The Permutations Paper for non Algebraic Speakers.

David Ponarovsky

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Abstract

A guide for reading Becker, Lubotzky, and Mosheiff's paper for computer scientists. The goal is to help the reader by providing analogs and examples from the combinatorics field.

1 Motivation (Use Cases List).

We start by presenting several use cases that may be of interest to computer scientists.

1. Testing candidates for LTC/QLDPC codes. One of the resources needed for the available constructions is a square complex in which the encoding associates each bit with a face. We can obtain these structures by taking the Left-Right Cayley graph generated by a pair of generator sets A, B, such that $[A, B] = 0$.

2. Testing if a set of stabilizers forms a stabilizer code. Here the stabilizers are subsets of the Pauli group and they form a code only if they all commute.

3. Classical toy-version of QMA complete problem. It's known that decide if two quantum circuits over n qubits are $1/poly$-equivalent is QMA-complete problem. So, One question that could be interesting is to ask given prem' $P = \prod p_i$ and $Q = \prod q_i$ such that p_i (q_i) act over a constant size of bits (assuming binary encoding), then ask whether $P = Q$.

2 Example.

Let us define the permutations f, g over $n = 2m$ elements defined as follow:

$$f(i) = \begin{cases} i+1 & i < m \\ i-1 & i \geq m \end{cases}$$

$$g(i) = \overbrace{n-i}^{\text{reflection}} +1$$

3 The S-graph.

Theorem 1. *Denote by $FGSol_E$ union over the finite connected entities of $GSol$, and by α the asymptotic (ifimum) Cheeger constant of them. If the α is positive, then E is not testable.*