Groverize Monotone Local Search. (Short Note)

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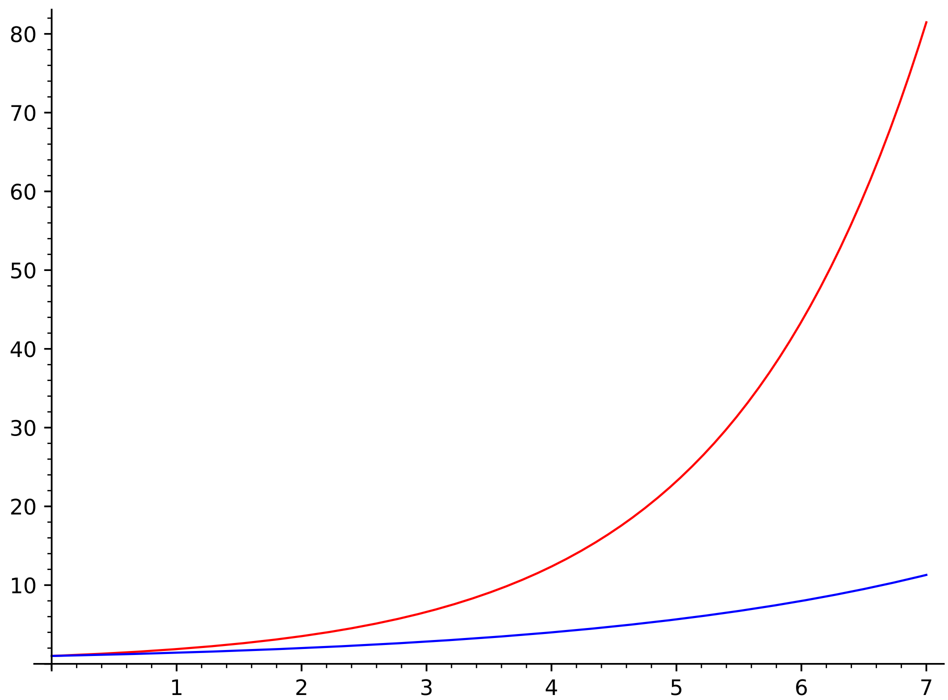
1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\sum_{k' \le k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \le \max_{k' \le k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \le k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\left(k'-t\right)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \le n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \le$$

$$\Rightarrow \left(2 - \frac{1}{c^2}\right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



Problem Name	Parameterized		New bound	Previous Bound	
Feedback Vertex Set	3^k (r)	[**cut-and-count**]	1.6667^n (r)		
Feedback Vertex Set	3.592^k	[**KociumakaP13**]	1.7217^n	1.7347^n	
Subset Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.7500^n	1.8638^n	
Feedback Vertex Set in Tournaments	1.6181^k	[**KumarL16**]	1.3820^n	1.4656^n	
Group Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.7500^n	NPR	
Node Unique Label Cover	$\lvert\Sigma\rvert^{2k}$	[**Wahlstrom14**]	$(2 - \frac{1}{\lvert\Sigma\rvert^2})^n$	NPR	
Vertex (r, ℓ)-Partization $(r, \ell \leq 2)$	3.3146^k	[**BasteFKS15**; **KolayP15**]	1.6984^n	NPR	
Interval Vertex Deletion	8^k	[**Cao8kinterval**]	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	
Proper Interval Vertex Deletion	6^k	[**HofV13**; **Cao15**]	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	
Block Graph Vertex Deletion	4^k	[**AgrawalLKS16**]	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	
Cluster Vertex Deletion	1.9102^k	[**BoralCKP14**]	1.4765^n	1.6181^n	
Thread Graph Vertex Deletion	8^k	[**Kante0KP15**]	1.8750^n	NPR	
Multicut on Trees	1.5538^k	[**KanjLLTXXYZZZ14**]	1.3565^n	NPR	
3-Hitting Set	2.0755^k	[**MagnusPhD07**]	1.5182^n	1.6278^n	
4-Hitting Set	3.0755^k	[**FominGKLS10**]	1.6750^n	1.8704^n	
d-Hitting Set $(d \geq 3)$	$(d - 0.9245)^k$	[**FominGKLS10**]	$(2 - \frac{1}{(d-0.9245)})^n$	[**CochefertCGK16**	
Min-Ones 3-SAT	2.562^k	[**abs-1007-1166**]	1.6097^n	NPR	
Min-Ones d-SAT $(d \geq 4)$	d^k		$(2 - \frac{1}{d})^n$	NPR	
Weighted d-SAT $(d \geq 3)$	d^k		$(2 - \frac{1}{d})^n$	NPR	
Weighted Feedback Vertex Set	3.6181^k	[**AgrawalLKS16**]	1.7237^n	1.8638^n	[**F**
Weighted 3-Hitting Set	2.168^k	[**ShachnaiZ15**]	1.5388^n	1.6755^n	[**C**
Weighted d-Hitting Set $(d \geq 4)$	$(d - 0.832)^k$	[**FominGKLS10**; **ShachnaiZ15**]	$(2 - \frac{1}{d-0.932})^n$		[**C**

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

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