

# From classical to good quantum LDPC codes.

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# Today.

- Brif Review of Coding.

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- Brief Review of Coding. Tanner and Expander codes.
- Quantum Error Correction Codes.

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- Brief Review of Coding. Tanner and Expander codes.
- Quantum Error Correction Codes.
- Good Classical Locally Testable Codes and Good Quantum LDPC.

# Classical Vs Quantum Encoding.

Classical:



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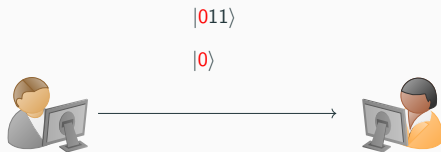
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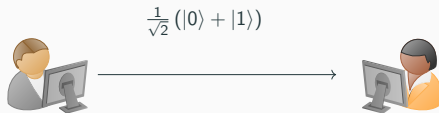


# Classical Vs Quantum Encoding.

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Quantum:



# Classical Vs Quantum Encoding.

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Quantum:



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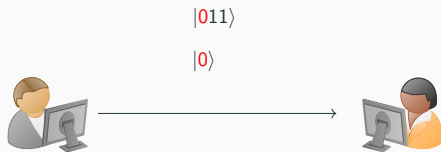


Quantum:



# Classical Vs Quantum Encoding.

Classical:



Quantum:



## The C.S Questions.

In the asymptotic regime, can we encode quantum states in codes robust against many errors, as our original message grows? And in what costs?

## Good Classical LDPC Code.

### Definition

Let  $n \in \mathbb{N}$  and  $\rho, \delta \in (0, 1)$ . We say that  $C$  is a **binary linear code** with parameters  $[n, \rho n, \delta n]$ . If  $C$  is a subspace of  $\mathbb{F}_2^n$ , and the dimension of  $C$  is at least  $\rho n$  and any pair of distinct elements in  $C$  differ in at least  $\delta n$  coordinates. We call to the vectors belong to  $C$  *codewords*, to  $\rho n$  the dimension of the code, and to  $\delta n$  the distance of the code.

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We will say that a family of codes is a **good code** if its parameters converge into positive values.

## Good Classical LDPC Code.

### Parity Check Matrix.

Code  $C$  is a linear subspace  $\Rightarrow$  There is a matrix  $H$  such:

$$x \in C \Leftrightarrow Hx = 0$$

We will call  $H$  the parity check matrix.

### Definition

A codes family will be called LDPC code if weight of any row (col) in  $H$  is  $O(1)$ .

### Example. Repetition code.

Let the Repetition code,  $[n, 1, n]$  be the mapping  $0 \rightarrow 0^n$  and  $1 \rightarrow 1^n$ .

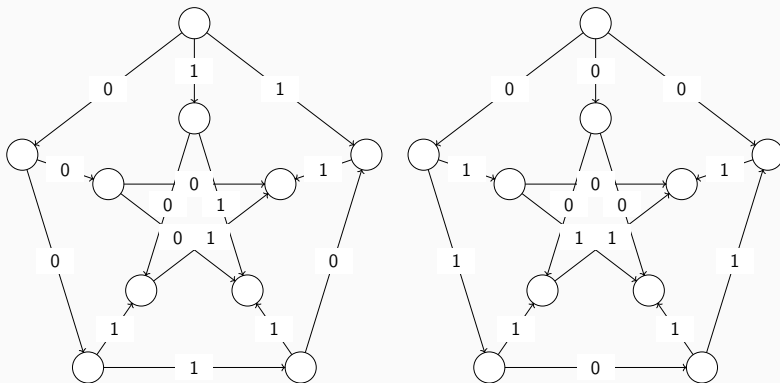
Technic for design LDPC families with positive rate.

### Definition

Let  $\Gamma$  be a graph and  $C_0$  be a “small” linear code with finite parameters  $[\Delta, \rho\Delta, \delta\Delta]$ . Let  $C = \mathcal{T}(\Gamma, C_0)$  be all the codewords which, for any vertex  $v \in \Gamma$ , the local view of  $v$  is a codeword of  $C_0$ . We say that  $C$  is a **Tanner code** of  $\Gamma, C_0$ . Notice that if  $C_0$  is a binary linear code, So  $C$  is.

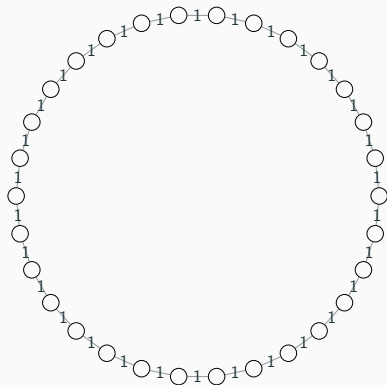
## Good Classical LDPC Code.

Example, the parity code on the Peterson graph.



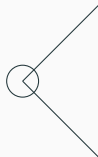
## Good Classical LDPC Code.

Another example, the repetition code can be thought as the tanner graph defined by the parity code on the cycle graph.



parity check matrix of  $C_0$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$



Parity check matrix of  $\mathcal{T}(\Gamma, C_0)$   
Each row associated with vertex check.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

## Good Classical LDPC Code.

### Lemma

*Tanner codes have a rate of at least  $2\rho - 1$ .*

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### Proof.

The dimension of the subspace is bounded by the dimension of the container minus the number of restrictions. So assuming non-degeneration of the small code restrictions, we have that any vertex count exactly  $(1 - \rho) \Delta$  restrictions. Hence,

$$\dim C \geq \frac{1}{2}n\Delta - (1 - \rho) \Delta n = \frac{1}{2}n\Delta (2\rho - 1)$$

Clearly, any small code with rate  $> \frac{1}{2}$  will yield a code with an asymptotically positive rate □



## Good Classical LDPC Code.

Technic for design LDPC families with positive relative distance.

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### Definition

Denote by  $\lambda$  the second eigenvalue of the adjacency matrix of the  $\Delta$ -regular graph. For our uses, it will be satisfied to define  $\lambda$ -Expander as a graph  $G = (V, E)$  such that for any two subsets of vertices  $T, S \subset V$ , the number of edges between  $S$  and  $T$  is at most:

$$|E(S, T) - \frac{\Delta}{n}|S||T|| \leq \lambda\sqrt{|S||T|}$$

## Good Classical LDPC Code.

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*Using  $\lambda$ -Expander, the Tanner Code defined bit is a good LDPC code.*

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### Proof.

Fix a codeword  $x \in C$  and denote By  $S$  the support of  $x$  over the edges. Namely, a vertex  $v \in V$  belongs to  $S$  if it connects to nonzero edges regarding the assignment by  $x$ , Assume towards contradiction that  $|x| = o(n)$ . And notice that  $|S|$  is at most  $2|x|$ , Then by The Expander Mixing Lemma we have that:

$$\begin{aligned} \text{bits seen by any } v \in S &\leq \text{average degree of } v \in G \text{ restricted to } S \\ &= \frac{E(S, S)}{|S|} \leq \frac{\Delta}{n}|S| + \lambda \\ &\leq_{n \rightarrow \infty} o(1) + \lambda \end{aligned}$$



## Good Classical LDPC Code.









## Idea I - (Uncertainty) Clouds as States.



## 'Idea II' - Tanner Checks are 'Too Much' Interdependence.

## 'Idea III' - Impossibility of Both $C_X, C_Z$ being Good.

# Quantum Tanner Code Construction.

