# Fourmlas Sheet.

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## Probability.

**Multiplicative Chernoff bound.** Suppose $X_1, ..., X_n$ are independence random variables taking values in $\{0,1\}$ Let $X$ denote their sum and let $\mu = \mathbf{E}\left[\sum_i^n X_i\right]$ denote the sum's expected value. Then for any $\delta > 0$:

$$\begin{aligned}
&\mathbf{Pr}\left[X \geq (1+\delta)\mu\right] \leq e^{-2\frac{\delta^2\mu^2}{n}} \\
&\mathbf{Pr}\left[|X-\mu| \geq \delta\mu\right] \leq 2e^{-\delta^2\mu/3}, \qquad 0 \leq \delta \leq 1
\end{aligned}$$

**Jensen's inequality.** If $X$ is a random variable and $\phi$ is a convex function, then:

$$\begin{aligned}
\phi\left(\mathbf{E}[X]\right) \leq \mathbf{E}\left[\phi(X)\right] \Rightarrow &\,\mathbf{E}[X] \leq \phi^{-1}\left(\mathbf{E}\left[\phi(X)\right]\right) \\
&\,\mathbf{E}[X] \leq \ln\left(\mathbf{E}\left[e^X\right]\right) \\
&\,\mathbf{E}[X] \geq e^{\mathbf{E}[\ln(X)]}
\end{aligned}$$