Groverize Monotone Local Search. (Short Note)

David Ponarovsky

April 25, 2023

1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

Definition 1 (Implicit Set System)**.** *We define an* implicit set system *as a function* Φ *that takes as input a string* $I \in \{0,1\}^*$ *and outputs a set system* $(U_I, \mathcal{F}_I)$*, where* U_I *is a universe and* $\mathcal{F}_I$ *is a collection of subsets of* U_I*. The string* I *is referred to as an* instance *and we denote by* $|U_I| = n$ *the size of the universe and by* $|I| = N$ *the size of the instance.*

We assume that $N \geq n$. The implicit set system Φ is said to be *polynomial time computable* if (a) there exists a polynomial time algorithm that given I produces U_I, and (b) there exists a polynomial time algorithm that given I, U_I and a subset S of U_I determines whether $S \in \mathcal{F}_I$. All implicit set systems discussed in this paper are polynomial time computable, except for the minimal satisfying assignments of d-CNF formulas which are not polynomial time computable unless P=NP [**YatoS03**].

An implicit set system Φ naturally leads to some problems about the set system $(U_I, \mathcal{F}_I)$. Find a set in $\mathcal{F}_I$. Is $\mathcal{F}_I$ non-empty? What is the cardinality of $\mathcal{F}_I$? In this paper we will primarily focus on the first and last problems. Examples of implicit sets systems include the set of all feedback vertex sets of a graph of size at most k, the set of all satisfying assignments of a CNF formula of weight at most W, and the set of all minimal hitting sets of a set system. Next we formally define subset problems.

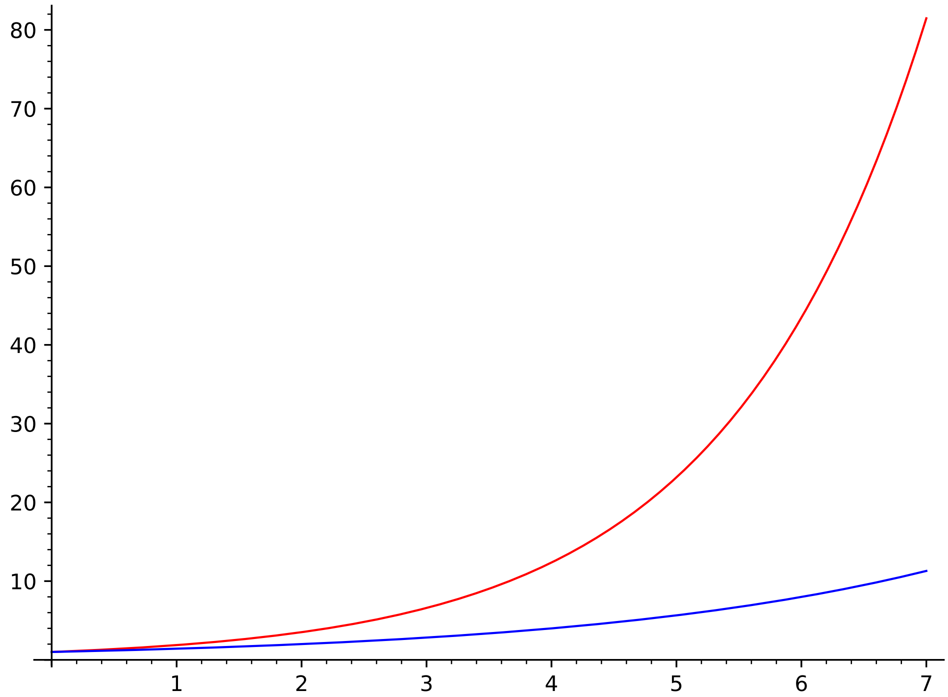
An instance IA set $S \in \mathcal{F}_I$ if one exists.

An example of a subset problem is Min-Ones d-Sat. Here for an integer k and a propositional formula F in conjunctive normal form (CNF) where each clause has at most d literals, the task is to determine whether F has a satisfying assignment with Hamming weight (number of 1s) at most k, i.e., setting at most k variables to 1. In our setting, the instance I consists of the input formula F and the integer k, encoded as a string over 0s and 1s. The implicit set system Φ is a function from I to $(U_I, \mathcal{F}_I)$, where U_I is the set of variables of F, and $\mathcal{F}_I$ is the set of all satisfying assignments of Hamming weight at most k.

$$\sum_{k' \le k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \le \max_{k' \le k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \le k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\left(k'-t\right)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \le n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \le$$

$$\Rightarrow \left(2 - \frac{1}{c^2}\right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



References

[Gro96] Lov K. Grover. *A fast quantum mechanical algorithm for database search.* 1996. arXiv: quant-ph/9605043 [quant-ph].

