State Synthesis Using PRS.

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Abstract

We studies the complexity of synthesis quantum states using PRS, our reasch continues the work by [Ira+22], [Ros23], [RY21], [MY23], [Del+23].

Claim 0.1. *Let G be a PRS generator, than one can assume that G takes as input two register, the first contains n ancille qubits initiliazied to $|0\rangle$ and the seconed contain a classic string initiliezed to be the seed k.*

Claim 0.2. *Let $G : |0\rangle^n \otimes \mathbb{F}_2^k \rightarrow \{|\psi_k\rangle\}_{k\in\mathcal{K}}$ be a PRS generator uses n- ancilles and k classicl bits. Then for any unitery $V : \mathcal{H}_n \rightarrow \mathcal{H}_n$ it holds that $(V \otimes I^{\otimes k})G$ is also a PRS.*

Proof. □

Claim 0.3 (Levis Lemma for PRS)**.** *Let $f : \mathcal{H} \rightarrow R$ be a **BQP**-computible fuction on the n-qubits hilbert space, and let $g : (0, 1) \rightarrow \mathbb{R}$ a function such that:*

$$\mathbf{Pr}_{|\psi\rangle\sim U}\left[f\left(|\psi\rangle\right) > \varepsilon\right] < g(\varepsilon)$$

Then, a similar inequality also holds for states sampled by the PRS, when the probability for the measure f-value grater than ε is bounded by $g(2\varepsilon)$. Namely,

$$\mathbf{Pr}_{|\psi\rangle\sim|\psi_k\rangle}\left[f\left(|\psi\rangle\right) > \varepsilon\right] < g(2\varepsilon)$$

In praticular, Levi's lemma has a version that capture consetration of states sampled by PRS generator, states the following: Assume there exsists K such that for any $|\psi\rangle, |\phi\rangle \in \mathcal{S}(\mathbb{C}^d)$ $|f(|\psi\rangle) - f(|\phi\rangle)| < K|\,|\psi\rangle - |\phi\rangle\,|$. Then there exsists a universal constant $C > 0$ such:

$$\mathbf{Pr}_{|\psi\rangle\sim|\psi_k\rangle}\left[\left|f\left(|\psi\rangle\right) - \mathbf{E}_{|\phi\rangle\sim U}\left[f\left(|\phi\rangle\right)\right]\right| > \varepsilon\right] < exp\left(-\frac{Cd}{K^2}4\varepsilon^2\right)$$

Proof. □

Claim 0.4. *Probablisitc counting argument and ε-net over PRS.*

Claim 0.5. *exsistness of poly(n) gates $G_1, G_2..$ such that, any G_i has a polynomial depth, $\langle p(G_i)|\tau\rangle > a$ and $\langle\tau^\perp|p(G_j)\rangle\,\langle p(G_i)|\tau^\perp\rangle < b$ for any $i \neq j$.*

Proof. □

Claim 0.6. *bla bla bla*

References

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