

Groverize Monotone Local Search. (Short Note)

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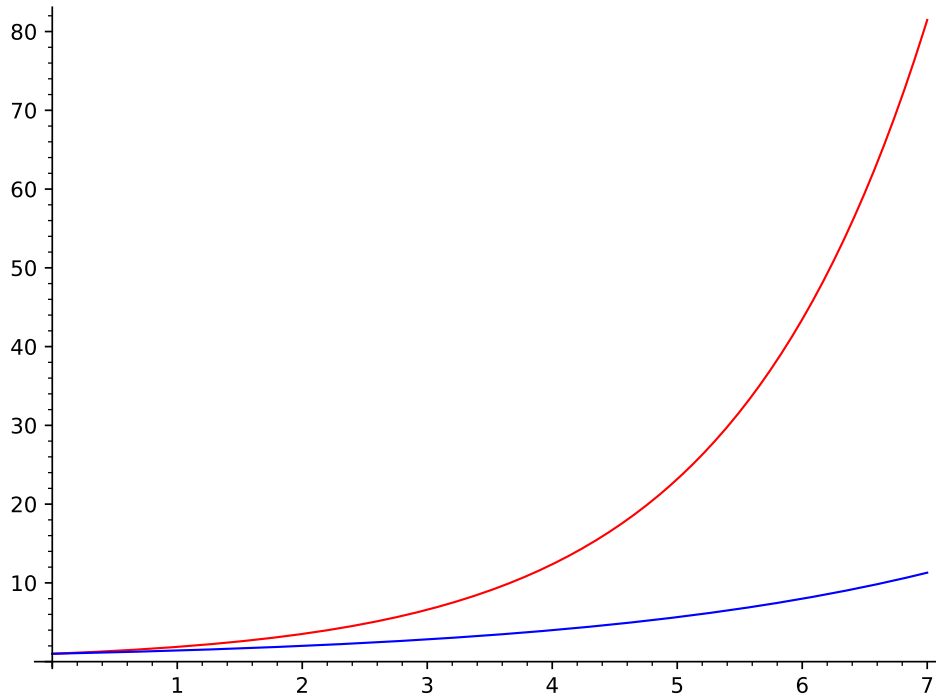
1 Todo.

1. Write the table (sage script).
2. Add definitions. Problem description.
3. Complete the 'proof'.
4. Prove lower bound.

2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the tree-width of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\begin{aligned} \sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} &\leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} = \\ \left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2(k'-t)} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} &= \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq \\ \Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)} \end{aligned}$$



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