Heaps - Recitation 4

Correctness, Loop Invariants And Heaps.

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Apart of quantify the resource requirement of our algorithms we are also intersted to prove that they indeed work. In this Recitation we will demonstrate how to prove correctness via the notation of loop invariant. In addition we will present the first (non-trival) data structre in course, and prove that it allow us to compute the maximum efficintly.

Correctness And Loop Invariant.

In this course, any algorithm is defined relative to a task/problem/function, And it will be said correct if for any input it compute diserable output. For example, suppose that our task is to extract the maximum element from a given array. So the input space are all the arrays of numbers, and proving that a given algorithm is correct, requires to prove that for an aribtrary array the algorithm's out put is the the maximal number. Formally:

Correctness.

We will say that an algorithm $\mathcal{A}$ (an ordered set of operations) computes $f : D_1 \to D_2$ if for every $x \in D_1 \Rightarrow f(x) = \mathcal{A}(x)$. Sometimes when it's obvious what is the goal function f, we will abbreviate and say that $\mathcal{A}$ is correct.

Other Examples of functions f might be including any compution taks: file saving, summing numbers, posting a message in the forum, etc. Let's dive back into the maximum extraction problem and see how correctenss should be prove in practice.

Task: Maximum Finding. *Given $n \in \mathbb{N}$ numbers $a_1, a_2, \cdots a_n \in \mathbb{R}$ write an Algorithm which returns their maximum.*

Consider the follows suggestion. How would you prove it correct?

Maximum finding.

Result: returns the maximum of $a_1 ... a_n \in \mathbb{R}^n$

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1 let b ← a_1
2 for i ∈ [2, n] do
3     b ← max(b, a_i)
4 return b
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Usually it will be convinant to divide the algortims into subsections and then characteristic, and prove correctness for each of them seapratly. One main technique is usning the notation of Loop Invariants. Loop Invariant is a property that characteristic a loop segment code and satisfy the following conditions: