Final Recitation – Information Theory, Application.

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Introduction

- ► Brief overview of the topic
- ► Importance and relevance
- Objectives of the presentation

Key Points

- ▶ Main point 1
- ► Main point 2
- ► Main point 3

Claim

Let Y be a bit given by moving X trough BSC(p). Then:

$$1 - H(Y) \le (1 - p^2) (1 - H(X))$$

Denote by δ the parameter for which X distributed as $\sim Bin(\frac{1+\delta}{2})$. First notice that:

$$\Pr(Y = 1) = \frac{1+\delta}{2}p + \frac{1-\delta}{2}(1-p) = \frac{1+\delta p}{2}$$

Claim

Let Y be a bit given by moving X trough BSC(p). Then:

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Claim

Let $Y = (Y_1, Y_2, ..., Y_m)$ be a bit given by moving each of $X_i \in X = (X_1, X_2, ..., X_m)$ trough BSC(p). Then:

$$m - H(Y) \le (1 - p^2) (m - H(X))$$

$$m - H(Y_1, Y_2, ..., Y_m) = m - \sum_{i} H(Y_i | Y_1, Y_2, ..., Y_{i-1})$$

$$\leq m - \sum_{i} H(Y_i | X_1, X_2, ..., X_{i-1})$$

$$\leq \sum_{i} 1 - H(Y_i | X_1, X_2, ..., X_{i-1})$$

$$\leq \sum_{i} (1 - p^2) (1 - H(X_i | X_1, X_2, ..., X_{i-1}))$$

$$\leq (1 - p^2) \sum_{i} (1 - H(X_i | X_1, X_2, ..., X_{i-1}))$$

$$= (1 - p^2) (m - H(X))$$