$\sqrt{n} \mapsto \Theta(n)$ Magic States 'Distillation' Using Quantum LDPC Codes.

David Ponarovsky

August 15, 2024

1 The Construction.

Let x_0 be a codeword of $C_X/C_Z^{\perp}$, Denote by $w \in \mathbb{F}_2^n$ the binary string presents Z-generator that anti commute with the X-generator corresponds to x_0. Let $\mathcal{X} = \{x_0, x_1, ..x_{k'}\} \in \mathbb{F}_2^n$ be a subset of a base for the code $C_X/C_Z^{\perp}$. Such (span $\mathcal{X}/x_0$) $|_w$ is Triorthogonalcode. Let us denote by $\mathcal{X}'$ the base $\{y_1, y_2, .., y_{k'}\} \in \mathbb{F}_2^n$ defined such: $y_i = x_j + x_0$.

Denote by E the circuit that encodes the logical ith bit to y_i, by $T^{(w)}$ the application of T gates on the qubits for which w act non trivial, means $T^{(w)}$ is a tensor product of T's and identity where on the ith qubit $T^{(w)}$ apply T if w_i is 1 and identity otherwise. And finally by D denote the gate that decode binary strings in $\mathbb{F}_2^n$ back into the logical space.

2 Proof of Theorem 1.

Claim 2.1. *There exists quantum codes such the code **span** $(\mathcal{X}')$ chosen respect to them define a code with positive rate.*

Proof. □

Claim 2.2. *Let* $|\mathcal{X}'\rangle \propto \sum_{x \in span\ \mathcal{X}'} |x\rangle$. *Then* $T^{(w)} |\mathcal{X}'\rangle \propto \sum_{x \in span\ i} x$