

Bucket Sort When You Know The Distribution.

David Ponnarovsky

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC code with testability query complexity of $\Theta(n^{1-\varepsilon})$ for any $\varepsilon > 0$.

The problem. Let $f : [0, 1] \rightarrow [0, 1]$ a fixed distribution function. Write an algorithm that sort n draws $x_1 \dots x_n$ at linear expectation time.

Solution. We will define a partition of the input into a seira of n buckets $\mathcal{B} = \{B_k = [t_k, t_{k+1}] : k \in [n]\}$ such that $\Pr[x \in B_i] = \frac{1}{n}$ for any bucket.

$$\begin{aligned}\mathbf{E}[B_i^2] &= \mathbf{E}\left[\left(\sum_j X_{ij}\right)^2\right] \\ &= \mathbf{E}\left[\sum_{j,j'} X_{ij} X_{ij'}\right] = \sum_{j,j'} \mathbf{E}[X_{ij}] \mathbf{E}[X_{ij'}] \\ &= \sum_{j \neq j'} \mathbf{E}[X_{ij}] \mathbf{E}[X_{ij'}] + \sum_j \mathbf{E}[X_{ij}]^2 \\ &= \frac{1}{n^2} \binom{n}{2} + 1 = O(1)\end{aligned}$$

$$\begin{aligned}\mathbf{E}[B_i^4] &= \mathbf{E}\left[\left(\sum_j X_{ij}\right)^4\right] \\ &= \mathbf{E}\left[\sum_{j,j'} \prod X_{ij}\right] = \sum_{j,j'} \mathbf{E}[X_{ij}] \mathbf{E}[X_{ij'}] \\ &= \sum_{j \neq j'} \mathbf{E}[X_{ij}] \mathbf{E}[X_{ij'}] + \sum_j \mathbf{E}[X_{ij}]^2 \\ &= \sum_{l \in [4]} \frac{1}{n^l} \binom{n}{l} = O(1)\end{aligned}$$

$$\mathbf{V}[B_i^2] = \sum_{l \in [4]} \binom{n}{l} \left(\frac{1}{n^l} - \frac{1}{n^4}\right) \leq e$$

$$\begin{aligned}\frac{1}{n} &= \Pr[x \in B_k] = f(t_{k+1}) - f(t_k) \\ &\Rightarrow t_{k+1} \leftarrow f^{-1}\left(\frac{1}{n} + f(t_k)\right)\end{aligned}$$