IDL Exercise 2.

David Ponarovsky

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1 Theoretical Questions:

1.1 LTI.

Show that a convolution with respect to any filter h is time/space invariant.

Solution. Let's denote by $\mathcal{L} : \text{funcs} \to funcs$ the convolution by the filter h, namely $\mathcal{L}[f] = f * h = \sum_x h(x)f(y-x)$. To show that $\mathcal{L}$ is an LTI operation we have to show that $\mathcal{L}[f(x-t)](y) = \mathcal{L}[f](y+t)$. (Notes that the linearity we got for free by the linearity of convolution, So only the time invarent part is left). So:

$$\begin{aligned}\mathcal{L}[f(x-t)](y) = f(x-t) * h = \sum_x h(x)f(\overbrace{y-t}^{\tau}-x) = \sum_x h(x)f(\tau - x) \\ = \mathcal{L}[f](\tau) = \mathcal{L}[f](y+t)\end{aligned}$$

1.2 TI.

Explain whether each of the following layers are time/space invariant or not:

1. Additive constant
2. Pointwise nonlinearity (such as ReLU)
3. Strided pooling by a factor > 1
4. As a result, is a CNN composed of all these operators (+convolution) time invariant?

1.3 Layers' Jacobian.

Calculate the Jacobian matrix of the following layers:

1. Additive bias vector
2. General Matrix multiplication
3. Convolution layer