Memory.

Michael Ben-Or David Ponarovsky

August 7, 2025

0.1 Definitions.

0.2 Idea.

 $\mathbf{Pr}\left[\mathbf{Sup}\left(E_{2}\right)=S\right] \leq \mathbf{Pr}\left[\text{any bit }v \in S_{c_{1}} \text{ sees majority of unstatisfied stabilizers }\right] \leq q^{\Delta|S|_{c_{1}}}$

$$\mathbf{Pr} \left[\mathbf{Sup} \left(E_{3} \right) = S \right] = \sum_{S' \subset S} \mathbf{Pr} \left[\mathbf{Sup} \left(E_{2} \right) = S' \cap \mathbf{Sup} \left(E_{3} / E_{2} \right) = S / S' \right] \\
\leq \sum_{S' \subset S} q^{\Delta |S'_{c_{1}}|} p^{|S / S'_{c_{1}}|} \leq \sum_{S' \subset S} q^{\Delta |S'_{c_{1}}|} p^{|S_{c_{1}}| - |S'_{c_{1}}|} \\
\leq \left(q^{\Delta} + p \right)^{|S_{c_{1}}|} \leq \begin{cases} \left(q^{\Delta} + p \right)^{\frac{1}{4}|S|} & \text{if } |S_{c_{1}}| \geq \frac{1}{4}|S| \\
\star & \text{else} \end{cases}$$

Let $S^t = \mathbf{Sup}(E)$ at time t and denote by \mathcal{P}_t the probability that $|S_{c_1}^t| > \frac{1}{4}|S_t|$.