

Quantum Information Theory - 67749  
Guided Exercise on Recitation, June 12, 2025

## 1 CSS codes.

1. Prove that the relation  $C_X \subset C_Z^\perp$  implies  $H_Z H_X^\top = 0$ , where  $H_Z$  and  $H_X$  are the parity check matrices of the codes  $C_X, C_Z$ .

**[Solution.]**  $H_X^\top$  is the generator matrix of the subspace spanned by its columns, namely by  $H_X$  rows, which, by definition, are all the vectors perpendicular to codewords in  $C_X$ . Thus,  $H_X^\top$  is the generator matrix for the code  $C_X^\perp$ . Since  $C_X^\perp \subset C_Z$ , we get the relation  $H_Z H_X^\top = 0$ .