Quantum LTC With Positive Rate

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preamble. preamble.

Claim *for any ?* $[[n, k, d]]$ ***CSS*** *code property 1 holds*
. **Proof.** let $y \in \{0, 1\}^n$ be a vector such $y \in G_z^{\delta}$, let assume that $|y|_{c^{x\perp}} \leq C_2 d$ then for any $c \in C_x^{\perp}$:

$$\delta r_z \geq |H_z y| = |H_z (y + c)|$$

Robusstness *Let* $\omega \leq \Delta^2$. *Let* C_A *and* C_B *be codes of length* Δ *with minimum distance* d_A *and* d_B. *We shall say that the dual tensor code* $C = C_A \otimes \mathbb{F}_2^B + \mathbb{F}_2^A \otimes C_B$ *is* ω*-robust, if for any codeword* $c \in C$ *of Hamming weight* $|c| \leq \omega$, *there exist* $A' \subset A, B' \subset B, |A'| \leq |c|/d_B, |B'| \leq |c|/d_A$, *such that* $c_{ab} = 0$ *whenever* $a \notin A', b \notin B'$.

Definition. Sub-Tensor Pair *We will say that* C'_A, C'_B *are sub-tensor pair of* C_A, C_B *if each of the code is subspace of* C_A, C_B *respeactivly and in addition one of the minimal codeword in* C_A *is also contained in* C'_A *(and similar to* C'_B.

Claim. Subcode Robusstness. *Consider the subspaces* $C'_A \subset C_A, C'_B \subset C_B$, *such that the dual tensor of* C_A, C_B *is* ω*-robust then the dual tensor of* C'_A, C'_B *is also* ω*-robust.*

Proof. Let c be a codeword in the dual tensor of C'_A, C'_B then it's clear that c is also in the dual tensor of C_A, C_B and therfore there exists V, Usubsets of A, B respectivly such that c supported only on them, and their size is less then $|c|/d_B, |c|/d_A$. As the length's space of the each of the subcode is indentical to his container, and by the fact that the minimal weight of codeward is grater then the minimal weight in the containig code it's follow that (1) $U \subset A' = A$ and $|c|/d_A$.