

# Fourmlas Sheet.

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January 21, 2023

## Probability.

**Multiplicative Chernoff bound.** Suppose  $X_1, \dots, X_n$  are independence random variables taking values in  $\{0, 1\}$ . Let  $X$  denote their sum and let  $\mu = \mathbf{E}[\sum_i^n X_i]$  denote the sum's expected value. Then for any  $\delta > 0$ :

$$\Pr[X \geq (1 + \delta)\mu] \leq e^{-2\frac{\delta^2\mu^2}{n}}$$

$$\Pr[|X - \mu| \geq \delta\mu] \leq 2e^{-\delta^2\mu/3}, \quad 0 \leq \delta \leq 1$$

**Jensen's inequality.** If  $X$  is a random variable and  $\phi$  is a convex function, then:

$$\phi(\mathbf{E}[X]) \leq \mathbf{E}[\phi(X)] \Rightarrow \mathbf{E}[X] \leq \phi^{-1}(\mathbf{E}[\phi(X)])$$

$$\mathbf{E}[X] \leq e^{\mathbf{E}[\ln(X)]}$$

$$\mathbf{E}[X] \geq \ln(\mathbf{E}[e^X])$$

