References

[1] A. Agrawal, D. Lokshtanov, S. Kolay, and S. Saurabh, *A faster FPT algorithm and a smaller kernel for block graph vertex deletion*, in Proceedings of the 12th Latin American Theoretical Informatics Symposium (LATIN 2016), Lecture Notes in Computer Science, Springer, to appear. Available as arXiv CoRR abs/1510.08154.

[2] N. Alon, L. Babai, and A. Itai, *A fast and simple randomized parallel algorithm for the maximal independent set problem*, J. Algorithms, 7 (1986), pp. 567–583.

[3] J. Baste, L. Faria, S. Klein, and I. Sau, *Parameterized complexity dichotomy for (r, ℓ)-vertex deletion*, Tech. Report abs/1504.05515, arXiv CoRR, 2015.

[4] I. Bliznets, F. V. Fomin, M. Pilipczuk, and Y. Villanger, *Largest chordal and interval subgraphs faster than 2^n*, in Proceedings of the 21st Annual European Symposium on Algorithms (ESA), vol. 8125 of Lecture Notes in Comput. Sci., Springer, 2013, pp. 193–204.

[5] A. Boral, M. Cygan, T. Kociumaka, and M. Pilipczuk, *A fast branching algorithm for cluster vertex deletion*, in Proceedings of the 9th International Computer Science Symposium in Russia (CSR), vol. 8476 of Lecture Notes in Computer Science, Springer, 2014, pp. 111–124.

[6] A. Brandstädt, V. B. Le, and J. P. Spinrad, *Graph Classes: A Survey*, Discrete Mathematics and Applications, SIAM, 1999.

[7] Y. Cao, *Unit interval editing is fixed-parameter tractable*, in Proceedings of the 42nd International Colloquium of Automata, Languages and Programming (ICALP), vol. 9134 of Lecture Notes in Comput. Sci., Springer, 2015, pp. 306–317.

[8] Y. Cao, *Linear recognition of almost interval graphs*, in Proceedings of the 27th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA 2016), to appear. Available as arXiv CoRR abs/1403.1515.

[9] Y. Cao, J. Chen, and Y. Liu, *On feedback vertex set: New measure and new structures*, Algorithmica, 73 (2015), pp. 63–86.

[10] J. Chen, F. V. Fomin, Y. Liu, S. Lu, and Y. Villanger, *Improved algorithms for feedback vertex set problems*, J. Computer and System Sciences, 74 (2008), pp. 1188–1198.

[11] M. Cochefert, J.-F. Couturier, S. Gaspers, and D. Kratsch, *Faster algorithms to enumerate hypergraph transversals*, in Proceedings of the 12th Latin American Theoretical Informatics Symposium (LATIN 2016), Lecture Notes in Computer Science, Springer, to appear. Available as arXiv CoRR abs/1510.05093.

[12] M. Cygan, F. V. Fomin, L. Kowalik, D. Lokshtanov, D. Marx, M. Pilipczuk, M. Pilipczuk, and S. Saurabh, *Parameterized Algorithms*, Springer, 2015.

[13] M. Cygan, J. Nederlof, M. Pilipczuk, M. Pilipczuk, J. M. M. van Rooij, and J. O. Wojtaszczyk, *Solving connectivity problems parameterized by treewidth in single exponential time*, in Proceedings of the 52nd Annual Symposium on Foundations of Computer Science (FOCS), IEEE, 2011, pp. 150–159.

[14] E. Dantsin, A. Goerdt, E. A. Hirsch, R. Kannan, J. Kleinberg, C. Papadimitriou, P. Raghavan, and U. Schöning, *A deterministic $(2 - 2/(k + 1))^n$ algorithm for k-SAT based on local search*, Theoretical Computer Science, 289 (2002), pp. 69–83.

[20] F. V. FOMIN AND D. KRATSCH, *Exact Exponential Algorithms*, Springer, 2010. An EATCS Series: Texts in Theoretical Computer Science.

[21] F. V. FOMIN, I. TODINCA, AND Y. VILLANGER, *Large induced subgraphs via triangulations and CMSO*, SIAM J. Comput., 44 (2015), pp. 54–87.

[22] S. GASPERS, E. J. KIM, S. ORDYNIAK, S. SAURABH, AND S. SZEIDER, *Don't be strict in local search!*, in Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence (AAAI), AAAI Press, 2012.

[23] S. GASPERS AND M. MNICH, *Feedback vertex sets in tournaments*, Journal of Graph Theory, 72 (2013), pp. 72–89.

[24] P. A. GOLOVACH, P. VAN 'T HOF, AND D. PAULUSMA, *Obtaining planarity by contracting few edges*, Theoretical Computer Science, 476 (2013), pp. 38–46.

[25] J. GUO, J. GRAMM, F. HÜFFNER, R. NIEDERMEIER, AND S. WERNICKE, *Compression-based fixed-parameter algorithms for feedback vertex set and edge bipartization*, J. Computer and System Sciences, 72 (2006), pp. 1386–1396.

