Groverize Monotone Local Search. (Short Note)

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1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

Definition 1 (Implicit Set System)**.** *We define an* implicit set system *as a function Φ that takes as input a string $I \in \{0,1\}^*$ and outputs a set system $(U_I, \mathcal{F}_I)$, where U_I is a universe and $\mathcal{F}_I$ is a collection of subsets of U_I. The string I is referred to as an* instance *and we denote by $|U_I| = n$ the size of the universe and by $|I| = N$ the size of the instance.*

We assume that $N \geq n$. The implicit set system Φ is said to be *polynomial time computable* if (a) there exists a polynomial time algorithm that given I produces U_I, and (b) there exists a polynomial time algorithm that given I, U_I and a subset S of U_I determines whether $S \in \mathcal{F}_I$. All implicit set systems discussed in this paper are polynomial time computable, except for the minimal satisfying assignments of d-CNF formulas which are not polynomial time computable unless P=NP [**YatoS03**].

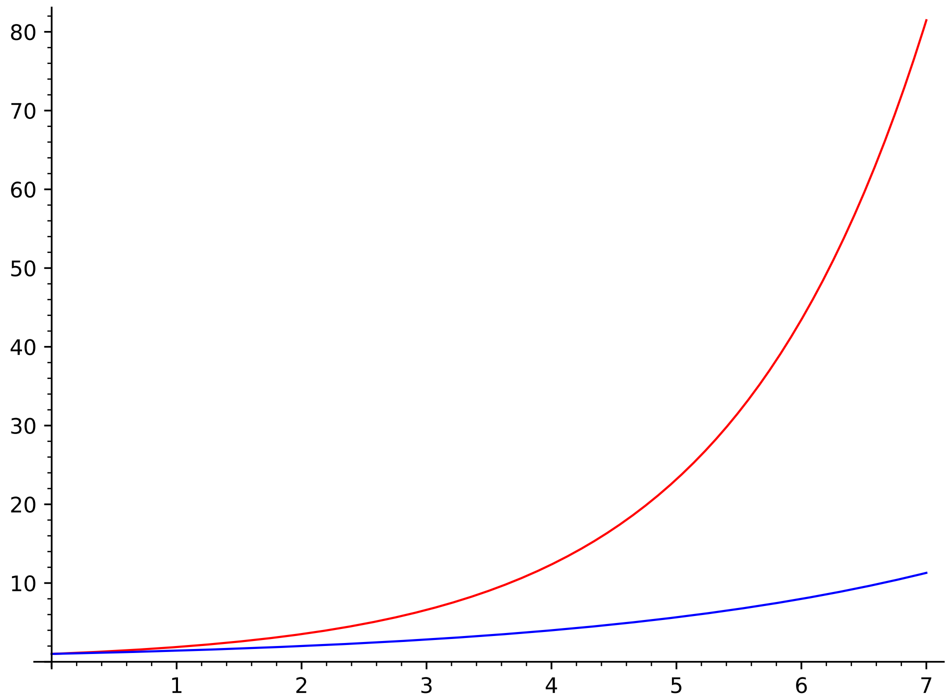
An implicit set system Φ naturally leads to some problems about the set system $(U_I, \mathcal{F}_I)$. Find a set in $\mathcal{F}_I$. Is $\mathcal{F}_I$ non-empty? What is the cardinality of $\mathcal{F}_I$? In this paper we will primarily focus on the first and last problems. Examples of implicit sets systems include the set of all feedback vertex sets of a graph of size at most k, the set of all satisfying assignments of a CNF formula of weight at most W, and the set of all minimal hitting sets of a set system. Next we formally define subset problems.

An instance IA set $S \in \mathcal{F}_I$ if one exists.
An example of a subset problem is Min-Ones d-Sat. Here for an integer k and a propositional formula F in conjunctive normal form (CNF) where each clause has at most d literals, the task is to determine whether F has a satisfying assignment with Hamming weight (number of 1s) at most k, i.e., setting at most k variables to 1. In our setting, the instance I consists of the input formula F and the integer k, encoded as a string over 0s and 1s. The implicit set system Φ is a function from I to $(U_I, \mathcal{F}_I)$, where U_I is the set of variables of F, and $\mathcal{F}_I$ is the set of all satisfying assignments of Hamming weight at most k.

$$\sum_{k'\leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k'\leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k'\leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\left(k'-t\right)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k\leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq$$

$$\Rightarrow \left(2 - \frac{1}{c^2}\right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$



Problem Name	Parameterized	Groverize	New bound	Previous Bound
Feedback Vertex Set	3^k (r) [Cyg+11]	1.3744^k	1.6667^n (r)	
Feedback Vertex Set	3.592^k [KP14]	1.3865^k	1.7217^n	1.7347^n [FTV15
Subset Feedback Vertex Set	4^k [**Wahlstrom14**]	1.3919^k	1.7500^n	1.8638^n [Fom+14
Feedback Vertex Set in Tournaments	1.6181^k [KL16]	1.2720^k	1.3820^n	1.4656^n [KL16
Group Feedback Vertex Set	4^k [**Wahlstrom14**]	1.3919^k	1.7500^n	NPR
Node Unique Label Cover	$\|\Sigma\|^{2k}$ [**Wahlstrom14**]	1.3919^k	$(2 - \frac{1}{\|\Sigma\|^2})^n$	NPR
Vertex (r, ℓ)-Partization $(r, \ell \leq 2)$	3.3146^k [**KolayP15**; Bas+17]	1.3817^k	1.6984^n	NPR
Interval Vertex Deletion	8^k [Cao16]	1.3466^k	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$ [BFP13
Proper Interval Vertex Deletion	6^k [tV13; Cao15]	1.4087^k	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$ [BFP13
Block Graph Vertex Deletion	4^k [Agr+16]	1.4044^k	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$ [BFP13
Cluster Vertex Deletion	1.9102^k [Bor+14]	1.3919^k	1.4765^n	1.6181^n [Fom+10
Thread Graph Vertex Deletion	8^k [Kan+15]	1.3919^k	1.8750^n	NPR
Multicut on Trees	1.5538^k [Kan+14]	1.3138^k	1.3565^n	NPR
3-Hitting Set	2.0755^k [**MagnusPhD07**]	1.4087^k	1.5182^n	1.6278^n [**MagnusPhD07**
4-Hitting Set	3.0755^k [Fom+10]	1.2593^k	1.6750^n	1.8704^n [Fom+10
d-Hitting Set $(d \geq 3)$	$(d - 0.9245)^k$ [Fom+10]	1.1763^k	$(2 - \frac{1}{(d-0.9245)})^n$	[Coc+16; Fom+10
Min-Ones 3-SAT	2.562^k [**abs-1007-1166**]	1.3296^k	1.6097^n	NPR
Min-Ones d-SAT $(d \geq 4)$	d^k	1.3763^k	$(2 - \frac{1}{d})^n$	NPR
Weighted d-SAT $(d \geq 3)$	d^k	1.3763^k	$(2 - \frac{1}{d})^n$	NPR
Weighted Feedback Vertex Set	3.6181^k [Agr+16]	1.1763^k	1.7237^n	1.8638^n [Fom+08
Weighted 3-Hitting Set	2.168^k [SZ15]	1.3593^k	1.5388^n	1.6755^n [Coc+16
Weighted d-Hitting Set $(d \geq 4)$	$(d - 0.832)^k$ [Fom+10; SZ15]	1.3919^k	$(2 - \frac{1}{d-0.932})^n$	[Coc+16

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

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