Fourmlas Sheet.

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Probability.

Multiplicative Chernoff bound. Suppose $X_1,...,X_n$ are independence random variables taking values in $\{0,1\}$ Let X denote their sum and let $\mu = \mathbf{E}\left[\sum_i^n X_i\right]$ denote the sum's expected value. Then for any $\delta > 0$:

$$\begin{aligned} &\mathbf{Pr}\left[X \geq \left(1 + \delta\right)\mu\right] \leq e^{-2\frac{\delta^2\mu^2}{n}} \\ &\mathbf{Pr}\left[|X - \mu| \geq \delta\mu\right] \leq 2e^{-\delta^2\mu/3}, \qquad 0 \leq \delta \leq 1 \end{aligned}$$

Jensen's inequality. If X is a random variable and ϕ is a convex function, then:

$$\begin{split} \phi\left(\mathbf{E}\left[X\right]\right) \leq &\mathbf{E}\left[\phi\left(X\right)\right] \Rightarrow \mathbf{E}\left[X\right] \leq \phi^{-1}\left(\mathbf{E}\left[\phi\left(X\right)\right]\right) \\ &\mathbf{E}\left[X\right] \leq e^{\mathbf{E}\left[\ln\left(X\right)\right]} \\ &\mathbf{E}\left[X\right] \geq \ln\left(\mathbf{E}\left[e^{X}\right]\right) \end{split}$$