Quantum Information Theory - 67749 Guided Exercise on Recitation, June 12, 2025

1 CSS codes.

1. Prove that the relation $C_X \subset C_Z^{\perp}$ implies $H_Z H_X^{\perp} = 0$, where H_Z and H_X are the parity check matrices of the codes C_X, C_Z .

[Solution.] H_X^{\top} is the generator matrix of the subspace spanned by its columns (True for any matrix), namely by H_X rows, which, by definition, are all the vectors perpendicular to codewords in C_X . Thus, H_X^{\top} is the generator matrix for the code C_X^{\perp} . Since $C_X^{\perp} \subset C_Z$, we get the relation $H_Z H_X^{\top} = 0$.

2. Prove that it cannot hold that both of the codes C_X , C_Z are LDPC codes with non-constant distance, and that they compose a CSS code.

[Solution.] By the relation $H_Z H_X^{\top} = 0$, we have that any check of H_X is a codeword of C_Z , so requiring that C_X is an LDPC code implies that C_Z has codewords at weight O(1).