The Dual-Tensor Polynomial Code Is Not w-Robustness.

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1 The Polynomial-Code Is Not w-Robust.

One idea for constructing is to use the polynomial code instead C_0, The follow form the fact that if one pick degree strictly greater than $\Delta/2$ then $C_0^{\perp} \subset C_0$ and therefore one could choose C_z to be the same code defined on the negative vertices of the graph.

Here we prove that the dual-tensor code, in that case, is not w-robust, meaning that any such construction should be consider other way for proving the reduction Lemma.

Claim 1. *Let C_0 be the $[\Delta, d, \Delta - d]$ polynomial code. Then any code word in $\left(C_0^{\perp} \otimes C_0^{\perp}\right)^{\perp}$ is a polynomial in $F[x, y]$ at degree at most $\Delta + d$*

Proof. Consider base element $C_0 \otimes \mathbb{F}$, denote it by $c = g_i \otimes e_j$. And notice that c has representation in $F[x, y]$ of $\prod_{y' \neq j} (y - y')g_i(x)$. By the fact that $g_i(x) \in C_0$ we have that degree of c is at most $\Delta + \delta$. Hence any element in the subspace of $C_0 \otimes \mathbb{F}$ is a polynomial at degree at most $\Delta + d$. □

Claim 2. *The dual-tensor polynomial code is not w-robust.*

Proof.

$$P(x, y) = \prod_{i \neq \Delta - 1} (x + iy) = \prod_{i \neq 1} (x - iy)$$
$$P(x, x) = \prod_{i \neq \Delta - 1} (x + ix) = x^{\Delta - 1} \prod_{i \neq \Delta - 1} (1 + i) = (\Delta - 1)! =_{\Delta} -1 \neq_{\Delta} 0$$