Groverize Monotone Local Search. (Short Note)

David Ponarovsky

April 25, 2023

1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

We will simplify the definitions given at [Fom+15] and use the following definitions instead:

Definition 1 (extension problems)**.** *Consider a decision problem inside **NP**, in this paper, we will associate two verifiers U, V with each language. U stands for input validation, conceptually it uses for checking that the solution 'live' inside the problem world. For example, for the 3-**SAT**, U checks that the input indeed encode an assignment. Formally the role of U is to restrict the inputs to certain form. And V responsible to verify that the word indeed in the langauge, ie check that the assignment satisfies the formula. We will say that a problem is an* extension problem *if requiring any of the input bits to be 1 could reduced to another instance of the problem. For example, consider 3-**SAT**, fixing an aribtray bit x_i to be 1 could reduced to another 3-**SAT** formula by erase any of the closures contain x_i and replacing any of the occureents of $\bar{x}_i$ by other termianl on the same clouser (i.e $\bar{x}_i \wedge \bar{y} \wedge z \mapsto \bar{y} \wedge \bar{y} \wedge z$). the input any instance of the problem could be representated as the bit-wise union of two strings which pass U verification. For example, any assignment satisfies a 3-**SAT** instance could be write as or-wise of two assignments.*

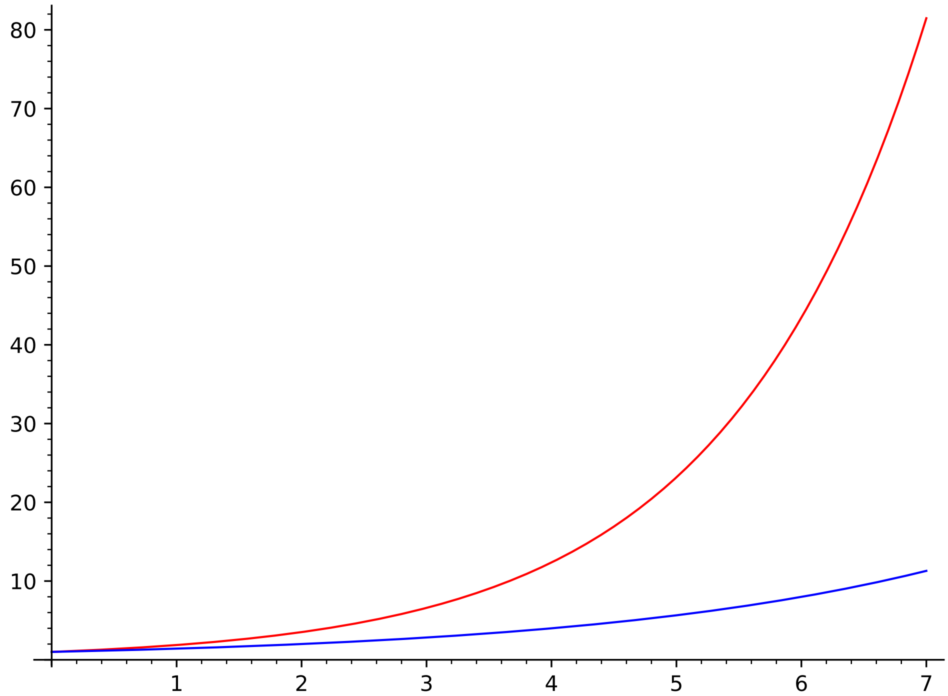
Definition 2. *A* directed graph *G is a pair (V, E) where V is a set of* vertices *and E is a set of* directed edges*.*

Definition 3. *The* directed shortest path problem *is the problem of finding the directed path with the minimum weight between two given vertices in a directed weighted graph.*

$$\sum_{k'\le k}\frac{1}{\sqrt{p(k')}}\cdot c^{k'-t}N^{\mathcal{O}(1)} \le \max_{k'\le k}\left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}}\right)^{\frac{1}{2}}\cdot c^{k'-t}N^{\mathcal{O}(1)} =$$

$$\left(\max_{k'\le k}\frac{\binom{n-|X|}{t}}{\binom{k'}{t}}\cdot c^{2\left(k'-t\right)}\right)^{\frac{1}{2}}N^{\mathcal{O}(1)} = \left(\max_{k\le n-|X|}\frac{\binom{n-|X|}{t}}{\binom{k}{t}}\right)^{\frac{1}{2}}\cdot c^{2(k-t)}N^{\mathcal{O}(1)} \le$$

