$\sqrt{n}\mapsto \Theta(n)$ Magic States 'Distillation' Using Quantum LDPC Codes.

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August 15, 2024

1 The Construction.

Let $\mathcal{X}=\{x_0,x_1,...x_{k-1}\}\in\mathbb{F}_2^n$ be a base for the code C_X/C_Z^\perp . Denote by $w\in\mathbb{F}_2^n$ the binary string presents Z-generator that anti commute with the X-generator corresponds to x_0 , So $x_0\cdot w=1$ and for any other $x'\in\mathcal{X}/x_0$ it holds that $x'\cdot w=0$. Let us denote by \mathcal{X}' the base $\{y_1,y_2,...,y_{k-1}\}\in\mathbb{F}_2^n$ such $y_i=x_i+x_0$. Denote by E the circuit that encodes the logical ith bit to y_i , by $T^{(w)}$ the application of T gates on the qubits for which w act non trivial, means $T^{(w)}$ is a tensor product of T's and identity where on the ith qubit $T^{(w)}$ apply T if w_i is 1 and identity otherwise. And finally by D denote the gate that decode binary strings in \mathbb{F}_2^n back into the logical space,

2 Proof of Theorem 1.

Claim 2.1. Let $|\mathcal{X}'\rangle \sum_{x\in\mathcal{X}'} |x\rangle$.