

Bucket Sort When You Know The Distribution.

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC code with testability query complexity of $\Theta(n^{1-\varepsilon})$ for any $\varepsilon > 0$.

The problem. Let $f : [0, 1] \rightarrow [0, 1]$ a fixed distribution function. Write an algorithm that sort n draws $x_1 \dots x_n$ at linear expectation time.

Solution. We will define a partition of the input into a seira of n buckets $\mathcal{B} = \{B_k = [t_k, t_{k+1}] : k \in [n]\}$ such that $\Pr[x \in B_i] = \frac{1}{n}$ for any bucket.

Claim. The probability that the size of the i th bucket exceeds $t \in \mathbb{N}$ is bounded by: $\Pr[B_i \geq t] \leq \frac{e}{t^k}$ for every intrger $k \leq n$.

Proof. Let the X_{ij} be the indecator of the event that x_j belongs to B_i . Then we have:

$$\begin{aligned}
 \mathbf{E}[B_i^k] &= \mathbf{E}\left[\left(\sum_j X_{ij}\right)^k\right] = \mathbf{E}\left[\sum_{J \in [n]^k} \prod_{l \in [k]} X_{iJ_l}\right] \\
 &= \mathbf{E}\left[\sum_{\substack{l \in [k] \\ |J|=l}} \sum_{J \subset [n]} \prod_{j \in J} X_{ij}\right] \\
 &= \mathbf{E}\left[\sum_{l \in [k]} \sum_{\binom{n}{l} \frac{l!}{n^l}}\right] \\
 &= \sum_{j \neq j'} \mathbf{E}[X_{ij}] \mathbf{E}[X_{ij'}] + \sum_j \mathbf{E}[X_{ij}] \\
 &= \sum_{l \in [4]} \frac{1}{n^l} \binom{n}{l} = O(1) \\
 \mathbf{V}[B_i^2] &= \sum_{l \in [4]} \binom{n}{l} \left(\frac{1}{n^l} - \frac{1}{n^4}\right) \leq e \\
 \mathbf{E}[(B_i^2)^k] &\leq \left(1 + \frac{1}{n}\right)^n \leq e \\
 \Pr[B_i \geq t] &\leq \frac{e}{t^k}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{n} &= \Pr[x \in B_k] = f(t_{k+1}) - f(t_k) \\
 &\Rightarrow t_{k+1} \leftarrow f^{-1}\left(\frac{1}{n} + f(t_k)\right)
 \end{aligned}$$