

Quantum Information Theory - 67749
Guided Exercise on Recitation, June 12, 2025

1 CSS codes.

1. Prove that the relation $C_X \subset C_Z^\perp$ implies $H_Z H_X^\top = 0$, where H_Z and H_X are the parity check matrices of the codes C_X, C_Z .

[**Solution.**] H_X^\top is the generator matrix of the subspace spanned by its columns (True for any matrix), namely by H_X rows, which, by definition, are all the vectors perpendicular to codewords in C_X . Thus, H_X^\top is the generator matrix for the code C_X^\perp . Since $C_X^\perp \subset C_Z$, we get the relation $H_Z H_X^\top = 0$.

2. Prove that it cannot hold that both of the codes C_X, C_Z are LDPC codes with non-constant distance, and that they compose a CSS code.

[**Solution.**] By the relation $H_Z H_X^\top = 0$, we have that any check of H_X is a codeword of C_Z , so requiring that C_X is an LDPC code implies that C_Z has codewords at weight $O(1)$.

3. Take a minute to think about the result above.