IDL Exercise 2.

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1 Theoretical Questions:

1.1 LTI.

Show that a convolution with respect to any filter h is time/space invariant.

Solution. Let's denote by $\mathcal{L}$: funcs $\rightarrow funcs$ the convolution by the filter h, namely $\mathcal{L}[f] = f * h = \sum_x h(x)f(y-x)$. To show that $\mathcal{L}$ is an LTI operation we have to show that $\mathcal{L}[f(x-t)](y) = \mathcal{L}[f](y-t)$. (Notes that the linearity we got for free by the linearity of convolution, So only the time invarent part is left). So:

$$\begin{aligned}\mathcal{L}[f(x-t)](y) = f(x-t) * h = \sum_x h(x)f(\overbrace{y-t}^{\tau}-x) = \sum_x h(x)f(\tau - x) \\ = \mathcal{L}[f](\tau) = \mathcal{L}[f](y-t)\end{aligned}$$

And we got that the convolution is an LTI operation.

1.2 TI.

Explain whether each of the following layers are time/space invariant or not:

1. Additive constant.

   Solution. TRUE.

   $$\mathcal{L}[f] = f(x) + c \Rightarrow \mathcal{L}[f(x-t)] = f(x-t) + c = \mathcal{L}[f](x-t)$$

2. Pointwise nonlinearity (such as ReLU) **Solution. FALSE.** Denote $\mathcal{L}[f] = Relu(f(x))$, now consider the ReLU action over the linear function $f(x) = x$.

   $$\Rightarrow \mathcal{L}[f(x-t)] = Relu(f(x-t)) = \begin{cases} 0 & \text{if} x < t \\ x-t & \text{else} \end{cases} \neq \mathcal{L}[f](x-t)$$

3. Strided pooling by a factor > 1 **Solution.**

4. As a result, is a CNN composed of all these operators (+convolution) time invariant? **Solution.**

1.3 Layers' Jacobian.

Calculate the Jacobian matrix of the following layers:

1. Additive bias vector

2. General Matrix multiplication

3. Convolution layer