Research Proposal - Fault-Tolerainzing Shallow Circuits.

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Abstract

In this work we study the overall depth overhead cost required for constructing fault tolerance circuits. We focus on shallow depth circuits classes, In particular, $\mathbf{QAC}_0, \mathbf{QNC}_{0,f}$ and $\mathbf{QNC}_1$ and certain knowns problem candidates for demonstrating quantum advantage such as factoring [Sho97] and Instantaneous Quantum Polynomial-time [BMS17], [Pal+24]. We only give a partial answers, Yet, clues that might pave the way towards a full understanding of the complexity versus fault tolerance trade-off.

1 Introduction.

The Quantum Computation model is widely believed to be superior to classical models, offering asymptotic speedup in tasks such as factoring [Sho97], searching [Gro96], simulating, and more relative to the best-known classical solutions. Yet even though there is almost full agreement about the superiority of the ideal quantum model, there is still a debate over whether it is possible to implement complex computation in the real world, where the qubits and gates are subject to faults. Similarly, the feasibility of realizing classical computation has also been an open question, In fact the question about the feasibility of computation under noise is almost as ancient as the computer science field itself, initialized by Von Neumann [Neu56] at the time that classical computation putted in debuts. Time been pass and the followed works had pointed that not even a polynomial computation in the presence of noise is still reasonable but one can implement a fault tolerance version at a most constant times cost at the circuits depth [Pip85]. Or in asymptotic sense, classical computation in the presence of noise is as exactly hard as computation in ideal environment.

Once again, the feasibility question raised again, this time regarding quantum computing, and while an intensive work has been done, and also succeed to prove that polynomial quantum computation can be made fault tolerance, [AB99],[Got14] and even with only constant overhead at the original circuit width [Gro19], the required depth over-head is still not well understood. We stress out that in all the familiar constructions, in construct to Pippenger [Pip85], original constant-depth gates are mapped to asymptotically grow[1] depth gates.

Moreover, even the deapth overhead is particularly intersting as the nowadays quantum machines are challennged to keep a quantum states for a long time, the limited machines motivated reaseach to define the NISQ, stands for Noisy Intermediate-Scale Quantum, referring to the current era of quantum computing characterized by quantum processors that have a limited number of qubits and are prone to errors due to noise. In addition to NISQ, anthoer common characterizion for limited computation, is computation without resets gates, proved to be impossible when restricted to polynomail space [**<empty citation>**], having a costant deapth-over head fault tolearance schema would impaly the feasibility of log deapth compuation in that model.

This work address the above, We ask whether a magnitude depth overhead is an unavoidable price that one has to pay. And, in particular, whether an ideal $\mathbf{QNC}_1$ circuits can be computed in noisy-$\mathbf{QNC}_1$ circuits. We show how using the ideas presented in [Gro19] and [Pip85] gives almost immediately $\mathbf{QNC}_{0,f} \subset$ noisy-$\mathbf{QNC}_1$ and that sampling from IQP [BMS17] also can be done in logarithmic depth circuits.

[1] Note, that here, classical computation is also counted in the overall depth cost

The proposal is organized as follows. Section 2 presents the notations, formal definitions, and states the open problems that will be studied through the research. Then, Section 3 describes strategies to prove CDFT . In particular, it lists primitives that can be used to achieve it and discusses how far we are from obtaining them. Having said that, Section 4 presents the first cues against the possibility of CDFT and provides the entry points to prove the impossibility claim. Finally, Section 5 discusses the applications and implications of either the correctness of CDFT or the impossibility of CDFT , from both theoretical and practical views.

2 Notations.

In the following, we present the notations used in the paper. Readers who are familiar with the literature of coding theory and quantum fault tolerance might skip **??** and **??** and continue directly to **??**, which introduces less standard notations.

3 Strategies to get CDFT .

4 Cues against CDFT .

5 Applications.