## Magic States Distillation Using $\Delta$ -Toric (good qLDPC?).

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## January 1, 2024

Let  $|f\rangle$  be a codeword in  $C_X$ , and let  $X_g$  be the indicator that equals 1 if f has support on  $X_g$ , and 0 otherwise. Observes that applying  $T^{\otimes}$  on  $|f\rangle$  yilds the state:

$$\begin{split} T^{\otimes n} \left| f \right\rangle &= T^{\otimes n} \left| \sum_{g} X_g g \right\rangle = \exp \left( i \pi / 4 \sum_{g} X_g |g| - 2 \cdot i \pi / 4 \sum_{g,h} X_g X_h |g \cdot h| \right. \\ &+ 4 \cdot i \pi / 4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| - 8 \cdot i \pi / 4 \cdot \text{ integers } \right) \left| f \right\rangle \\ &= \exp \left( i \pi / 4 \sum_{g} X_g |g| - 2 \cdot \pi / 4 \sum_{g,h} X_g X_h |g \cdot h| + 4 \cdot i \pi / 4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| \right) \left| f \right\rangle \end{split}$$

Now assume that the code  $C_X$  is the quantum Tanner code, denote by G,A,B the group and the two generator sets that are used for constructing the square complex. In addition, let us assume the existence of  $d \in G$  such that d is non-identity and commutes with any element in  $A \cup B$ . Then, observe that multiplying by d preserves adjacency on the complex. Namely, if  $\{u,v\} \in E$  then also  $\{du,dv\} \in E$ .

Consider  $|f\rangle$  such that if  $X_g$  is not zero, and g is associated with a local codeword  $c \in C_A \otimes C_B$  on vertex v, then the generator associated with the local codeword c on vertex  $d \cdot v$  also supports f, denoted by g'. Thus, the exponent above becomes:

$$\begin{split} &=\exp\left(i\pi/4\sum_{g}X_{g}|g|-2\cdot\pi/4\sum_{g,h\in G/a}X_{g}X_{h}|g\cdot h|+X_{g'}X_{h'}|g\cdot h|\\ &+4\cdot i\pi/4\sum_{g,h\in G/a}X_{g}X_{h}X_{l}|g\cdot h\cdot l|+X_{g'}X_{h'}X_{l'}|g\cdot h\cdot l|\right)|f\rangle\\ &=\exp\left(i\pi/4\sum_{g}X_{g}|g|-2\cdot2\cdot\pi/4\sum_{g,h\in G/a}X_{g}X_{h}|g\cdot h|+2\cdot4\cdot i\pi/4\sum_{g,h\in G/a}X_{g}X_{h}X_{l}|g\cdot h\cdot l|\right)|f\rangle\\ &=\exp\left(i\pi/4\sum_{g}X_{g}|g|-i\pi\sum_{g,h\in G/a}X_{g}X_{h}|g\cdot h|\right)|f\rangle \end{split}$$

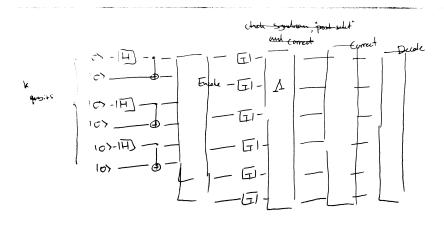


Figure 1: Quantum Circuit for distillation.