, A, B$  the group and the two generator sets that are used for constructing the square complex. In addition, let us assume the existence of  $d \in G$  such that  $d$  is non-identity and commutes with any element in  $A \cup B$ . Then, observe that multiplying by  $d$  preserves adjacency on the complex. Namely, if  $\{u, v\} \in E$  then also  $\{du, dv\} \in E$ .

Consider  $|f\rangle$  such that if  $X_g$  is not zero, and  $g$  is associated with a local codeword  $c \in C_A \otimes C_B$  on vertex  $v$ , then the generator associated with the local codeword  $c$  on vertex  $d \cdot v$  also supports  $f$ , denoted by  $g'$ . Thus, the exponent above becomes:

$$\begin{aligned} &= \exp \left( i\pi/4 \sum_g X_g |g| - 2 \cdot \pi/4 \sum_{g,h \in G/a} X_g X_h |g \cdot h| + X_{g'} X_{h'} |g \cdot h| \right. \\ &\quad \left. + 4 \cdot i\pi/4 \sum_{g,h \in G/a} X_g X_h X_l |g \cdot h \cdot l| + X_{g'} X_{h'} X_{l'} |g \cdot h \cdot l| \right) |f\rangle \\ &= \exp \left( i\pi/4 \sum_g X_g |g| - 2 \cdot 2 \cdot \pi/4 \sum_{g,h \in G/a} X_g X_h |g \cdot h| + 2 \cdot 4 \cdot i\pi/4 \sum_{g,h \in G/a} X_g X_h X_l |g \cdot h \cdot l| \right) |f\rangle \\ &= \exp \left( i\pi/4 \sum_g X_g |g| - i\pi \sum_{g,h \in G/a} X_g X_h |g \cdot h| \right) |f\rangle \end{aligned}$$

**Claim 0.1.** *The gate  $|f\rangle \mapsto \exp \left( -i\pi \sum_{g,h \in G/a} X_g X_h |g \cdot h| \right) |f\rangle$  is in the Clifford.*

*Proof.* Just decode  $f$  and apply **CZ** between any pair of qubits corresponding to the generators  $g, h$  such that  $g \cap h = 1$ . Then encode the state again. Observe that **CZ** is a Clifford gate, and by the fact that the code is a CSS code then the decoder and the encoder are both in the Clifford.  $\square$

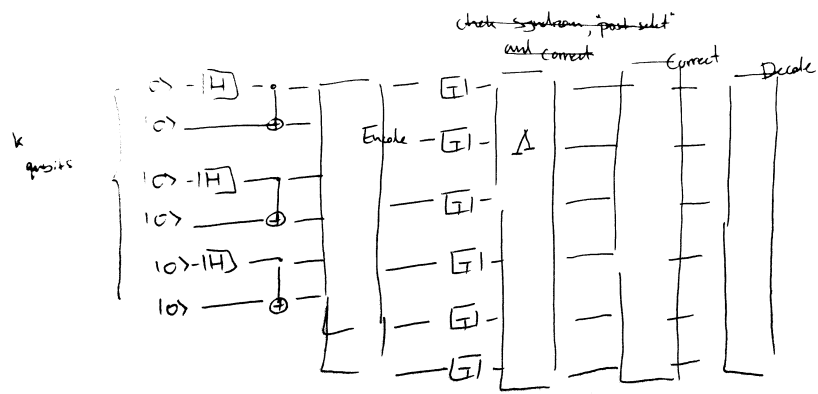


Figure 1: Quantum Circuit for distillation.