Dining Philosophers (Short Note)

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Abstract

This paper presents an alternative proof for the Dining Philosophers problem's impossibility of solving by deterministic protocol. The proof offered in this article is not novel; instead, it is an alternate way of providing the understanding that the protocols that intend to resolve the problem deterministically are bound to fail. By providing a new perspective, the paper aims to help computer science researchers and students understand the problem and the impossibility of deterministic protocols. Overall, the proof highlights the familiar idea that deterministic protocols for resolving the Dining Philosophers problem violate one or more of the necessary properties.

1 Todo.

1. Check the events suggested by GPT.

2. Draw a nice Tikz figure of Dining Philosophers.

3. Complete the base case for $n = 2$.

2 Introduction.

The Dining Philosophers problem is a classic synchronization problem in computer science, where a group of philosophers sit at a round table and alternate between thinking and eating. The problem arises when they share common resources (forks) and a set of rules must be established to prevent deadlocks and starvation.

The impossibility result for the deterministic case states that it is impossible to design a solution to the Dining Philosophers problem if the philosophers behave deterministically and the resource allocation is symmetric. This is because each philosopher would require the same resources as their neighbor at the same time, which leads to deadlocks. Michael Rabin proposed a solution to the randomized case, where the philosophers behave randomly in choosing which fork to pick up first. This randomization breaks the symmetry and prevents deadlocks.

Attempts have also been made to solve the problem using quantumness, with [COMMENT] Adi Shamir and Avi Wigderson proposing a quantum analog to the Dining Philosophers problem. In [COMMENT] 2003, Dorit Aharonov proposed a quantum solution to the problem, which involves using entanglement to share the forks between philosophers.

An example of a real-world application of the Dining Philosophers problem is in resource allocation in computer networks, where multiple nodes may need access to a shared resource. The problem can also be used as a teaching tool in computer science to illustrate the importance of synchronization and avoiding deadlocks.

An incident related to the problem occurred in [COMMENT] 2008, when a bug in the synchronization code of the iPhone's email application caused it to hang, leading to a flurry of frustrated complaints from iPhone users dubbed the "Dining Philosopher's bug".

Theorem 1. *It is impossible to solve the Dining Philosophers problem for any number of philosophers larger than one.*

3 The Proof.

We prove the theorem by induction. First, we show that the base case of two philosophers is impossible to solve.

Claim 1. *It is impossible to solve the Dining Philosophers problem for a pair of philosophers.*

Proof. Suppose there are two philosophers sitting at a table, each with a fork. Each philosopher needs both forks to eat, but only one fork is available to each philosopher. Thus, they will forever wait for the other philosopher to release the fork, resulting in a deadlock. $\square$

3.1 Validity for Odds Number Yields Validity for Even Ones.

Lemma 1. *If there exists an odd number $n > 1$ such that there is a valid protocol to solve the Dining Philosophers problem for n philosophers, then there exists an even number $n' = n + 1$ such that the problem can be solved by a protocol.*

Proof. Let $P_1, P_2, ..., P_{n+1}$ denote the $n+1$ philosophers sitting around the table, and let $F_1, F_2, ..., F_{n+1}$ denote the forks placed between them. Consider the running of the n-philosophers protocol over n'-philosophers. That it, each philosopher execute exactly the same code as it would execute in the n-philosophers protocol.

Claim 2. *Suppose there is a deadlock and all the forks are held, then either all the philosophers hold the fork to their right or all of them hold the fork to their left.*

Proof. Assume through contradiction otherwise, then there are must to be two adjoined philosophers P_i and P_{i+1} such one of them hold the fork to his left while the other hold the fork to his right. So either F_{i+1} is held by both of them, which by definition cannot be true, or F_i is free, which contradict the fact the second restriction of the claim. $\square$

Claim 3. *We can assume when deadlock occur then all the forks are held by the philosophers.*

The is given in the Appendix. Combining claim 2 we can assume w.l.g that each P_i holds F_{i+1}. Now denote by j the philosopher who is the last one to pick it's fork. We will show, that the protocol has a running on n philosophers in which for any $i \neq j$, the ith philosopher sees the same local view as P_i in the n'-protocol. $\square$

3.2 Back From $2m$ Protocol to 2-philosophers Protocol.

Lemma 2. *The existence of such even number follows a valid solution for the base case of only two philosophers.*

Proof. Suppose there is an even number $n = 2m$ for which the there is a valid protocol for n philosophers. We show that a protocol exists in which two philosophers simulate the n philosophers in the original protocol.

Let the original protocol have n philosophers and n forks, labeled $F_0, F_1, ..., F_{n-1}$, where F_i is between philosopher i and philosopher $(i + 1) \mod n$. In the new protocol, the two simulating Philosophers, P_1 and P_2, will each be responsible for simulating a continuous half of the original Philosophers, i.e., P_1 will simulate Philosophers 0 to $m - 1$ and P_2 will simulate Philosophers m to $n - 1$.

To do this, P_1 will define $m - 1$ imaginary forks, labeled $I_1, I_2, \ldots, I_{m-1}$, where I_i matches the original fork F_i for $i \leq m-1$. Similarly, P_2 will define $m-1$ imaginary forks, labeled $I'_1, I'_2, \ldots, I'_{m-1}$. Denote by J_0 and J'_0 the real shared forks between P_1 and P_2. Now each of the philosophers follows the n-protocol, such that when the ith philosopher picks a fork, then both P_1 and P_2 pick I_i and I'_i accordingly (or J_0, J'_0 if i is zero). Notice that the protocol is symmetric.

Now, observe that any running of the 2-philosophers protocol is also a valid instance of the n-philosophers protocol. Therefore any scenario in which deadlock or starvation occur either on real philosophers or imaginary ones matches a possible occurrence of the same incident while executing

the n-philosophers protocol. We can conclude that any protocols that guarantee correctness for even numbers of philosophers yield a correct protocol for the base case. $\square$

4 Appendix

proof of claim 3.

Proof. Let $\mathcal{A}$ be a protocol, and consider the protocol $\mathcal{A}'$ in which each time that one of the philosopher is get stack then he pass a number with initial value 0 through the net by sending it to the philosopher sit to his right. Then any philosopher upon receiving, increase the number by one, records it and resend the bit to his right sibling and so on. In the end the original writer of the message receive a number with value n. On recording n numbers $1, 2...n$ the philosopher knows that every one on the circle gets stuck (and it easy to see that is the only scenario in which a philosopher record n different values).

At this point, $\mathcal{A}'$ define the philosopher to pick any of the forks next to it if he can do so. Notice that if $\mathcal{A}$ is a deadlock free protocol so is $\mathcal{A}'$. $\square$