[26] I. A. KANJ, G. LIN, T. LIU, W. TONG, G. XIA, J. XU, B. YANG, F. ZHANG, P. ZHANG, AND B. ZHU, *Algorithms for cut problems on trees*, in Proceedings of the 8th International Conference on Combinatorial Optimization and Applications (COCOA), vol. 8881 of Lecture Notes in Computer Science, Springer, 2014, pp. 283–298.

[27] M. M. KANTÉ, E. J. KIM, O. KWON, AND C. PAUL, *An FPT algorithm and a polynomial kernel for linear rankwidth-1 vertex deletion*, in Proceedings of the 10th International Symposium on Parameterized and Exact Computation (IPEC 2015), vol. 43 of LIPIcs, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2015, pp. 138–150.

[28] T. KOCIUMAKA AND M. PILIPCZUK, *Faster deterministic feedback vertex set*, Inf. Process. Lett., 114 (2014), pp. 556–560.

[29] S. KOLAY AND F. PANOLAN, *Parameterized algorithms for deletion to (r, ℓ)-graphs*, in Proceedings of FSTTCS 2015, to appear. Available as arXiv CoRR abs/1504.08120.

[30] M. KUMAR AND D. LOKSHTANOV, *Faster exact and parameterized algorithm for feedback vertex set in tournaments*, in Proceedings of the 33rd International Symposium on Theoretical Aspects of Computer Science (STACS 2016), to appear. Available as arXiv CoRR abs/1510.07676.

[31] K. KUTZKOV AND D. SCHEDER, *Using CSP to improve deterministic 3-SAT*, CoRR, abs/1007.1166 (2010).

[32] D. MARX AND I. SCHLOTTER, *Stable assignment with couples: Parameterized complexity and local search*, Discrete Optimization, 8 (2011), pp. 25–40.

[33] J. W. MOON, *On maximal transitive subtournaments*, Proc. Edinburgh Math. Soc. (2), 17 (1971), pp. 345–349.

[34] R. A. MOSER AND D. SCHEDER, *A full derandomization of Schöning's k-SAT algorithm*, in Proceedings of the 43rd Annual ACM Symposium on Theory of Computing (STOC), ACM, 2011, pp. 245–252.

[35] M. Naor, L. J. Schulman, and A. Srinivasan, *Splitters and near-optimal derandomization*, in Proceedings of the 36th Annual Symposium on Foundations of Computer Science (FOCS), IEEE, 1995, pp. 182–191.

[36] I. Razgon, *Exact computation of maximum induced forest*, in Proceedings of the 10th Scandinavian Workshop on Algorithm Theory (SWAT 2006), vol. 4059 of Lecture Notes in Comput. Sci., Berlin, 2006, Springer, pp. 160–171.

[37] ——, *Computing minimum directed feedback vertex set in $o(1.9977^n)$*, in Proceedings of the 10th Italian Conference on Theoretical Computer Science (ICTCS), 2007, pp. 70–81.

[38] U. Schöning, *A probabilistic algorithm for k-SAT and constraint satisfaction problems*, in 40th Annual Symposium on Foundations of Computer Science (New York, 1999), IEEE Computer Soc., Los Alamitos, CA, 1999, pp. 410–414.

[39] H. Shachnai and M. Zehavi, *A multivariate approach for weighted FPT algorithms*, in Proceedigs of the 23rd Annual European Symposium on Algorithms (ESA), vol. 9294 of Lecture Notes in Computer Science, Springer, 2015, pp. 965–976.

[40] P. van 't Hof and Y. Villanger, *Proper interval vertex deletion*, Algorithmica, 65 (2013), pp. 845–867.

[41] M. Wahlström, *Algorithms, measures and upper bounds for satisfiability and related problems*, PhD thesis, Linköping University, Sweden, 2007.

[42] ——, *Half-integrality, LP-branching and FPT algorithms*, in Proceedings of the 24th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA), SIAM, 2014, pp. 1762–1781.

[43] M. Xiao and H. Nagamochi, *Exact algorithms for maximum independent set*, in Proceedings of the 24th International Symposium on Algorithms and Computation (ISAAC), vol. 8283 of Lecture Notes in Computer Science, Springer, 2013, pp. 328–338. See also arXiv CoRR abs/1312.6260.

[44] T. Yato and T. Seta, *Complexity and completeness of finding another solution and its application to puzzles*, IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, E86-A (2003), pp. 1052–1060.

