

Simple qLDPC For Near Future Fault Tolerance.

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Abstract

We propose a new simple construction for quantum LDPC codes and derive a new thresholds for wide regime of noise models.

1 Quantum Code.

Definition. [COMMENT] defintion of the square complex.

Construction. Let $x \in C$ a non-reducible codeword. Then it has a linear weight.

Proof. As shown eriler x has represntion as codeword of the disgreement code over the negative graph: $x = \sum_{u \in V} c_u$ where $c_u \in C_A \otimes C_B$. By x been non-reducible codeword and by Lemma [COMMENT] add number we have that at least linear fruction of the negative vertices contribute a non-trivial codeword.

Draft. By the Discrete Cheeger's inequality it follows that,

$$\frac{1}{2}\lambda' \leq \frac{E_{G'}(S', \cdot)}{|S|} \leq \left(1 - \frac{1}{2}\delta_1\right) \Delta \leq \frac{1}{2}\delta_2 \Delta$$

$$\Rightarrow |S| \geq \left(\delta_2 - \frac{\lambda'}{\Delta}\right) \Delta |T|$$

$$\frac{1}{2}\lambda' \leq \frac{\sum_{u \sim v} (x(u) - x(v))^2}{\sum x(v)^2} \leq \frac{(\beta + \alpha)^2 (1 - \frac{1}{2}\delta_1) \Delta |S|}{(\alpha^2 - \beta^2) |T|}$$

$$\delta_2 \Delta |S| \leq \langle \chi_{S'}, J \chi_S \rangle + \lambda' \sqrt{|S'| |S|}$$

$$\leq \frac{(1 - \frac{1}{2}\delta_1) \Delta^2 |S| + |S|^2 \Delta^2}{\frac{1}{2} |T| \Delta} + \lambda' \sqrt{|S'| |S|}$$

$$\Rightarrow |S| \geq \frac{|T|}{2} \left(\delta_2 - \left(1 - \frac{1}{2}\delta_1\right) - \frac{\lambda'}{\Delta} \right)$$

Lemma. Let C_1, C_2 be Tanner codes over the graph G and small codes $C_{0i} = \Delta[1, \rho_i, \delta_i]$. Let's define the code C to be all the non-reducible words in the intersection between C_1^\oplus and C_2 . Then C has linear distance.

Proof. Consider a vaild codeword $x \in C$ and denote by S the support of x on the vertecis which do not suggest a trivial codeword. We have seen that the degree of the vertices of S in the induced subgraph (T, \cdot) is at least $\frac{1}{2}\delta_1 \Delta$. Denote by $S' \subset T$ the vertices such their neighborhood is also contained in T and consider the subgraph $G' = (T, E')$ obtained by taking the vertices which suggested non-trivial codewords and the edges which are fully supported on those vertices.

and therefore the weight of any $v \in S$ upon the edges of the induced graph is at least $(\delta_2 - (1 - \frac{1}{2}\delta_1)) \Delta$. Otherwise

there exists a vertex which see less than $\delta_2 \Delta$ bits. Using the Expander Mixining Lemma we have that:

$$\left(\delta_2 - \left(1 - \frac{1}{2}\delta_1\right)\right) \Delta \leq \frac{E(S, S)}{|S|} \leq \frac{\Delta}{n} |S|^2 + \lambda |S|$$

$$|S| \geq \left(\delta_2 + \frac{1}{2}\delta_1 - 1 - \frac{\lambda}{\Delta}\right) n$$

[COMMENT] $\delta^2 + \frac{1}{2}\delta - 1 > 0 \Rightarrow \delta \in \left(0, \frac{\sqrt{2}-1}{2}\right)$. So in the end it will be fine. \square

In the following section we will construct a family of complexes on which we will define a pairs of Tanner Codes, eventually, they will used to compose a CSS pairs of good quantum codes.

Infinte Family Of Tanner Quantum Codes. Let p be a prime and $\delta \in (0, 1)$. Consider the Cayly graphs obtained by taking uniformly a $c(\delta) \log n$ generators of the cyclic group at order p , denote that set by S . It was shown by N.Alon that with high probability that process yield a Graph with δ -algebraic expansion. Now, consider the double cover of that graph and denote it by $G = (V = V^+ \cup V^-, E)$. And define the folowing graph denoted by $\Gamma^\pm = (V^\pm, E')$:

$$((u, \pm), (v, \pm)) \in E' \Leftrightarrow \exists a \neq b \in S \text{ s.t } abu = v$$

clearly $|E'| = \frac{1}{2} \binom{|S|}{2} |V|$. [COMMENT] We need to show expansion, One elgante way is first to pick $\sqrt{\log n}$ elements and then show that they match to expansion generator set.