[Fom+08] Fedor V. Fomin et al. “On the minimum feedback vertex set problem: exact and enumeration algorithms”. In: *Algorithmica* 52.2 (2008), pp. 293–307. ISSN: 0178-4617. DOI: 10.1007/s00453-007-9152-0. URL: https://doi.org/10.1007/s00453-007-9152-0.

[Fom+10] Fedor V. Fomin et al. “Iterative compression and exact algorithms”. In: *Theoret. Comput. Sci.* 411.7-9 (2010), pp. 1045–1053. ISSN: 0304-3975. DOI: 10.1016/j.tcs.2009.11.012. URL: https://doi.org/10.1016/j.tcs.2009.11.012.

[Cyg+11] Marek Cygan et al. “Solving connectivity problems parameterized by treewidth in single exponential time (extended abstract)”. In: *2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011.* IEEE Computer Soc., Los Alamitos, CA, 2011, pp. 150–159. DOI: 10.1109/FOCS.2011.23. URL: https://doi.org/10.1109/FOCS.2011.23.

[tV13] Pim van ’t Hof and Yngve Villanger. “Proper interval vertex deletion”. In: *Algorithmica* 65.4 (2013), pp. 845–867. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9661-3. URL: https://doi.org/10.1007/s00453-012-9661-3.

Problem Name	Parameterized		Groverize	New bound	Previous Bound	
Feedback Vertex Set	3^k (r)	[Cyg+11]	1.3744^k	1.6667^n (r)		
Feedback Vertex Set	3.592^k	[KP14]	1.3865^k	1.7217^n	1.7347^n	[FTV15
Subset Feedback Vertex Set	4^k	**[Wahlstrom14]**	1.3919^k	1.7500^n	1.8638^n	[Fom+14
Feedback Vertex Set in Tournaments	1.6181^k	[KL16]	1.2720^k	1.3820^n	1.4656^n	[KL16
Group Feedback Vertex Set	4^k	**[Wahlstrom14]**	1.3919^k	1.7500^n	NPR	
Node Unique Label Cover	$\|\Sigma\|^{2k}$	**[Wahlstrom14]**	1.3919^k	$(2-\frac{1}{\|\Sigma\|^2})^n$	NPR	
Vertex (r,ℓ)-Partization $(r,\ell \leq 2)$	3.3146^k	**[KolayP15**; Bas+17]	1.3817^k	1.6984^n	NPR	
Interval Vertex Deletion	8^k	[Cao16]	1.3466^k	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Proper Interval Vertex Deletion	6^k	[tV13; Cao15]	1.4087^k	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Block Graph Vertex Deletion	4^k	[Agr+16]	1.4044^k	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Cluster Vertex Deletion	1.9102^k	[Bor+14]	1.3919^k	1.4765^n	1.6181^n	[Fom+10
Thread Graph Vertex Deletion	8^k	[Kan+15]	1.3919^k	1.8750^n	NPR	
Multicut on Trees	1.5538^k	[Kan+14]	1.3138^k	1.3565^n	NPR	
3-Hitting Set	2.0755^k	**[MagnusPhD07]**	1.4087^k	1.5182^n	1.6278^n	**[MagnusPhD07**
4-Hitting Set	3.0755^k	[Fom+10]	1.2593^k	1.6750^n	1.8704^n	[Fom+10
d-Hitting Set $(d \geq 3)$	$(d-0.9245)^k$	[Fom+10]	1.1763^k	$(2-\frac{1}{(d-0.9245)})^n$		[Coc+16; Fom+10
Min-Ones 3-SAT	2.562^k	**[abs-1007-1166]**	1.3296^k	1.6097^n	NPR	
Min-Ones d-SAT $(d \geq 4)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted d-SAT $(d \geq 3)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted Feedback Vertex Set	3.6181^k	[Agr+16]	1.1763^k	1.7237^n	1.8638^n	[Fom+08
Weighted 3-Hitting Set	2.168^k	[SZ15]	1.3593^k	1.5388^n	1.6755^n	[Coc+16
Weighted d-Hitting Set $(d \geq 4)$	$(d-0.832)^k$	[Fom+10; SZ15]	1.3919^k	$(2-\frac{1}{d-0.932})^n$		[Coc+16

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

[BFP13] Ivan Bliznets, Fedor V. Fomin, and Yngve Pilipczuk Michałand Villanger. "Largest chordal and interval subgraphs faster than 2^n". In: *Algorithms—ESA 2013*. Vol. 8125. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2013, pp. 193–204. DOI: 10.1007/978-3-642-40450-4_17. URL: https://doi.org/10.1007/978-3-642-40450-4_17.