Groverize Monotone Local Search. (Short Note)

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1 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} =$$

$$\left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2(k'-t)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq$$

$$\Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)}$$
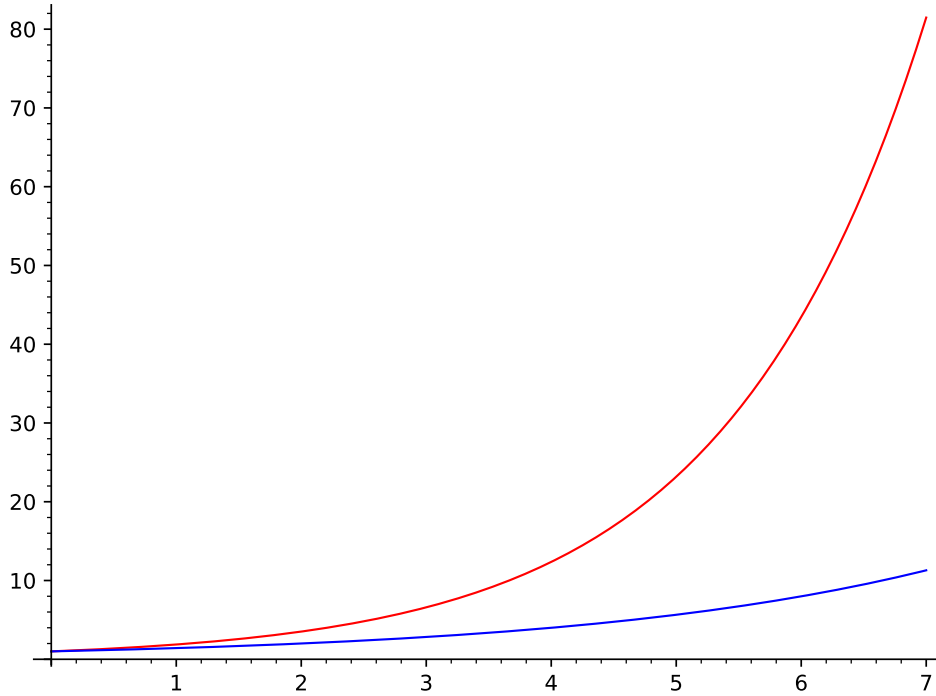
Problem Name	Parameterized		New bound	Previous Bound
Feedback Vertex Set	3^k (r)	[cut-and-count]	1.6667^n (r)	
Feedback Vertex Set	3.592^k	[KociumakaP13]	1.7217^n	1.7347^n
Subset Feedback Vertex Set	4^k	[Wahlstrom14]	1.7500^n	1.8638^n
Feedback Vertex Set in Tournaments	1.6181^k	[KumarL16]	1.3820^n	1.4656^n
Group Feedback Vertex Set	4^k	[Wahlstrom14]	1.7500^n	NPR
Node Unique Label Cover	$|\Sigma|^{2k}$	[Wahlstrom14]	$(2-\frac{1}{|\Sigma|^2})^n$	NPR
Vertex (r,ℓ)-Partization $(r,\ell \le 2)$	3.3146^k	[BasteFKS15; KolayP15]	1.6984^n	NPR
Interval Vertex Deletion	8^k	[Cao8kinterval]	1.8750^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
Proper Interval Vertex Deletion	6^k	[HofV13; Cao15]	1.8334^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
Block Graph Vertex Deletion	4^k	[AgrawalLKS16]	1.7500^n	$(2-\varepsilon)^n$ for $\varepsilon < 10^{-20}$
Cluster Vertex Deletion	1.9102^k	[BoralCKP14]	1.4765^n	1.6181^n
Thread Graph Vertex Deletion	8^k	[Kante0KP15]	1.8750^n	NPR
Multicut on Trees	1.5538^k	[KanjLLTXXYZZZ14]	1.3565^n	NPR
3-Hitting Set	2.0755^k	[MagnusPhD07]	1.5182^n	1.6278^n
4-Hitting Set	3.0755^k	[FominGKLS10]	1.6750^n	1.8704^n
d-Hitting Set $(d \ge 3)$	$(d-0.9245)^k$	[FominGKLS10]	$(2-\frac{1}{(d-0.9245)})^n$	[CochefertCGK16
Min-Ones 3-SAT	2.562^k	[abs-1007-1166]	1.6097^n	NPR
Min-Ones d-SAT $(d \ge 4)$	d^k		$(2-\frac{1}{d})^n$	NPR
Weighted d-SAT $(d \ge 3)$	d^k		$(2-\frac{1}{d})^n$	NPR
Weighted Feedback Vertex Set	3.6181^k	[AgrawalLKS16]	1.7237^n	1.8638^n [F
Weighted 3-Hitting Set	2.168^k	[ShachnaiZ15]	1.5388^n	1.6755^n [C
Weighted d-Hitting Set $(d \ge 4)$	$(d-0.832)^k$ [FominGKLS10; ShachnaiZ15]		$(2-\frac{1}{d-0.932})^n$	[C

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).

References

[Gro96]   Lov K. Grover. *A fast quantum mechanical algorithm for database search*. 1996. arXiv: quant-ph/9605043 [quant-ph].

[Fom+15]   Fedor V. Fomin et al. *Exact Algorithms via Monotone Local Search*. 2015. arXiv: 1512.01621 [cs.DS].