

# Recursion Code.

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February 23, 2023

## Abstract

None

## 1 Construction.

**Definition 1.** Let  $\Delta$  be an integer greater than 2 and consider an algorithm  $\mathcal{A}$  that for any  $n$  that is power of 3 construct a  $\Delta$ -regular graph over  $n$  vertices. Now, let  $G$  be  $\Delta$ -regular graph over  $n$  vertices generated by  $\mathcal{A}$ . Define the **third graph obtained by  $G$** , labeled by  $G^\sim$  to be the graph which  $\mathcal{A}$  returns over  $\frac{1}{3}n$  such that any of the edges could be associate by puncturing a  $\frac{2}{3}$  fraction of the edges of each vertex.

**Definition 2.** Let  $G$  be a  $\Delta$ -regular graph, such that each edge is associated with integer in  $[\frac{1}{2}\Delta]$  and no vertex adjoins to two different indexed edges. For example consider a Cayley graph defined by  $\frac{1}{2}\Delta$  generators  $\{g_0, g_1, g_2, \dots, g_{\frac{1}{2}\Delta}\}$ , then the undirected graph is  $\Delta$ -regular and any edge could be labeled by the function  $f(g_i, g_i^{-1}) \rightarrow i$ . Define the  **$[a, b]$ -fraction graph obtained by  $G$** , labeled by  $G^{[a, b]}$  to be the graph which obtained taken only the edges such their label's are in the range  $[a, b]$ .

For convenient, we will denote by  $G^{\frac{1}{3}}$ ,  $G^{\frac{2}{3}+}$  and  $G^{\frac{2}{3}-}$  the fraction graph correspond to taking the middle third edges and the higher and the lower 2-third edges.

**Definition 3** (Recursion Code). Let  $C_0 = \Delta[1, \rho_0, \delta_0]$  be a binary linear code. We will define the recursion code in recursive manner. First for a sufficiently large integer  $n_0$ , which is also power of 3,  $C(n_0)$  defined to be the Tanner code defined by the  $C_0$  and graph  $\mathcal{A}(n_0)$ . Then let  $n$  be any power of 3, such that  $n > n_0$ , denote by  $G$  the graph that constructed by the running of  $\mathcal{A}(n)$ . Then let  $C(n)$  be the code obtained by the joining the parity check matrix of the Tanner code  $\mathcal{T}(G, C_0)$  and by the checks of the  $C(n/3)$  over the bits associated with the  $G^\sim$ . We will call to that code family the **recursion code**.

**Lemma 1.** If  $\rho_0 > \frac{2}{3}$ , then the recursion code has a positive rate.

*Proof.* By counting the restrictions we have that:

$$H(n) = \Delta n (1 - \rho_0) + H(n/3) \leq \frac{3}{2} \Delta (1 - \rho_0) \Delta n$$

So we dimension of the code is at least  $\frac{1}{2} \Delta n - H(n)$  which is

$$\frac{1}{2} n \Delta - \frac{3}{2} \Delta (1 - \rho_0) \Delta n = \frac{1}{2} \Delta n (3\rho_0 - 2)$$

So for any  $\rho_0 > \frac{2}{3}$  we have that the rate of the  $C_n$  is grater than constant.  $\square$

**Recursion Decoder.** balabla

- 1 Decode  $G^{\frac{2}{3}+}$
- 2 Decode  $G^{\frac{2}{3}-}$
- 3 Decode  $G^{\frac{1}{3}}$