$$\Rightarrow\left(2-\frac{1}{c^2}\right)^{\frac{n-|X|}{2}}N^{\mathcal{O}(1)}$$



Problem Name	Parameterized		Groverize	New bound	Previous Bound	
Feedback Vertex Set	3^k (r)	[Cyg+11]	1.3744^k	1.6667^n (r)		
Feedback Vertex Set	3.592^k	[KP14]	1.3865^k	1.7217^n	1.7347^n	[FTV15
Subset Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.3919^k	1.7500^n	1.8638^n	[Fom+14
Feedback Vertex Set in Tournaments	1.6181^k	[KL16]	1.2720^k	1.3820^n	1.4656^n	[KL16
Group Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.3919^k	1.7500^n	NPR	
Node Unique Label Cover	$\lvert\Sigma\rvert^{2k}$	[**Wahlstrom14**]	1.3919^k	$(2-\frac{1}{\lvert\Sigma\rvert^2})^n$	NPR	
Vertex (r,ℓ)-Partization $(r,\ell \leq 2)$	3.3146^k	[**KolayP15**; Bas+17]	1.3817^k	1.6984^n	NPR	
Interval Vertex Deletion	8^k	[Cao16]	1.3466^k	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Proper Interval Vertex Deletion	6^k	[tV13; Cao15]	1.4087^k	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Block Graph Vertex Deletion	4^k	[Agr+16]	1.4044^k	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Cluster Vertex Deletion	1.9102^k	[Bor+14]	1.3919^k	1.4765^n	1.6181^n	[Fom+10
Thread Graph Vertex Deletion	8^k	[Kan+15]	1.3919^k	1.8750^n	NPR	
Multicut on Trees	1.5538^k	[Kan+14]	1.3138^k	1.3565^n	NPR	
3-Hitting Set	2.0755^k	[**MagnusPhD07**]	1.4087^k	1.5182^n	1.6278^n	[**MagnusPhD07**
4-Hitting Set	3.0755^k	[Fom+10]	1.2593^k	1.6750^n	1.8704^n	[Fom+10
d-Hitting Set $(d \geq 3)$	$(d-0.9245)^k$	[Fom+10]	1.1763^k	$(2-\frac{1}{(d-0.9245)})^n$		[Coc+16; Fom+10
Min-Ones 3-SAT	2.562^k	[**abs-1007-1166**]	1.3296^k	1.6097^n	NPR	
Min-Ones d-SAT $(d \geq 4)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted d-SAT $(d \geq 3)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted Feedback Vertex Set	3.6181^k	[Agr+16]	1.1763^k	1.7237^n	1.8638^n	[Fom+08
Weighted 3-Hitting Set	2.168^k	[SZ15]	1.3593^k	1.5388^n	1.6755^n	[Coc+16
Weighted d-Hitting Set $(d \geq 4)$	$(d-0.832)^k$	[Fom+10; SZ15]	1.3919^k	$(2-\frac{1}{d-0.932})^n$		[Coc+16

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

References

[Gro96] Lov K. Grover. *A fast quantum mechanical algorithm for database search.* 1996. arXiv: quant-ph/9605043 [quant-ph].

[Fom+08] Fedor V. Fomin et al. "On the minimum feedback vertex set problem: exact and enumeration algorithms". In: *Algorithmica* 52.2 (2008), pp. 293–307. ISSN: 0178-4617. DOI: 10.1007/s00453-007-9152-0. URL: https://doi.org/10.1007/s00453-007-9152-0.

[Fom+10] Fedor V. Fomin et al. "Iterative compression and exact algorithms". In: *Theoret. Comput. Sci.* 411.7-9 (2010), pp. 1045–1053. ISSN: 0304-3975. DOI: 10.1016/j.tcs.2009.11.012. URL: https://doi.org/10.1016/j.tcs.2009.11.012.

[Cyg+11] Marek Cygan et al. "Solving connectivity problems parameterized by treewidth in single exponential time (extended abstract)". In: *2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011.* IEEE Computer Soc., Los Alamitos, CA, 2011, pp. 150–159. DOI: 10.1109/FOCS.2011.23. URL: https://doi.org/10.1109/FOCS.2011.23.

[tV13] Pim van 't Hof and Yngve Villanger. "Proper interval vertex deletion". In: *Algorithmica* 65.4 (2013), pp. 845–867. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9661-3. URL: https://doi.org/10.1007/s00453-012-9661-3.

[BFP13] Ivan Bliznets, Fedor V. Fomin, and Yngve Pilipczuk Michałand Villanger. "Largest chordal and interval subgraphs faster than 2^n". In: *Algorithms—ESA 2013.* Vol. 8125. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2013, pp. 193–204. DOI: 10.1007/978-3-642-40450-4_17. URL: https://doi.org/10.1007/978-3-642-40450-4_17.

