$\sqrt{n} \mapsto \Theta(n)$ Magic States 'Distillation' Using Quantum LDPC Codes.

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August 15, 2024

1 The Construction.

Let $\mathcal{X} = \{x_0, x_1, ..x_{k-1}\} \in \mathbb{F}_2^n$ be a base for the code $C_X/C_Z^{\perp}$. Denote by $w \in \mathbb{F}_2^n$ the binary string presents Z-generator that anti commute with the X-generator corresponds to x_0, So $x_0 \cdot w = 1$ and for any other $x' \in \mathcal{X}/x_0$ it holds that $x' \cdot w = 0$. Let us denote by $\mathcal{X}'$ the base $\{y_1, y_2, .., y_{k-1}\} \in \mathbb{F}_2^n$ such $y_i = x_i + x_0$. Denote by E the circuit that encodes the logical ith bit to y_i, by $T^{(w)}$ the application of T gates on the qubits for which w act non trivial, means $T^{(w)}$ is a tensor product of T's and identity where on the ith qubit $T^{(w)}$ apply T if w_i is 1 and identity otherwise. And finally by D denote the gate that decode binary strings in $\mathbb{F}_2^n$ back into the logical space,

2 Proof of Theorem 1.

Claim 2.1. *Let* $|\mathcal{X}'\rangle \sum_{x \in span\ \mathcal{X}'} |x\rangle$.