

Final Recitation – Information Theory, Application.

David Ponarovsky

June 19, 2025

Introduction

- ▶ Brief overview of the topic
- ▶ Importance and relevance
- ▶ Objectives of the presentation

Key Points

- ▶ Main point 1
- ▶ Main point 2
- ▶ Main point 3

Your Title Here

Claim

Let Y be a bit given by moving X through $\text{BSC}(p)$. Then:

$$1 - H(Y) \leq (1 - p^2) (1 - H(X))$$

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Denote by δ the parameter for which X distributed as $\sim \text{Bin}(\frac{1+\delta}{2})$.
First notice that:

$$\Pr(Y = 1) = \frac{1+\delta}{2}(1-p) + \frac{1-\delta}{2}p = \frac{1-2\delta p}{2}$$

So $Y \sim \text{Bin}(\frac{1-2\delta p}{2})$, Or $\delta \mapsto -2p\delta$.

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Now expand $1 - H(X)$ to it's Taylor Seryias at δ gives:

$$\begin{aligned} 1 - H(X) &= 1 - \frac{1}{2} \left((1 + \delta) \log \left(\frac{1 + \delta}{2} \right) + (1 - \delta) \log \left(\frac{1 - \delta}{2} \right) \right) \\ &= -\frac{1}{2} \left((1 + \delta) \log \left(\frac{1 + \delta}{2} \right) + (1 - \delta) \log \left(\frac{1 - \delta}{2} \right) \right) \end{aligned}$$

Denote the above by $K(\delta)$

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Now, observes that:

$$\begin{aligned} 1 - H(Y) &= K(2p\delta) = \sum_{i=1}^{\infty} \frac{(2p\delta)^{2n}}{2n(2n-1)} \\ &\leq 2p^2 K(\delta) = 2p^2(1 - H(X)) \end{aligned}$$

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Claim

Let $Y = (Y_1, Y_2, \dots, Y_m)$ be a bit given by moving each of $X_i \in X = (X_1, X_2, \dots, X_m)$ through $\text{BSC}(p)$. Then:

$$m - H(Y) \leq (1 - p^2) (m - H(X))$$

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$$\begin{aligned} m - H(Y_1, Y_2, \dots, Y_m) &= m - \sum_i H(Y_i | Y_1, Y_2, \dots, Y_{i-1}) \\ &\leq m - \sum_i H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i 1 - H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i (1 - p^2) (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &\leq (1 - p^2) \sum_i (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &= (1 - p^2) (m - H(X)) \end{aligned}$$

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