

Recycling Quantum Computation.

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Consider the CSS code composed by C_x, C_z^\perp at length n . Define the 1-**SWAP** test on $|\psi\rangle \otimes |\phi\rangle$ to be:

1. Apply the hadamard gate on ancilla.
2. Pick a random coordinate $i \sim [n]$.
3. conditional on the ancilla a swap between the i th qubit of $|\psi\rangle$ to the i th qubit of $|\phi\rangle$.
4. Apply the hadamard again on the ancilla and measure. If $|0\rangle$ measured then accept, otherwise reject.

suppose for the moment that $|\psi\rangle$ and $|\phi\rangle$ are in the code. Thus:

$$\begin{aligned}
 |\psi\rangle &= \frac{1}{\sqrt{|C_z^\perp|}} \sum_{z \in C_z} |\psi + z\rangle \\
 (1 - \mathbf{SWAP}) |0\rangle |\psi\rangle |\phi\rangle &= \sum_{z, \xi \in C_z} (1 - \mathbf{SWAP}) |0\rangle |\psi + z\rangle |\phi + \xi\rangle \\
 &= \sum_{z, \xi \in C_z} H |\pm\rangle \left(|\psi + z\rangle |\phi + \xi\rangle \pm |(\phi + \xi)_i (\psi + z)_{/i}\rangle |(\psi + z)_i (\phi + \xi)_{/i}\rangle \right) \\
 \Rightarrow \mathbf{Pr}[|0\rangle] &= \sum_{z, \xi \in C_z} A
 \end{aligned}$$