

Magic States Distillation Using Δ -Toric (good qLDPC?).

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Let $|f\rangle$ be a codeword in C_X , and let X_g be the indicator that equals 1 if f has support on X_g , and 0 otherwise. Observe that applying T^\otimes on $|f\rangle$ yields the state:

$$\begin{aligned} T^{\otimes n} |f\rangle &= T^{\otimes n} \left| \sum_g X_g g \right\rangle = \exp \left(i\pi/4 \sum_g X_g |g| - 2 \cdot i\pi/4 \sum_{g,h} X_g X_h |g \cdot h| \right. \\ &\quad \left. + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| - 8 \cdot i\pi/4 \cdot \text{integers} \right) |f\rangle \\ &= \exp \left(i\pi/4 \sum_g X_g |g| - 2 \cdot \pi/4 \sum_{g,h} X_g X_h |g \cdot h| + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| \right) |f\rangle \end{aligned}$$

1 Many to One.

Assume that f is supported on exactly one generator. Then we have that $T^{\otimes n} |f\rangle = e^{i\pi|g|/4} |f\rangle$. Therefore, if $|g| = 4k + 1$ then we are done.

2 Using Quantum Error Correction Codes.

Now assume that the code C_X is the quantum Tanner code, denote by G, A, B the group and the two generator sets that are used for constructing the square complex.

Claim 2.1. *Consider g, h that are supported on the same $v \in V$. We will call such a pair a source-sharing pair. Suppose that for any v we have that $|g \cdot h|$ is even. Then there is a Clifford gate that computes $|f\rangle \mapsto \exp \left(-i\pi \sum_{g,h \text{ source-sharing}} X_g X_h |g \cdot h| \right) |f\rangle$.*

3 Fail Attempt.

In addition, let us assume the existence of $d \in G$ such that d is non-identity and commutes with any element in $A \cup B$. Then, observe that multiplying by d preserves adjacency on the complex. Namely, if $\{u, v\} \in E$ then also $\{du, dv\} \in E$.

Consider $|f\rangle$ such that if X_g is not zero, and g is associated with a local codeword $c \in C_A \otimes C_B$ on vertex v , then the generator associated with the local codeword c on vertex $d \cdot v$ also supports f , denoted by g' . Thus, the exponent above becomes:

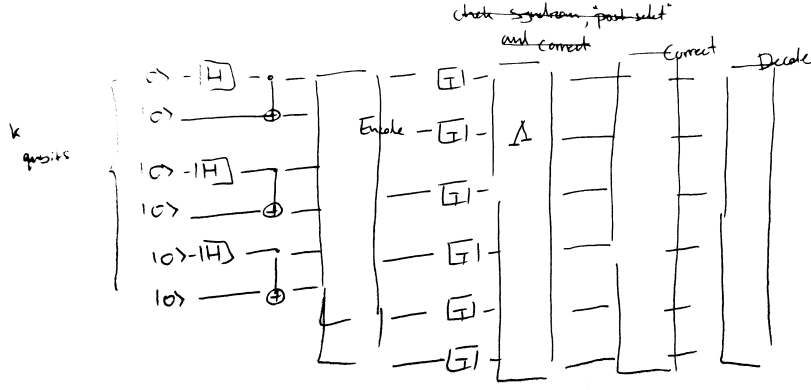


Figure 1: Quantum Circuit for distillation.

$$\begin{aligned}
&= \exp \left(i\pi/4 \sum_g X_g |g\rangle - 2 \cdot \pi/4 \sum_{g,h \in G/a} X_g X_h |g \cdot h\rangle + X_{g'} X_{h'} |g \cdot h\rangle \right. \\
&\quad \left. + 4 \cdot i\pi/4 \sum_{g,h \in G/a} X_g X_h X_l |g \cdot h \cdot l\rangle + X_{g'} X_{h'} X_{l'} |g \cdot h \cdot l\rangle \right) |f\rangle \\
&= \exp \left(i\pi/4 \sum_g X_g |g\rangle - 2 \cdot 2 \cdot \pi/4 \sum_{g,h \in G/a} X_g X_h |g \cdot h\rangle + 2 \cdot 4 \cdot i\pi/4 \sum_{g,h \in G/a} X_g X_h X_l |g \cdot h \cdot l\rangle \right) |f\rangle \\
&= \exp \left(i\pi/4 \sum_g X_g |g\rangle - i\pi \sum_{g,h \in G/a} X_g X_h |g \cdot h\rangle \right) |f\rangle
\end{aligned}$$

Claim 3.1. *The gate $|f\rangle \mapsto \exp \left(-i\pi \sum_{g,h \in G/a} X_g X_h |g \cdot h\rangle \right) |f\rangle$ is in the Clifford.*

Proof. Just decode f and apply **CZ** between any pair of qubits corresponding to the generators g, h such that $g \cap h = 1$. Then encode the state again. Observes that **CZ** is a Clifford gate, and by the fact that the code is a CSS code then the decoder and the encoder are both in the Clifford. \square

Let's denote the circuit defined in Claim 3.1 by Λ . So we have that:

$$\begin{aligned}
&\Lambda^\dagger \exp \left(i\pi/4 \sum_g X_g |g\rangle - i\pi \sum_{g,h \in G/a} X_g X_h |g \cdot h\rangle \right) |f\rangle \\
&= \exp \left(i\pi/4 \sum_g X_g |g\rangle \right) |f\rangle
\end{aligned}$$

Maybe what do we need is to arrange in some way $|g| + |g'| = 4k + 1$ and $\langle g, f \rangle = \langle g', f' \rangle$