[Bor+14] Anudhyan Boral et al. "A fast branching algorithm for cluster vertex deletion". In: *Computer science—theory and applications*. Vol. 8476. Lecture Notes in Comput. Sci. Springer, Cham, 2014, pp. 111–124. DOI: 10.1007/978-3-319-06686-8_9. URL: https://doi.org/10.1007/978-3-319-06686-8_9.

[Fom+14] Fedor V. Fomin et al. "Enumerating minimal subset feedback vertex sets". In: *Algorithmica* 69.1 (2014), pp. 216–231. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9731-6. URL: https://doi.org/10.1007/s00453-012-9731-6.

[Kan+14] Iyad Kanj et al. "Algorithms for cut problems on trees". In: *Combinatorial optimization and applications*. Vol. 8881. Lecture Notes in Comput. Sci. Springer, Cham, 2014, pp. 283–298. DOI: 10.1007/978-3-319-12691-3_22. URL: https://doi.org/10.1007/978-3-319-12691-3_22.

[KP14] Tomasz Kociumaka and Marcin Pilipczuk. "Faster deterministic Feedback Vertex Set". In: *Inform. Process. Lett.* 114.10 (2014), pp. 556–560. ISSN: 0020-0190. DOI: 10.1016/j.ipl.2014.05.001. URL: https://doi.org/10.1016/j.ipl.2014.05.001.

[Cao15] Yixin Cao. "Unit interval editing is fixed-parameter tractable". In: *Automata, languages, and programming. Part I*. Vol. 9134. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2015, pp. 306–317. DOI: 10.1007/978-3-662-47672-7_25. URL: https://doi.org/10.1007/978-3-662-47672-7_25.

[FTV15] Fedor V. Fomin, Ioan Todinca, and Yngve Villanger. "Large induced subgraphs via triangulations and CMSO". In: *SIAM J. Comput.* 44.1 (2015), pp. 54–87. ISSN: 0097-5397. DOI: 10.1137/140964801. URL: https://doi.org/10.1137/140964801.

[Fom+15] Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: 1512.01621 [cs.DS].

[Kan+15] Mamadou Moustapha Kanté et al. "An FPT algorithm and a polynomial kernel for linear rankwidth-1 vertex deletion". In: *10th International Symposium on Parameterized and Exact Computation*. Vol. 43. LIPIcs. Leibniz Int. Proc. Inform. Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2015, pp. 138–150.

[SZ15] Hadas Shachnai and Meirav Zehavi. "A multivariate approach for weighted FPT algorithms". In: *Algorithms—ESA 2015*. Vol. 9294. Lecture Notes in Comput. Sci. Springer, Heidelberg, 2015, pp. 965–976. DOI: 10.1007/978-3-662-48350-3_80. URL: https://doi.org/10.1007/978-3-662-48350-3_80.

[Agr+16] Akanksha Agrawal et al. "A faster FPT algorithm and a smaller kernel for block graph vertex deletion". In: *LATIN 2016: theoretical informatics*. Vol. 9644. Lecture Notes in Comput. Sci. Springer, Berlin, 2016, pp. 1–13. DOI: 10.1007/978-3-662-49529-2_1. URL: https://doi.org/10.1007/978-3-662-49529-2_1.

[Cao16] Yixin Cao. "Linear recognition of almost interval graphs". In: *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms*. ACM, New York, 2016, pp. 1096–1115. DOI: 10.1137/1.9781611974331.ch77. URL: https://doi.org/10.1137/1.9781611974331.ch77.

[Coc+16] Manfred Cochefert et al. "Faster algorithms to enumerate hypergraph transversals". In: *LATIN 2016: theoretical informatics*. Vol. 9644. Lecture Notes in Comput. Sci. Springer, Berlin, 2016, pp. 306–318. DOI: 10.1007/978-3-662-49529-2_23. URL: https://doi.org/10.1007/978-3-662-49529-2_23.

[KL16] Mithilesh Kumar and Daniel Lokshtanov. "Faster exact and parameterized algorithm for feedback vertex set in tournaments". In: *33rd Symposium on Theoretical Aspects of Computer Science*. Vol. 47. LIPIcs. Leibniz Int. Proc. Inform. Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2016, Art. No. 49, 13.

[Bas+17] Julien Baste et al. "Parameterized complexity dichotomy for (r, ℓ)-vertex deletion". In: *Theory Comput. Syst.* 61.3 (2017), pp. 777–794. ISSN: 1432-4350. DOI: 10.1007/s00224-016-9716-y. URL: https://doi.org/10.1007/s00224-016-9716-y.