Fourmlas Sheet.

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January 21, 2023

Probability.

Multiplicative Chernoff bound. Suppose $X_1, ..., X_n$ are independence random variables taking values in $\{0, 1\}$ Let X denote their sum and let $\mu = \mathbf{E}\left[\sum_i^n X_i\right]$ denote the sum's expected value. Then for any $\delta > 0$:

$$\mathbf{Pr}\left[X \geq (1+\delta)\,\mu\right] \leq e^{-2\frac{\delta^2\mu^2}{n}}$$
$$\mathbf{Pr}\left[|X-\mu| \geq \delta\mu\right] \leq 2e^{-\delta^2\mu/3}, \qquad 0 \leq \delta \leq 1$$

Jensen's inequality. If X is a random variable and ϕ is a convex function, then:

$$\phi\left(\mathbf{E}\left[X\right]\right) \leq \mathbf{E}\left[\phi\left(X\right)\right] \Rightarrow \mathbf{E}\left[X\right] \leq \phi^{-1}\left(\mathbf{E}\left[\phi\left(X\right)\right]\right)$$
$$\mathbf{E}\left[X\right] \leq e^{\mathbf{E}[\ln(X)]}$$
$$\mathbf{E}\left[X\right] \geq \ln\left(\mathbf{E}\left[e^X\right]\right)$$