*Submission Guidelines.
Due date - [21].
Make sure your submission is clear. Unreadable assignments will get zero score.
Using any Generative AI tool is forbidden but not enforced. Yet, please keep in mind that we might call you for an inte

Entropy.

Information Quantties Properties.

Entropy Upper Bound.

Show that the classical entropy is bounded from above by the logarithm of the dimension of the alphabet. Use the non-

Similarly to the previews section, bound the von Neumann entropy using the non-negativity of the divergence.

Mutual Information Upper Bound. Let d_A, d_B be the dimensions of systems $\mathcal{H}_A, \mathcal{H}_B$. Prove that $S(A;B) \leq 2\min$

Mutual Information Chain Rule. Prove the chain rule for the mutual infomration: S(AB;C) = S(A;B) + S(B;C—

Entropy of Reduced Entangled State. Consider the sate ρ over the system $\mathcal{H}_A \otimes \mathcal{H}_B$.

Prove that if $S(A) \geq S(AB)$ then ρ is entangled.

Show, that the condition is not necessary, that is, give an example of an entangled state for which the reduced state ρ_A

Conditional Mutual Information. consider a joint system ABX where X is classical, show that the conditional mut

Classical-Quantum States. Recall the monotonicity of entropy with respect to revealing classical information.

Claim 1 *Let X be a classical random variable, and let $\mathcal{H}_B$ be a subsystem for which, upon X's value, the state ρ_x is ind*

with equality if and only if the states ρ_x are orthogonal, i.e., they have supports on orthogonal spaces.

In the lecture, we saw the equality when the quantum states induced on B have orthogonal supports. Prove the inequal

Computing Entropy. Compute the entropy of the following densitiy matrices.

Let V be a subset of 2^n. Denote by V the uniform superposition over V, defined as $V = \sum_{v \in V} v$ (up to normalization).

Consider the fully entangled state $\Omega = \sum_x x, x$ (up to normalization) over the system $\mathcal{H}_A \otimes \mathcal{H}_B$. Compute the entropy

Fidelity. Compute the fidelity between ρ and σ in the following cases:

When ρ and σ commute.

When ρ is mixed and σ is pure.

When $\rho = \alpha I + \beta X$ and $\sigma = \gamma I + \delta Z$.

Remark 1 *Observes that the normalization condition* $\mathbf{Tr}\rho = 1$ *implies* $\alpha = \gamma = 1$. *Yet, for the sake of practice, we will k*

Schmedit. Prove that a pure state is entangled if and only if its Schmidt number is greater than one.