Groverize Monotone Local Search. (Short Note)

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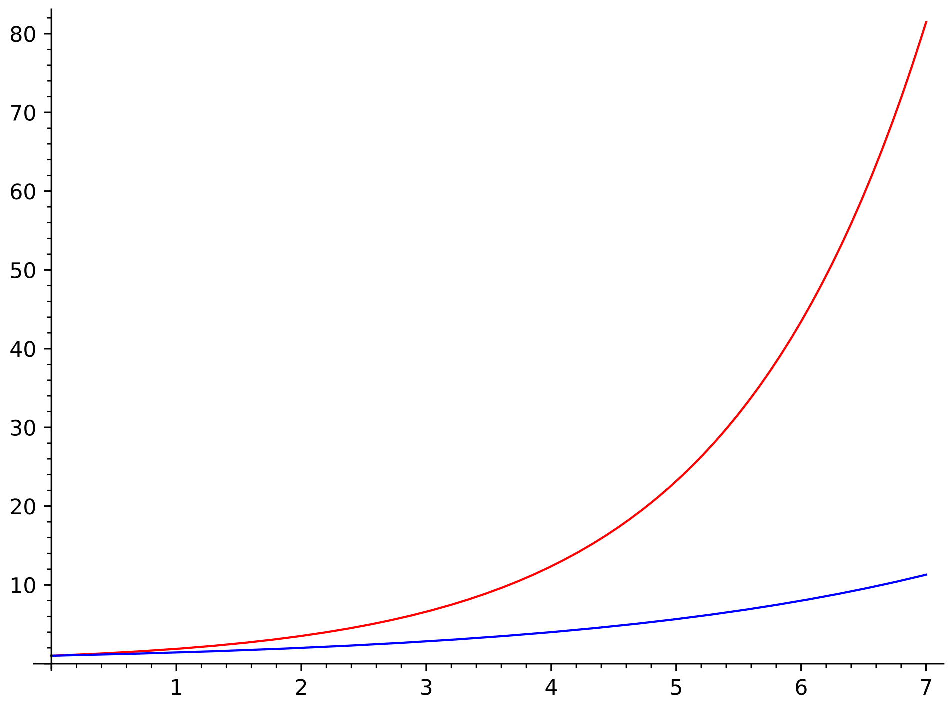
1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\left(k'-t\right)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq$$

$$\Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



Problem Name	Parameterized		New bound	Previous Bound	
FEEDBACK VERTEX SET	3^k (r)	[Cyg+11]	1.6667^n (r)		
FEEDBACK VERTEX SET	3.592^k	[KP14]	1.7217^n	1.7347^n	[FTV15]
SUBSET FEEDBACK VERTEX SET	4^k	**[Wahlstrom14]**	1.7500^n	1.8638^n	[Fom+14]
FEEDBACK VERTEX SET IN TOURNAMENTS	1.6181^k	[KL16]	1.3820^n	1.4656^n	[KL16]
GROUP FEEDBACK VERTEX SET	4^k	**[Wahlstrom14]**	1.7500^n	NPR	
NODE UNIQUE LABEL COVER	$\|\Sigma\|^{2k}$	**[Wahlstrom14]**	$(2-\frac{1}{\|\Sigma\|^2})^n$	NPR	
VERTEX (r,ℓ)-PARTIZATION $(r,\ell \leq 2)$	3.3146^k	**[KolayP15**; Bas+17]	1.6984^n	NPR	
INTERVAL VERTEX DELETION	8^k	[Cao16]	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13]
PROPER INTERVAL VERTEX DELETION	6^k	[tV13; Cao15]	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13]
BLOCK GRAPH VERTEX DELETION	4^k	[Agr+16]	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13]
CLUSTER VERTEX DELETION	1.9102^k	[Bor+14]	1.4765^n	1.6181^n	[Fom+10]
THREAD GRAPH VERTEX DELETION	8^k	[Kan+15]	1.8750^n	NPR	
MULTICUT ON TREES	1.5538^k	[Kan+14]	1.3565^n	NPR	
3-HITTING SET	2.0755^k	**[MagnusPhD07]**	1.5182^n	1.6278^n	**[MagnusPhD07]**
4-HITTING SET	3.0755^k	[Fom+10]	1.6750^n	1.8704^n	[Fom+10]
d-HITTING SET $(d \geq 3)$	$(d-0.9245)^k$	[Fom+10]	$(2-\frac{1}{(d-0.9245)})^n$		[Coc+16; Fom+10]
MIN-ONES 3-SAT	2.562^k	**[abs-1007-1166]**	1.6097^n	NPR	
MIN-ONES d-SAT $(d \geq 4)$	d^k		$(2-\frac{1}{d})^n$	NPR	
WEIGHTED d-SAT $(d \geq 3)$	d^k		$(2-\frac{1}{d})^n$	NPR	
WEIGHTED FEEDBACK VERTEX SET	3.6181^k	[Agr+16]	1.7237^n	1.8638^n	[Fom+08]
WEIGHTED 3-HITTING SET	2.168^k	[SZ15]	1.5388^n	1.6755^n	[Coc+16]
WEIGHTED d-HITTING SET $(d \geq 4)$	$(d-0.832)^k$	[Fom+10; SZ15]	$(2-\frac{1}{d-0.932})^n$		[Coc+16]

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

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