

Recursion Code.

David Ponarovsky

February 23, 2023

Abstract

None

1 Construction.

Definition 1. Let Δ be an integer greater than 2 and consider an algorithm \mathcal{A} that for any n that is power of 3 construct a Δ -regular graph over n vertices. Now, let G be Δ -regular graph over n vertices generated by \mathcal{A} . Define the **third graph obtained by G** , labeled by G^\sim to be the graph which \mathcal{A} returns over $\frac{1}{3}n$ such that any of the edges could be associate by puncturing a $\frac{2}{3}$ fraction of the edges of each vertex.

Definition 2 (Recursion Code). Let $C_0 = \Delta[1, \rho_0, \delta_0]$ be a binary linear code. We will define the recursion code in recursive manner. First for a sufficiently large integer n_0 , which is also power of 3, $C(n_0)$ defined to be the Tanner code defined by the C_0 and graph $\mathcal{A}(n_0)$. Then let n be any power of 3, such that $n > n_0$, denote by G the graph that constructed by the running of $\mathcal{A}(n)$. Then let $C(n)$ be the code obtained by the joining the parity check matrix of the Tanner code $\mathcal{T}(G, C_0)$ and by the checks of the $C(n/3)$ over the bits associated with the G^\sim . We will call to that code family the **recursion code**.

Lemma 1. If $\rho_0 > \frac{2}{3}$, then the recursion code has a positive rate.

Proof. By counting the restrictions we have that:

$$H(n) = \Delta n (1 - \rho_0) + H(n/3) \leq \frac{3}{2} \Delta (1 - \rho_0) \Delta n$$

So we dimension of the code is at least $\frac{1}{2} \Delta n - H(n)$ which is

$$\frac{1}{2} n \Delta - \frac{3}{2} \Delta (1 - \rho_0) \Delta n = \frac{1}{2} \Delta n (3\rho_0 - 2)$$

So for any $\rho_0 > \frac{2}{3}$ we have that the rate of the C_n is grater than constant. □

Recursion Decoder.

-
- 1 Decode $G^{\frac{2}{3}+}$
 - 2 Decode $G^{\frac{2}{3}-}$
 - 3 Decode $G^{\frac{1}{3}}$
-