Bucket Sort When You Know The Distribution.

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC code with testability query complexity of $\Theta\left(n^{1-\varepsilon}\right)$ for any $\varepsilon > 0$.

The problem. Let $f : [0,1] \rightarrow [0,1]$ a fixed distribution function. Write an algorithm that sort n draws $x_1 ... x_n$ at linear expectation time.

Solution. We will define a partition of the input into a seira of n buckets $\mathcal{B} = \{B_k = [t_k, t_{k+1}] : k \in [n]\}$ such that $\mathbf{Pr}[x \in B_i] = \frac{1}{n}$ for any bucket.

$$\begin{aligned}
\mathbf{E}\left[B_i^2\right] &= \mathbf{E}\left[\left(\sum_j X_{ij}\right)^2\right] \\
&= \mathbf{E}\left[\sum_{j,j'} X_{ij} X_{ij'}\right] = \sum_{j,j'} \mathbf{E}\left[X_{ij}\right] \mathbf{E}\left[X_{ij'}\right] \\
&= \sum_{j \neq j'} \mathbf{E}\left[X_{ij}\right] \mathbf{E}\left[X_{ij'}\right] + \sum_j \mathbf{E}\left[X_{ij}\right] \\
&= \frac{1}{n}\binom{n}{2} + 1 = O\left(1\right)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{n} &= \mathbf{Pr}\left[x \in B_k\right] = f\left(t_{k+1}\right) - f\left(t_k\right) \\
\Rightarrow t_{k+1} &= f^{-1}\left(\frac{1}{n} + f\left(t_k\right)\right)
\end{aligned}$$