

# $\sqrt{n} \mapsto \Theta(n)$ Magic States 'Distillation' Using Quantum LDPC Codes.

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## 1 The Construction.

Let  $x_0$  be a codeword of  $C_X/C_Z^\perp$ , Denote by  $w \in \mathbb{F}_2^n$  the binary string presents  $Z$ -generator that anti commute with the  $X$ -generator corresponds to  $x_0$ . Let  $\mathcal{X} = \{x_0, x_1, \dots, x_{k'}\} \in \mathbb{F}_2^n$  be a subset of a base for the code  $C_X/C_Z^\perp$ . Such  $(\text{span } \mathcal{X}/x_0) \mid_w$  is Triorthogonalcode. Let us denote by  $\mathcal{X}'$  the base  $\{y_1, y_2, \dots, y_{k'}\} \in \mathbb{F}_2^n$  defined such:  $y_i = x_j + x_0$ .

Denote by  $E$  the circuit that encodes the logical  $i$ th bit to  $y_i$ , by  $T^{(w)}$  the application of  $T$  gates on the qubits for which  $w$  act non trivial, means  $T^{(w)}$  is a tensor product of  $T$ 's and identity where on the  $i$ th qubit  $T^{(w)}$  apply  $T$  if  $w_i$  is 1 and identity otherwise. And finally by  $D$  denote the gate that decode binary strings in  $\mathbb{F}_2^n$  back into the logical space.

## 2 Proof of Theorem 1.

**Claim 2.1.** *There exists quantum codes such the set  $C_X/C_Z^\perp$  chosen respect to them define a code with positive rate.*

*Proof.*

□

**Claim 2.2.** *Let  $|\mathcal{X}'\rangle \propto \sum_{x \in \text{span } \mathcal{X}'} |x\rangle$ . Then  $T^{(w)} |\mathcal{X}'\rangle \propto \sum_{x \in \text{span } i} x$*