# Groverize Monotone Local Search. (Short Note)

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April 25, 2023

### 1 Todo.

- 1. Write the table (sage script).
- 2. Add definitions. Problem description.
- 3. Complete the 'proof'.
- 4. Prove lower bound.

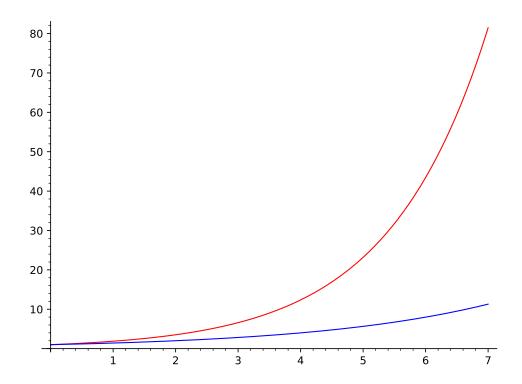
# 2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the treewidth of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\begin{split} & \sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left( \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} = \\ & \left( \max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\binom{k'-t}{t}} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left( \max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq \\ & \Rightarrow \left( 2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)} \end{split}$$

Problem Name	Parameterized	Groverize	New bound	Previous Bound
FEEDBACK VERTEX SET FEEDBACK VERTEX SET SUBSET FEEDBACK VERTEX SET	$3^{k}$ (r) [Cyg+11] $3.592^{k}$ [KP14] $4^{k}$ [Wahlstrom14]	??	$1.6667^{n}$ (r) $1.7217^{n}$ $1.7500^{n}$	$1.7347^{n}$ [FTV15] $1.8638^{n}$ [Fom+14]

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).



#### References

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- [Cyg+11] Marek Cygan et al. "Solving connectivity problems parameterized by treewidth in single exponential time (extended abstract)". In: 2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011. IEEE Computer Soc., Los Alamitos, CA, 2011, pp. 150–159. DOI: 10.1109/FOCS.2011.23. URL: https://doi.org/10.1109/FOCS.2011.23.
- [Fom+14] Fedor V. Fomin et al. "Enumerating minimal subset feedback vertex sets". In: Algorithmica 69.1 (2014), pp. 216–231. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9731-6. URL: https://doi.org/10.1007/s00453-012-9731-6.
- [KP14] Tomasz Kociumaka and Marcin Pilipczuk. "Faster deterministic Feedback Vertex Set".
  In: Inform. Process. Lett. 114.10 (2014), pp. 556–560. ISSN: 0020-0190. DOI: 10.1016/j.ipl.2014.05.001. URL: https://doi.org/10.1016/j.ipl.2014.05.001.
- [FTV15] Fedor V. Fomin, Ioan Todinca, and Yngve Villanger. "Large induced subgraphs via triangulations and CMSO". In: SIAM J. Comput. 44.1 (2015), pp. 54–87. ISSN: 0097-5397. DOI: 10.1137/140964801. URL: https://doi.org/10.1137/140964801.

 $[Fom+15] \quad \text{Fedor V. Fomin et al. } \textit{Exact Algorithms via Monotone Local Search. 2015. arXiv: 1512. } \\ 01621 \ [\text{cs.DS}].$