

Groverize Monotone Local Search. (Short Note)

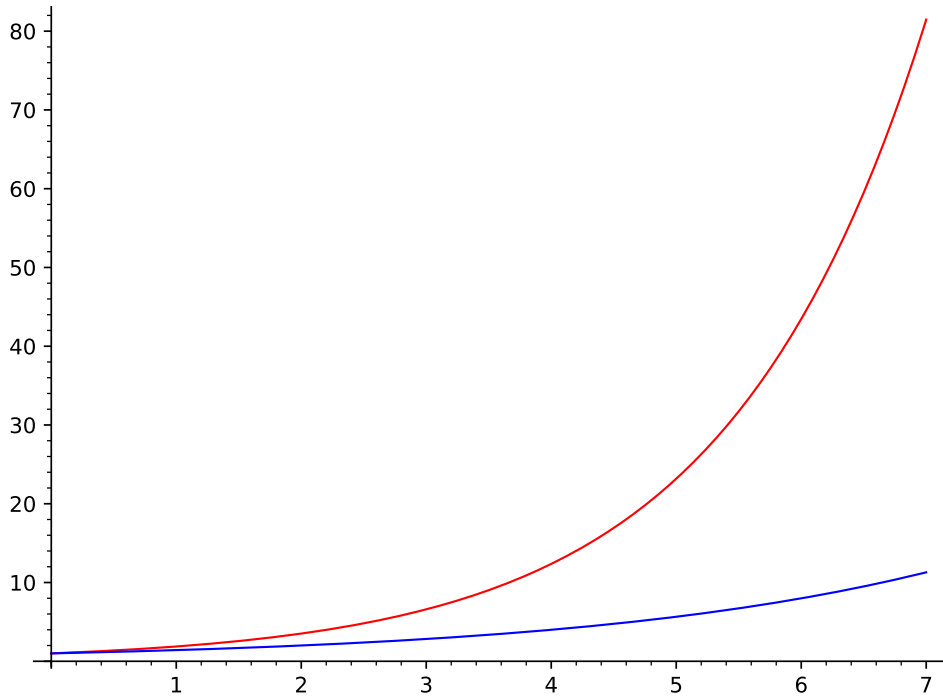
David Ponnarovsky

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1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\begin{aligned} \sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} &\leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} = \\ \left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2(k'-t)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} &= \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq \\ \Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)} \end{aligned}$$



Problem Name	Parameterized		New bound	Previous Bound
FEEDBACK VERTEX SET	3^k (r)	[cut-and-count]	1.6667^n (r)	
FEEDBACK VERTEX SET	3.592^k	[KociumakaP13]	1.7217^n	1.7347^n
SUBSET FEEDBACK VERTEX SET	4^k	[Wahlstrom14]	1.7500^n	1.8638^n
FEEDBACK VERTEX SET IN TOURNAMENTS	1.6181^k	[KumarL16]	1.3820^n	1.4656^n
GROUP FEEDBACK VERTEX SET	4^k	[Wahlstrom14]	1.7500^n	NPR
NODE UNIQUE LABEL COVER	$ \Sigma ^{2k}$	[Wahlstrom14]	$(2 - \frac{1}{ \Sigma ^2})^n$	NPR
$(r, \ell \leq 2)$	3.3146^k	[BasteFKS15; KolayP15]	1.6984^n	NPR
INTERVAL VERTEX DELETION	8^k	[Cao8kinterval]	1.8750^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$
PROPER INTERVAL VERTEX DELETION	6^k	[HofV13; Cao15]	1.8334^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$
BLOCK GRAPH VERTEX DELETION	4^k	[AgrawalLKS16]	1.7500^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$
CLUSTER VERTEX DELETION	1.9102^k	[BoralCKP14]	1.4765^n	1.6181^n
THREAD GRAPH VERTEX DELETION	8^k	[Kante0KP15]	1.8750^n	NPR
MULTICUT ON TREES	1.5538^k	[KanjLLTXXYZZZ14]	1.3565^n	NPR
3-HITTING SET	2.0755^k	[MagnusPhD07]	1.5182^n	1.6278^n
4-HITTING SET	3.0755^k	[FominGKLS10]	1.6750^n	1.8704^n
d -HITTING SET ($d \geq 3$)	$(d - 0.9245)^k$	[FominGKLS10]	$(2 - \frac{1}{(d-0.9245)})^n$	[CochefertCGK16]
MIN-ONES 3-SAT	2.562^k	[abs-1007-1166]	1.6097^n	NPR
MIN-ONES d -SAT ($d \geq 4$)	d^k		$(2 - \frac{1}{d})^n$	NPR
WEIGHTED d -SAT ($d \geq 3$)	d^k		$(2 - \frac{1}{d})^n$	NPR
WEIGHTED FEEDBACK VERTEX SET	3.6181^k	[AgrawalLKS16]	1.7237^n	1.8638^n
WEIGHTED 3-HITTING SET	2.168^k	[ShachnaiZ15]	1.5388^n	1.6755^n
WEIGHTED d -HITTING SET ($d \geq 4$)	$(d - 0.832)^k$	[FominGKLS10; ShachnaiZ15]	$(2 - \frac{1}{d-0.932})^n$	

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N . The algorithms in the first row are randomized (r).

References

- [Gro96] Lov K. Grover. *A fast quantum mechanical algorithm for database search*. 1996. arXiv: [quant-ph/9605043](#) [quant-ph].
- [Fom+15] Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: [1512.01621](#) [cs.DS].