[Bor+14] Anudhyan Boral et al. "A fast branching algorithm for cluster vertex deletion". In: *Computer science—theory and applications.* Vol. 8476. Lecture Notes in Comput. Sci. Springer, Cham, 2014, pp. 111–124. DOI: 10.1007/978-3-319-06686-8_9. URL: https://doi.org/10.1007/978-3-319-06686-8_9.

[Fom+14] Fedor V. Fomin et al. "Enumerating minimal subset feedback vertex sets". In: *Algorithmica* 69.1 (2014), pp. 216–231. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9731-6. URL: https://doi.org/10.1007/s00453-012-9731-6.

[Kan+14] Iyad Kanj et al. "Algorithms for cut problems on trees". In: *Combinatorial optimization and applications*. Vol. 8881. Lecture Notes in Comput. Sci. Springer, Cham, 2014, pp. 283–298. DOI: 10.1007/978-3-319-12691-3_22. URL: https://doi.org/10.1007/978-3-319-12691-3_22.

[KP14] Tomasz Kociumaka and Marcin Pilipczuk. "Faster deterministic Feedback Vertex Set". In: *Inform. Process. Lett.* 114.10 (2014), pp. 556–560. ISSN: 0020-0190. DOI: 10.1016/j.ipl.2014.05.001. URL: https://doi.org/10.1016/j.ipl.2014.05.001.

[Cao15] Yixin Cao. "Unit interval editing is fixed-parameter tractable". In: *Automata, languages, and programming. Part I*. Vol. 9134. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2015, pp. 306–317. DOI: 10.1007/978-3-662-47672-7_25. URL: https://doi.org/10.1007/978-3-662-47672-7_25.

[FTV15] Fedor V. Fomin, Ioan Todinca, and Yngve Villanger. "Large induced subgraphs via triangulations and CMSO". In: *SIAM J. Comput.* 44.1 (2015), pp. 54–87. ISSN: 0097-5397. DOI: 10.1137/140964801. URL: https://doi.org/10.1137/140964801.

[Fom+15] Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: 1512.01621 [cs.DS].

[Kan+15] Mamadou Moustapha Kanté et al. "An FPT algorithm and a polynomial kernel for linear rankwidth-1 vertex deletion". In: *10th International Symposium on Parameterized and Exact Computation*. Vol. 43. LIPIcs. Leibniz Int. Proc. Inform. Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2015, pp. 138–150.

[SZ15] Hadas Shachnai and Meirav Zehavi. "A multivariate approach for weighted FPT algorithms". In: *Algorithms—ESA 2015*. Vol. 9294. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2015, pp. 965–976. DOI: 10.1007/978-3-662-48350-3_80. URL: https://doi.org/10.1007/978-3-662-48350-3_80.

[Agr+16] Akanksha Agrawal et al. "A faster FPT algorithm and a smaller kernel for block graph vertex deletion". In: *LATIN 2016: theoretical informatics*. Vol. 9644. Lecture Notes in Comput. Sci. Springer, Berlin, 2016, pp. 1–13. DOI: 10.1007/978-3-662-49529-2_1. URL: https://doi.org/10.1007/978-3-662-49529-2_1.

[Cao16] Yixin Cao. "Linear recognition of almost interval graphs". In: *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms*. ACM, New York, 2016, pp. 1096–1115. DOI: 10.1137/1.9781611974331.ch77. URL: https://doi.org/10.1137/1.9781611974331.ch77.

[Coc+16] Manfred Cochefert et al. "Faster algorithms to enumerate hypergraph transversals". In: *LATIN 2016: theoretical informatics*. Vol. 9644. Lecture Notes in Comput. Sci. Springer, Berlin, 2016, pp. 306–318. DOI: 10.1007/978-3-662-49529-2_23. URL: https://doi.org/10.1007/978-3-662-49529-2_23.

[KL16] Mithilesh Kumar and Daniel Lokshtanov. "Faster exact and parameterized algorithm for feedback vertex set in tournaments". In: *33rd Symposium on Theoretical Aspects of Computer Science*. Vol. 47. LIPIcs. Leibniz Int. Proc. Inform. Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2016, Art. No. 49, 13.

[Bas+17] Julien Baste et al. "Parameterized complexity dichotomy for (r, ℓ)-vertex deletion". In: *Theory Comput. Syst.* 61.3 (2017), pp. 777–794. ISSN: 1432-4350. DOI: 10.1007/s00224-016-9716-y. URL: https://doi.org/10.1007/s00224-016-9716-y.