Recursion Code.

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Abstract

None

1 Construction.

Definition 1. *Let Δ be an integer greater than 2 and consider an algorithm $\mathcal{A}$ that for any n that is power of 3 construct a Δ-regular graph over n vertices. Now, let G be Δ-regular graph over n vertices generated by $\mathcal{A}$. Define the* ***third graph obtained by*** *G , labeled by $G^{\sim}$ to be the graph which $\mathcal{A}$ returns over $\frac{1}{3}n$ such that any of the edges could be associate by puncturing a $\frac{2}{3}$ fraction of the edges of each vertex.*

Definition 2. *Let G be a Δ-regular graph, such that each edge is associated with integer in $[\frac{1}{2}\Delta]$ and no vertex adjoins to two different indexed edges. For example consider a Cayley graph defined by $\frac{1}{2}\Delta$ generators $\{g_0, g_1, g_2, \cdots g_{\frac{1}{2}\Delta}\}$, then the undirected graph is Δ-regular and any edge could be labeled by the function $f\left(g_i, g_i^{-1}\right) \rightarrow i$.*

*Define the $[a,b]$**-fraction graph obtained by** G , labeled by $G^{[a,b]}$ to be the graph which obtained taken only the edges such their label's are in the range $[a,b]$.*

For convenient, we will denote by $G^{\frac{1}{3}}, G^{\frac{2}{3}+}$ and $G^{\frac{2}{3}-}$ the fraction graph correspond to taking the middle third edges and the higher and the lower 2-third edges.

Definition 3 (Recursion Code)**.** *Let $C_0 = \Delta[1, \rho_0, \delta_0]$ be a binary linear code. We will define the recursion code in recursive manner. First for a sufficiently large integer n_0, which is also power of 3, $C\left(n_0\right)$ defined to be the Tanner code defined by the C_0 and graph $\mathcal{A}\left(n_0\right)$. Then let n be any power of 3, such that $n > n_0$, denote by G the graph that constructed by the running of $\mathcal{A}(n)$. Then let $C\left(n\right)$ be the code obtained by the joining the parity check matrix of the Tanner code $\mathcal{T}\left(G, C_0\right)$ and by the checks of the $C\left(n/3\right)$ over the bits associated with the $G^{\sim}$. We will call to that code family the* ***recursion code****.*

Lemma 1. *If $\rho_0 > \frac{2}{3}$, then the recursion code has a positive rate.*

Proof. By counting the restrictions we have that:

$$H\left(n\right) = \Delta n \left(1 - \rho_0\right) + H\left(n/3\right) \leq \frac{3}{2}\Delta\left(1 - \rho_0\right)\Delta n$$

So we dimension of the code is at least $\frac{1}{2}\Delta n - H\left(n\right)$ which is

$$\frac{1}{2}n\Delta - \frac{3}{2}\Delta\left(1 - \rho_0\right)\Delta n = \frac{1}{2}\Delta n\left(3\rho_0 - 2\right)$$

So for any $\rho_0 > \frac{2}{3}$ we have that the rate of the C_n is grater than constant. □

Recursion Decoder. balabla

1 Decode $G^{\frac{2}{3}+}$
2 Decode $G^{\frac{2}{3}-}$
3 Decode $G^{\frac{1}{3}}$