Magic States Distillation Using Δ-Toric (good qLDPC?).

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Let $|f\rangle$ be a codeword in C_X, and let X_g be the indicator that equals 1 if f has support on X_g, and 0 otherwise. Observes that applying $T^{\otimes}$ on $|f\rangle$ yilds the state:

$$\begin{aligned} T^{\otimes n}|f\rangle = T^{\otimes n}|\sum_g X_g g\rangle &= \exp\Big(i\pi/4\sum_g X_g|g| - 2\cdot i\pi/4\sum_{g,h} X_gX_h|g\cdot h| \\ &+ 4\cdot i\pi/4\sum_{g,h} X_gX_hX_l|g\cdot h\cdot l| - 8\cdot i\pi/4\cdot \text{ integers }\Big)|f\rangle \\ &= \exp\Big(i\pi/4\sum_g X_g|g| - 2\cdot \pi/4\sum_{g,h} X_gX_h|g\cdot h| + 4\cdot i\pi/4\sum_{g,h} X_gX_hX_l|g\cdot h\cdot l|\Big)|f\rangle \end{aligned}$$

Now assume that the code C_X is the quantum Tanner code, denote by G, A, B the group and the two generator sets that are used for constructing the square complex. In addition, let us assume the existence of $d \in G$ such that d is non-identity and commutes with any element in $A \cup B$. Then, observe that multiplying by d preserves adjacency on the complex. Namely, if $\{u, v\} \in E$ then also $\{du, dv\} \in E$.

Consider $|f\rangle$ such that if X_g is not zero, and g is associated with a local codeword $c \in C_A \otimes C_B$ on vertex v, then the generator associated with the local codeword c on vertex $d \cdot v$ also supports f, denoted by g'. Thus, the exponent above becomes:

$$\begin{aligned} &= \exp\Big(i\pi/4\sum_g X_g|g| - 2\cdot\pi/4\sum_{g,h\in G/a} X_gX_h|g\cdot h| + X_{g'}X_{h'}|g\cdot h| \\ &+ 4\cdot i\pi/4\sum_{g,h\in G/a} X_gX_hX_l|g\cdot h\cdot l| + X_{g'}X_{h'}X_{l'}|g\cdot h\cdot l|\Big)|f\rangle \\ &= \exp\Big(i\pi/4\sum_g X_g|g| - 2\cdot 2\cdot\pi/4\sum_{g,h\in G/a} X_gX_h|g\cdot h| + 2\cdot 4\cdot i\pi/4\sum_{g,h\in G/a} X_gX_hX_l|g\cdot h\cdot l|\Big)|f\rangle \\ &= \exp\Big(i\pi/4\sum_g X_g|g| - i\pi\sum_{g,h\in G/a} X_gX_h|g\cdot h|\Big)|f\rangle \end{aligned}$$

Claim 0.1. *The gate* $|f\rangle \mapsto \exp\Big(-i\pi\sum_{g,h\in G/a} X_gX_h|g\cdot h|\Big)|f\rangle$ *is in the Clifford.*

Proof. Just decode f and apply **CZ** between any pair of qubits corresponding to the generators g, h such that $g \cap h = 1$. Then encode the state again. Observes that **CZ** is a Clifford gate, and by the fact that the code is a CSS code then the decoder and the encoder are both in the Clifford. □

Let's denote the circuit defined in Claim 0.1 by Λ. So we have that:

$$\begin{aligned} \Lambda^\dagger \exp\Big(i\pi/4\sum_g X_g|g| - i\pi\sum_{g,h\in G/a} X_gX_h|g\cdot h|\Big)|f\rangle \\ = \exp\Big(i\pi/4\sum_g X_g|g|\Big)|f\rangle \end{aligned}$$

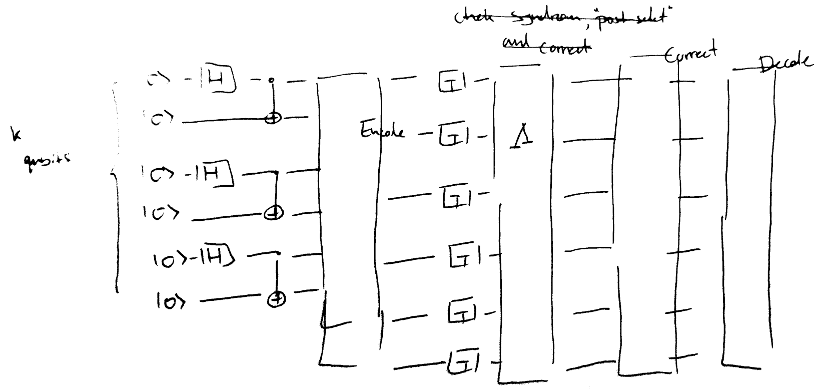

Figure 1: Quantum Circuit for distillation.