# Chapter 11

# Minimum Spanning Tree Recitation.

## 11.1 The Spanning Tree Problem.

**Definition 11.1.1.** A spanning tree $T$ of a graph $G = (V, E)$ is a subset of edges in $E$ such that $T$ is a tree (having no cycles), and the graph $(V, T)$ is connected.

*Problem* 11.1.1 (MST). Let $G = (V, E)$ be a weighted graph with weight function $w : E \to \mathbb{R}$. We extend the weight function to subsets of $E$ by defining the weight of $X \subset E$ to be $w(X) = \sum_{e \in X} w(e)$. The minimum spanning tree (MST) of $G$ is the spanning tree of $G$ that has the minimal weight according to $w$. Note that in general, there might be more than one MST for $G$.

**Definition 11.1.2.** Let $U \subset V$. We define the cut associated with $U$ as the set of outer edges of $U$, namely all the edges $(u, v) \in E$ such that $u \in U$ and $v \notin U$. We use the notation $X = (U, \bar{U})$ to represent the cut. We say that $E' \subset E$ respects the cut if $E' \cap X = \emptyset$.

**Lemma 11.1.1** (The Cut-Lemma). *Let $T$ be an MST of $G$. Consider a forest $F \subset T$ and a cut $X$ that respects $X$ (i.e. $F \cap X = \emptyset$). Then $F \cup \ arg\min_e w(e)$ is also contained in some MST. Note that it does not necessarily have to be the same tree $T$.*

In the lecture you saw the Kruskal algorithm for finding an MST.

**Result:** Returns MST of given S $G = (V, E, w)$

sorts the $E$ according to $w$
define $F_0 = \emptyset$ and $i \leftarrow 0$
**for** $e \in E$ *in sorted order* **do**
    **if** $F_i \cup \{e\}$ *has no cycle* **then**
        $F_{i+1} \leftarrow F_i \cup \{e\}$
        $i \leftarrow i + 1$
    **end**
**end**
**return** $F_i$

**Algorithm 1:** Kruskal alg.