

Fourmlas Sheet.

David Ponarovsky

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Probability.

Multiplicative Chernoff bound. Suppose X_1, \dots, X_n are independence random variables taking values in $\{0, 1\}$. Let X denote their sum and let $\mu = \mathbf{E}[\sum_i^n X_i]$ denote the sum's expected value. Then for any $\delta > 0$:

$$\Pr[X \geq (1 + \delta)\mu] \leq e^{-2\frac{\delta^2\mu^2}{n}}$$

$$\Pr[|X - \mu| \geq \delta\mu] \leq 2e^{-\delta^2\mu/3}, \quad 0 \leq \delta \leq 1$$

Jensen's inequality. If X is a random variable and ϕ is a convex function, then:

$$\phi(\mathbf{E}[X]) \leq \mathbf{E}[\phi(X)]$$

$$\mathbf{E}[X] \leq \ln(\mathbf{E}[e^X])$$

$$\mathbf{E}[X] \leq e^{\mathbf{E}[\ln(X)]}$$

