

Hardness of Computing Fault Tolerance.

David Ponnarovsky

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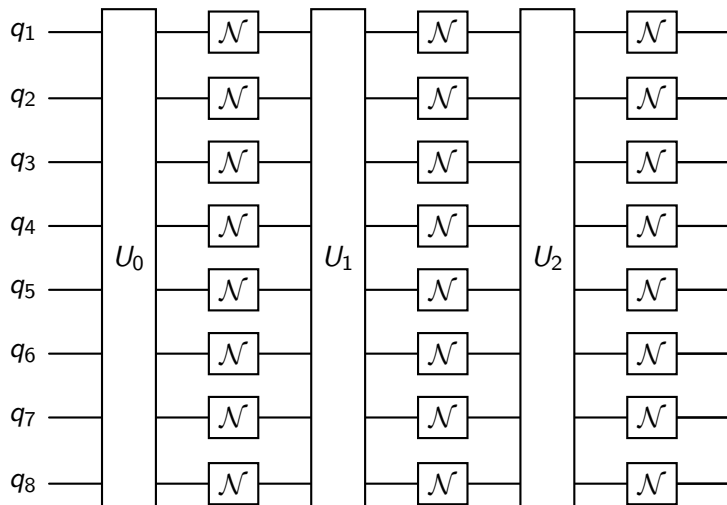
Introduction

- ▶ Brief overview of the topic
- ▶ Importance and relevance
- ▶ Objectives of the presentation

Key Points

- ▶ Main point 1
- ▶ Main point 2
- ▶ Main point 3

Nosiy Circuit.



Threshold Theorem.

Pippenger's Construction.

Encode each bit with the repetition code $0 \mapsto 0^m$, $1 \mapsto 1^m$. Now observe that any logical operation, without decoding, can be made in $O(1)$ depth.

For example, $\text{OR}(\bar{x}, \bar{y})$ can be computed by applying in parallel $\text{OR}(x_i, y_i)$ for each i .

The 'Decoding' trick.

Instead of completely decoding, we would apply only a single step of partial decoding. We assume that in each code block the bits are partitioned into random disjoint triples, and we will apply a local correction to each of the triples by majority.

Claim

There are constants $\alpha, \eta \in (0, 1)$ such that for any bit string x at a distance $\leq \alpha n$ from the code (Repetition Code), one cycle of local correction on x yields x' such that:

$$d(x', C) \leq d(x, C)$$

The 'Decoding' trick.

Suppose that a bit absorb a bit flip with probability p . So in expectation we expect that entire block at length n will absorb pn flips.

$$\eta(\beta + p)n \leq \beta n$$
$$\beta \geq \frac{p}{1 - \eta}$$

The Franch's Construction.

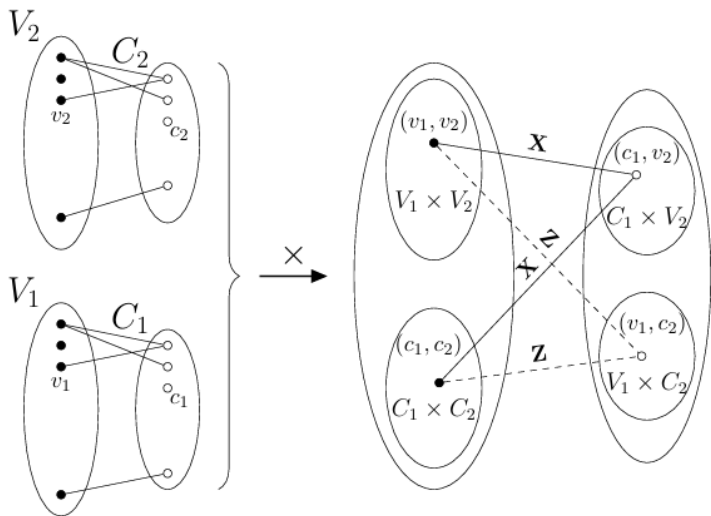


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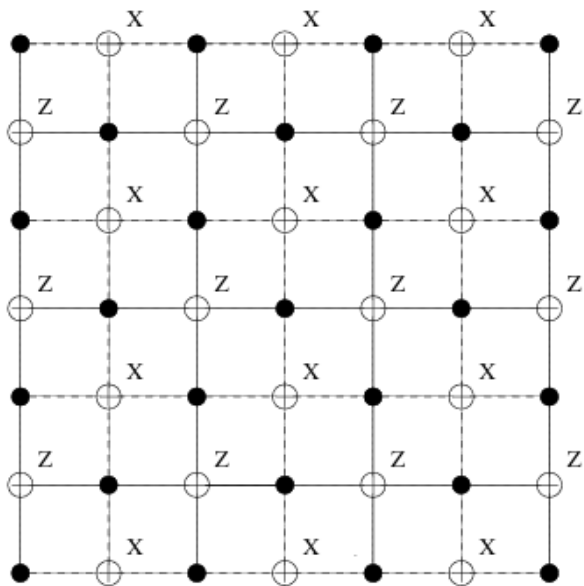


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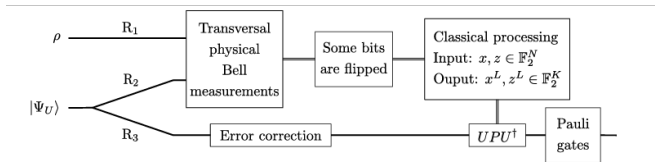


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