Magic States Distillation Using Δ-Toric (good qLDPC?).

David Ponarovsky

January 3, 2024

Let $|f\rangle$ be a codeword in C_X, and let X_g be the indicator that equals 1 if f has support on X_g, and 0 otherwise. Observes that applying $T^{\otimes}$ on $|f\rangle$ yilds the state:

$$\begin{aligned}
T^{\otimes n}|f\rangle = T^{\otimes n}|\sum_g X_g g\rangle = \exp\Big(& i\pi/4 \sum_g X_g |g| - 2 \cdot i\pi/4 \sum_{g,h} X_g X_h |g \cdot h| \\
& + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| - 8 \cdot i\pi/4 \cdot \text{ integers } \Big) |f\rangle \\
= \exp\Big(& i\pi/4 \sum_g X_g |g| - 2 \cdot \pi/4 \sum_{g,h} X_g X_h |g \cdot h| + 4 \cdot i\pi/4 \sum_{g,h} X_g X_h X_l |g \cdot h \cdot l| \Big) |f\rangle
\end{aligned}$$

1 Many to One.

Assume that f is supported on exactly one generator. Then we have that $T^{\otimes n}|f\rangle = e^{i\pi|g|/4}|f\rangle$ Therefore, if $|g| = 4k + 1$ then we are done.

2 Using Quntum Error Correction Codes.

Now assume that the code C_X is the quantum Tanner code, denote by G, A, B the group and the two generator sets that are used for constructing the square complex.

Claim 2.1. *Consider g, h that are supported on the same $v \in V$. We will call such a pair a source-sharing pair. Suppose that for any we have that $|g \cdot h|$ is even. Then there is a Clifford gate that computes* $|f\rangle \mapsto \exp\Big(- i\pi \sum_{g,h \text{ source-sharing}} X_g X_h |g \cdot h| \Big) |f\rangle$.

3 Fail Attempt.

In addition, let us assume the existence of $d \in G$ such that d is non-identity and commutes with any element in $A \cup B$. Then, observe that multiplying by d preserves adjacency on the complex. Namely, if $\{u, v\} \in E$ then also $\{du, dv\} \in E$.

Consider $|f\rangle$ such that if X_g is not zero, and g is associated with a local codeword $c \in C_A \otimes C_B$ on vertex v, then the generator associated with the local codeword c on vertex $d \cdot v$ also supports f, denoted by g'. Thus, the exponent above becomes:

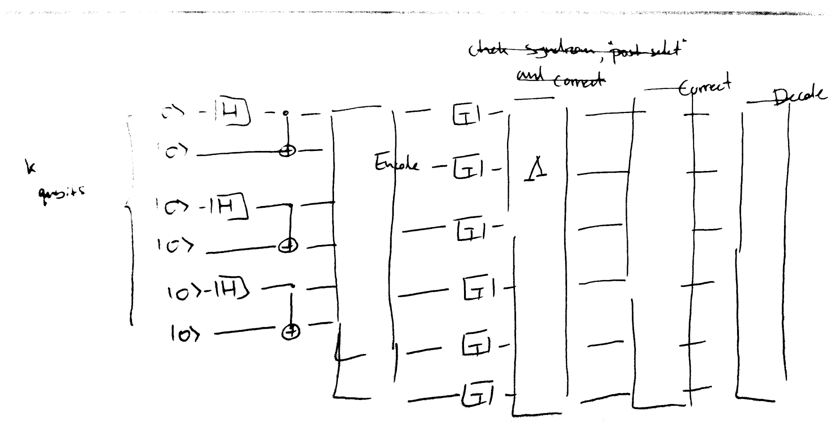

Figure 1: Quantum Circuit for distillation.

$$
\begin{aligned}
&= \exp\Big(i\pi/4 \sum_g X_g|g| - 2\cdot \pi/4 \sum_{g,h\in G/a} X_g X_h |g\cdot h| + X_{g'}X_{h'}|g\cdot h| \\
&+ 4\cdot i\pi/4 \sum_{g,h\in G/a} X_g X_h X_l |g\cdot h\cdot l| + X_{g'}X_{h'}X_{l'}|g\cdot h\cdot l|\Big)\,|f\rangle \\
&= \exp\Big(i\pi/4 \sum_g X_g|g| - 2\cdot 2\cdot \pi/4 \sum_{g,h\in G/a} X_g X_h |g\cdot h| + 2\cdot 4\cdot i\pi/4 \sum_{g,h\in G/a} X_g X_h X_l |g\cdot h\cdot l|\Big)\,|f\rangle \\
&= \exp\Big(i\pi/4 \sum_g X_g|g| - i\pi \sum_{g,h\in G/a} X_g X_h |g\cdot h|\Big)\,|f\rangle
\end{aligned}
$$

Claim 3.1. *The gate* $|f\rangle \mapsto \exp\Big(-i\pi \sum_{g,h\in G/a} X_g X_h |g\cdot h|\Big)\,|f\rangle$ *is in the Clifford.*

Proof. Just decode f and apply **CZ** between any pair of qubits corresponding to the generators g, h such that $g \cap h = 1$. Then encode the state again. Observes that **CZ** is a Clifford gate, and by the fact that the code is a CSS code then the decoder and the encoder are both in the Clifford. ☐

Let's denote the circuit defined in Claim 3.1 by Λ. So we have that:

$$
\begin{aligned}
\Lambda^\dagger \exp\Big(i\pi/4 \sum_g X_g|g| - i\pi \sum_{g,h\in G/a} X_g X_h |g\cdot h|\Big)\,|f\rangle \\
= \exp\Big(i\pi/4 \sum_g X_g|g|\Big)\,|f\rangle
\end{aligned}
$$

Maybe what do we need is to arrange in some way $|g| + |g'| = 4k+1$ and $\langle g, f\rangle = \langle g', f'\rangle$

Claim 3.2. *For any* m *codewords* $x_1..x_m$ *there is a set of coordinates* I *and* $|I| < \alpha n$*. Such that:*

$$
\sum_{j\in[n]/I} x_a^j x_b^j = 0
$$

For any pair x_a, x_b*.*