Groverize Monotone Local Search. (Short Note)

David Ponarovsky

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1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process. We will simplify the definitions given at [Fom+15] and use the following definitions instead:

Consider a decision problem inside **NP**, in this paper, we will associate two verifiers U, V with each language. U stands for input validation, conceptually it uses for checking that the solution 'live' inside the problem world. For example, for the 3-**SAT**, U checks that the input indeed encode an assignment. Formally the role of U is to restrict the inputs to certain form. And V responsible to verify that the word indeed in the langauge, ie check that the assignment satisfies the formula. We will say that a problem is an *extension problem* if requiring any of the input bits to be 1 could reduced to another instance of the problem. For example, consider 3-**SAT**, fixing an aribtray bit x_i to be 1 could reduced to another 3-**SAT** formula by erase any of the closures contain x_i and replacing any of the occureents of $\bar{x_i}$ by other termianl on the same clouser (i.e $\bar{x_i} \wedge \bar{y} \wedge z \mapsto \bar{y} \wedge \bar{y} \wedge z$). the input any instance of the problem could be representated as the bit-wise union of two strings which pass U verification. For example, any assignment satisfies a 3-**SAT** instance could be write as or-wise of two assignments.

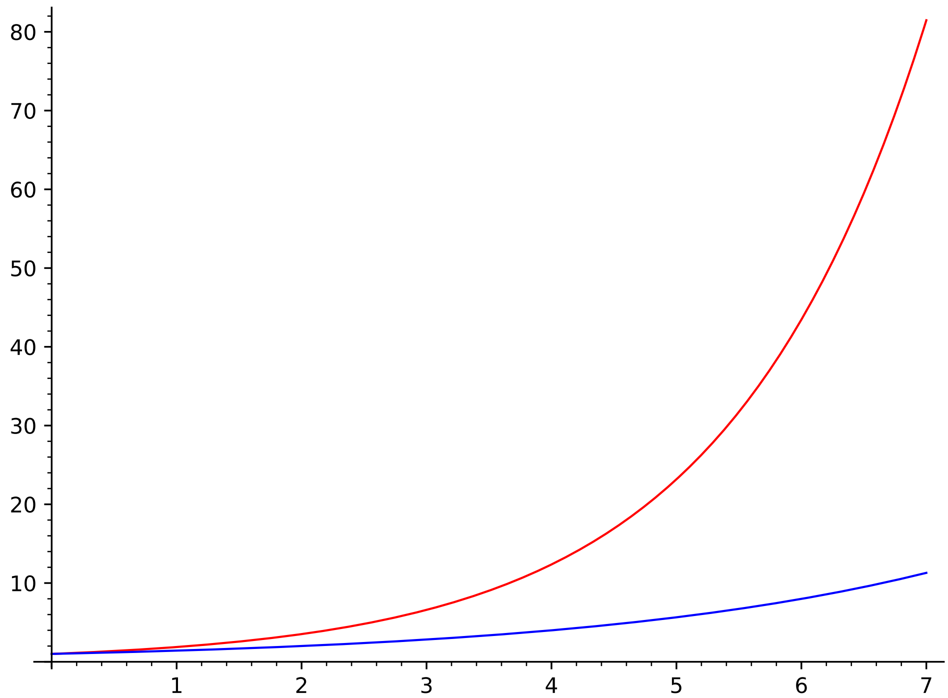
Definition 1. *A* directed graph *G is a pair (V, E) where V is a set of* vertices *and E is a set of* directed edges*.*

Definition 2. *The* directed shortest path problem *is the problem of finding the directed path with the minimum weight between two given vertices in a directed weighted graph.*

$$\sum_{k'\leq k}\frac{1}{\sqrt{p(k')}}\cdot c^{k'-t}N^{\mathcal{O}(1)}\leq \max_{k'\leq k}\left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}}\right)^{\frac{1}{2}}\cdot c^{k'-t}N^{\mathcal{O}(1)}=$$

$$\left(\max_{k'\leq k}\frac{\binom{n-|X|}{t}}{\binom{k'}{t}}\cdot c^{2\left(k'-t\right)}\right)^{\frac{1}{2}}N^{\mathcal{O}(1)}=\left(\max_{k\leq n-|X|}\frac{\binom{n-|X|}{t}}{\binom{k}{t}}\right)^{\frac{1}{2}}\cdot c^{2(k-t)}N^{\mathcal{O}(1)}\leq$$

$$\Rightarrow\left(2-\frac{1}{c^2}\right)^{\frac{n-|X|}{2}}N^{\mathcal{O}(1)}$$



Problem Name	Parameterized		Groverize	New bound	Previous Bound	
Feedback Vertex Set	3^k (r)	[Cyg+11]	1.3744^k	1.6667^n (r)		
Feedback Vertex Set	3.592^k	[KP14]	1.3865^k	1.7217^n	1.7347^n	[FTV15
Subset Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.3919^k	1.7500^n	1.8638^n	[Fom+14
Feedback Vertex Set in Tournaments	1.6181^k	[KL16]	1.2720^k	1.3820^n	1.4656^n	[KL16
Group Feedback Vertex Set	4^k	[**Wahlstrom14**]	1.3919^k	1.7500^n	NPR	
Node Unique Label Cover	$\lvert\Sigma\rvert^{2k}$	[**Wahlstrom14**]	1.3919^k	$(2-\frac{1}{\lvert\Sigma\rvert^2})^n$	NPR	
Vertex (r,ℓ)-Partization $(r,\ell \leq 2)$	3.3146^k	[**KolayP15**; Bas+17]	1.3817^k	1.6984^n	NPR	
Interval Vertex Deletion	8^k	[Cao16]	1.3466^k	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Proper Interval Vertex Deletion	6^k	[tV13; Cao15]	1.4087^k	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Block Graph Vertex Deletion	4^k	[Agr+16]	1.4044^k	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$	[BFP13
Cluster Vertex Deletion	1.9102^k	[Bor+14]	1.3919^k	1.4765^n	1.6181^n	[Fom+10
Thread Graph Vertex Deletion	8^k	[Kan+15]	1.3919^k	1.8750^n	NPR	
Multicut on Trees	1.5538^k	[Kan+14]	1.3138^k	1.3565^n	NPR	
3-Hitting Set	2.0755^k	[**MagnusPhD07**]	1.4087^k	1.5182^n	1.6278^n	[**MagnusPhD07**
4-Hitting Set	3.0755^k	[Fom+10]	1.2593^k	1.6750^n	1.8704^n	[Fom+10
d-Hitting Set $(d \geq 3)$	$(d-0.9245)^k$	[Fom+10]	1.1763^k	$(2-\frac{1}{(d-0.9245)})^n$		[Coc+16; Fom+10
Min-Ones 3-SAT	2.562^k	[**abs-1007-1166**]	1.3296^k	1.6097^n	NPR	
Min-Ones d-SAT $(d \geq 4)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted d-SAT $(d \geq 3)$	d^k		1.3763^k	$(2-\frac{1}{d})^n$	NPR	
Weighted Feedback Vertex Set	3.6181^k	[Agr+16]	1.1763^k	1.7237^n	1.8638^n	[Fom+08
Weighted 3-Hitting Set	2.168^k	[SZ15]	1.3593^k	1.5388^n	1.6755^n	[Coc+16
Weighted d-Hitting Set $(d \geq 4)$	$(d-0.832)^k$	[Fom+10; SZ15]	1.3919^k	$(2-\frac{1}{d-0.932})^n$		[Coc+16

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

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