

QNC₁ \subset noisy-BQP

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1 Notations.

C_g - good qLDPC, C_{ft} - concatenation code (ft stands for fault tolerance). For a code C_y we use Φ_y, E_y, D_y to denote the channel maps circuits into the circuits compute in the code space, the encoder, and the decoder. We use Φ_U to denote the 'Bell'-state storing the gate U .

2 The Noise Model

3 Fault Tolerance (With Resets gates) at Linear Depth.

Claim 3.1. *There is $p_{th} \in (0, 1)$ such that if $p < p_{th}$ then any quantum circuit C with depth D and width W can be computed by p -noisy, resets allowed, circuit C' , with a depth at most $\max\{D, \log(WD)\}$.*

3.1 Initializing Magic for Teleportation gates and encodes ancillaries.

The Protocol:

1. Initializing zeros. Divide the qubits into $|B|$ -size blocks. Encodes each block in C_g via $D_{ft}\Phi_{ft}[E_g] |0^{|B|}\rangle$.
2. Initializing Magic for Teleportation gates encoded in C_g via $D_{ft}\Phi_{ft}[E_g] |\Phi_U\rangle$ for each gate U in the original circuit.
3. Each gate is replaced by gate teleportation.
4. At any time tick, any block runs a single round of error reduction.

Claim 3.2. *The gate $D_{ft}\Phi_{ft}[E_g]$ initializes states encoded in C_g subject to p -noise channel.*

Claim 3.3. *With probability almost surely, the total amount of noise been absorb in a block is less than αn .*

Proof. Consider the i th block, denoted by B_i . Using the Hoeffding's inequality we bound the probability that more than $\beta|B|$ bits are flipped at time t .

$$\Pr[\beta n \text{ bits were flipped in } B_i \text{ at time } t] \leq 2e^{-2|B|(\beta-p)}$$

□