Problem

October 5, 2023

1 Problem.

Let $G = (L, R_0 \cup R_1, E)$ be bipartite graph. We think about the R_i vertices as the vertics that should applay 'logical-gate' X_i and a 'fake-gate' $X_{\bar{i}}$. Now let L' be additional vertices set at size $\Theta(|L|)$.

Claim 1.1. *There is a way to connect R_0, R_1 to L' such that:*

1. *Any vertex of R is connected by exactly single edge to L'.*
2. *(Strong.) The obtained graph is expnder. (Weak.) The expnasion of the new graph is not far way form the expnasion of the original graph.*
3. *(Computinal.) The reduction takes polynomial time.*

2 Idea.

Just connect R to L' such the graph (R, L') is expander. And then we get for $S \subset R$:

$$\begin{aligned}\Gamma'(S) &\geq \Gamma^{(1)}(S) + \Gamma^{(2)}(S) \geq \alpha\Delta|S| + \beta|S| \\ &\geq \min(\alpha, \beta)(\Delta + 1)|S|\end{aligned}$$

And for the left expansion $T = T_1 \cup T_2$ such that $T_1 \subset L$ and $T_2 \subset L'$.

$$\begin{aligned}\Gamma(T) &= \Gamma(T_1) + \Gamma(T_2) - \Gamma(T_1) \cap \Gamma(T_2) \\ &\geq \Gamma^{(1)}(T_1), \Gamma^{(2)}(T_2) \to \frac{1}{2}\left(\Gamma^{(1)}(T_1) + \Gamma^{(2)}(T_2)\right) \\ &\geq \frac{1}{2}\min(\gamma, \rho)\Delta|T|\end{aligned}$$