Bucket Sort When You Know The Distribution.

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC code with testability query complexity of $\Theta\left(n^{1-\varepsilon}\right)$ for any $\varepsilon > 0$.

The problem. Let $f:[0,1] \to [0,1]$ a fixed distribution function. Write an algorithm that sort n draws $x_1...x_n$ at linear expectation time.

Solution. We will define a partition of the input into a seira of n buckets $\mathcal{B} = \{B_k = [t_k, t_{k+1}] : k \in [n]\}$ such that $\mathbf{Pr}[x \in B_i] = \frac{1}{n}$ for any bucket.

$$\mathbf{E}\left[B_{i}^{2}\right] = \mathbf{E}\left[\left(\sum_{j}X_{ij}\right)^{2}\right]$$

$$= \mathbf{E}\left[\sum_{j,j'}X_{ij}X_{ij'}\right] = \sum_{j,j'}\mathbf{E}\left[X_{ij}\right]\mathbf{E}\left[X_{ij'}\right]$$

$$= \sum_{j\neq j'}\mathbf{E}\left[X_{ij}\right]\mathbf{E}\left[X_{ij'}\right] + \sum_{j}\mathbf{E}\left[X_{ij}\right]$$

$$= \frac{1}{n}\binom{n}{2} + 1 = O\left(1\right)$$