Groverize Monotone Local Search. (Short Note)

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1 Todo.

- 1. Write the table (sage script).
- 2. Add definitions. Problem description.
- 3. Complete the 'proof'.
- 4. Prove lower bound.

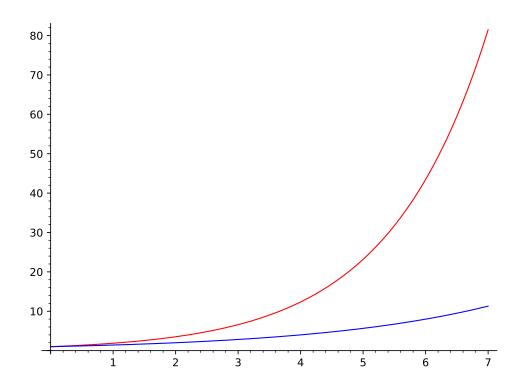
2 Introduction.

We follow the study of [Fom+15], who relate between the parametrized complexity to the general average case complexity. Crudely put, they shown that for particular wide range of **NP** hard problems, a solution which run exponentially at some complexity parameter, for example the treewidth of a graph, can be used to derive a batter than bruteforce solution for the general problem. We continue their work by plugin the Grover search [Gro96] routine instead the original sampling process.

$$\begin{split} & \sum_{k' \leq k} \frac{1}{\sqrt{p(k')}} \cdot c^{k'-t} N^{\mathcal{O}(1)} \leq \max_{k' \leq k} \left(\frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \right)^{\frac{1}{2}} \cdot c^{k'-t} N^{\mathcal{O}(1)} = \\ & \left(\max_{k' \leq k} \frac{\binom{n-|X|}{t}}{\binom{k'}{t}} \cdot c^{2\binom{k'-t}{t}} \right)^{\frac{1}{2}} N^{\mathcal{O}(1)} = \left(\max_{k \leq n-|X|} \frac{\binom{n-|X|}{t}}{\binom{k}{t}} \right)^{\frac{1}{2}} \cdot c^{2(k-t)} N^{\mathcal{O}(1)} \leq \\ & \Rightarrow \left(2 - \frac{1}{c^2} \right)^{\frac{n-|X|}{2}} N^{\mathcal{O}(1)} \end{split}$$

Problem Name	Parameterized	Groverize	New bound	Previous Bound
FEEDBACK VERTEX SET FEEDBACK VERTEX SET SUBSET FEEDBACK VERTEX SET	3^{k} (r) [Cyg+11] 3.592^{k} [KP14] 4^{k} [Wahlstrom14]		1.6667^{n} (r) 1.7217^{n} 1.7500^{n}	1.7347 ⁿ [FTV15] 1.8638 ⁿ [Fom+14]

Table 1: Summary of known and new results for different optimization problems. NPR means that we are not aware of any previous algorithms faster than brute-force. All bounds suppress factors polynomial in the input size N. The algorithms in the first row are randomized (r).



References

- [Gro96] Lov K. Grover. A fast quantum mechanical algorithm for database search. 1996. arXiv: quant-ph/9605043 [quant-ph].
- [Cyg+11] Marek Cygan et al. "Solving connectivity problems parameterized by treewidth in single exponential time (extended abstract)". In: 2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011. IEEE Computer Soc., Los Alamitos, CA, 2011, pp. 150–159. DOI: 10.1109/FOCS.2011.23. URL: https://doi.org/10.1109/FOCS.2011.23.
- [Fom+14] Fedor V. Fomin et al. "Enumerating minimal subset feedback vertex sets". In: Algorithmica 69.1 (2014), pp. 216–231. ISSN: 0178-4617. DOI: 10.1007/s00453-012-9731-6. URL: https://doi.org/10.1007/s00453-012-9731-6.
- [KP14] Tomasz Kociumaka and Marcin Pilipczuk. "Faster deterministic Feedback Vertex Set".
 In: Inform. Process. Lett. 114.10 (2014), pp. 556–560. ISSN: 0020-0190. DOI: 10.1016/j.ipl.2014.05.001. URL: https://doi.org/10.1016/j.ipl.2014.05.001.
- [FTV15] Fedor V. Fomin, Ioan Todinca, and Yngve Villanger. "Large induced subgraphs via triangulations and CMSO". In: SIAM J. Comput. 44.1 (2015), pp. 54–87. ISSN: 0097-5397. DOI: 10.1137/140964801. URL: https://doi.org/10.1137/140964801.

 $[Fom+15] \quad \text{Fedor V. Fomin et al. } \textit{Exact Algorithms via Monotone Local Search. 2015. arXiv: 1512. } \\ 01621 \ [\text{cs.DS}].$