

# Memory.

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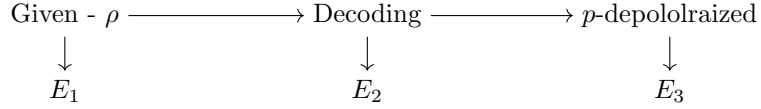
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## 0.1 Notations and Definitions.

Consider a code with a 2-colored ( $k$ -colored) Tanner graph, such that any two left bits of the same color share no stabilizer. For a subset of bits  $S$ , we denote by  $S_{c_1}$  its restriction to color  $c_1$ . We use the integer  $\Delta$  to denote half of the stabilizers connected to a single bit. (We assume fixed left and right degree in the graph). Our computation is subjected to  $p$ -depolarized noise. We denote by  $m$  the block length of the code. The decoder works as follows:

1. Pick a random color.
2. For any (q)bit at that color, check if flipping it decreases the syndrome. If so, then flip it.

## 0.2 Idea.



$$\Pr[\mathbf{Sup}(E_2) = S] \leq \Pr[\text{any bit } v \in S_{c_1} \text{ sees majority of unsatisfied stabilizers}] \leq q^{\Delta|S|_{c_1}}$$

$$\begin{aligned}
 \Pr[\mathbf{Sup}(E_3) = S] &= \sum_{S' \subset S} \Pr[\mathbf{Sup}(E_2) = S' \cap \mathbf{Sup}(E_3/E_2) = S/S'] \\
 &\leq \sum_{S' \subset S} q^{\Delta|S'_{c_1}|} p^{|S/S'_{c_1}|} \leq \sum_{S' \subset S} q^{\Delta|S'_{c_1}|} p^{|S_{c_1}| - |S'_{c_1}|} \\
 &\leq (q^\Delta + p)^{|S_{c_1}|} \leq \begin{cases} (q^\Delta + p)^{\frac{1}{4}|S|} & \text{if } |S_{c_1}| \geq \frac{1}{4}|S| \\ \star & \text{else} \end{cases}
 \end{aligned}$$

Let  $S^t = \mathbf{Sup}(E)$  at time  $t$  and denote by  $\mathcal{P}_t$  the probability that  $|S_{c_1}^t| > \frac{1}{4}|S_t|$ . Then:

$$\begin{aligned}
 \mathcal{P}_{t+1} &\geq \Pr\left[|S_{c_1}^t| > \frac{1}{4}|S_t| \text{ and } |(S_{t+1}/S_t)_{c_1}| \geq \frac{1}{4}|S_{t+1}/S_t|\right] \\
 &\geq \mathcal{P}_t \cdot (1 - e^{-\varepsilon} m) \geq \mathcal{P}_0 (1 - (t+1)e^{-\varepsilon m})
 \end{aligned}$$