

<p>Quantum Information Theory - 67749</p> <p>Exercise 2, May 20, 2025</p>

1 Submission Guidelines.

- Due date - May 31, 2025.
- Make sure your submission is clear. Unreadable assignments will get zero score.
- Using any Generative AI tool is forbidden but not enforced. Yet, please keep in mind that we might call you for an interview about your assignment.

2 AEP.

Recall the converse part of the classical AEP lemma¹:

Lemma 2.1. *Let P_X be a pmf on \mathcal{X} and let $\mathcal{B}^{(n)}$ be a set in \mathcal{X}^n of size at most $2^{n\alpha}$. Then for any $\varepsilon > 0$ and n large enough.*

$$\Pr[X^n \in \mathcal{B}^{(n)}] = P_X^{\otimes n}(\mathcal{B}^{(n)}) \leq \varepsilon + 2^{n(\alpha + \varepsilon - H(X))}$$

1. Rewrite the classical AEP using the density matrix notation, namely point out each entity defined in the above statement and link it to its analogy in the QAEF lemma. [Hint²]

3 Entropy.

3.1 Entropy Upper Bound.

1. Show that entropy is bounded from above by log alphabet size .
- 2.

$$\begin{aligned} 0 \leq S\left(\rho^A \left\| \frac{I}{d}\right.\right) &= -S(\rho^A) - \text{tr}\left(\rho^A \log \frac{I}{d}\right) \\ &= -S(\rho^A) - \log \frac{1}{d} \cdot \text{tr}(\rho^A) - \underbrace{\text{tr}(\rho^A \log I)}_{=0} = -S(\rho^A) + \log d, \end{aligned}$$

where in the last equality we used the fact that $\text{tr}(\rho^A) = 1$ for any density matrix

3.2 Computing Entropy.

Compute the entropy of the following density matrices.

1. The super position over linear subspace. (purestate).
2. The uniform distribution over linear subspace.

4 Fidelity.

Compute the fidelity between ρ and σ in the following cases:

1. When ρ and σ commute.
2. When ρ is mixed and σ is pure.

¹Taken from Prof. Ordentlich

²A good answer would be a single paragraph long.

3. When $\rho = \frac{1}{2} (I + X)$ and $\sigma = \alpha I + \beta Z$.
4. Use the construction presented in the proof of Uhlman's to calculate the fidelity between: $\rho = p|0\rangle\langle 0| + (1-p)|+\rangle\langle +|$ and $\sigma = (1-p)|0\rangle\langle 0| + p|+\rangle\langle +|$.

5 Coding Theorem.

Let ρ be the density matrix: $p|\beta_{00}\rangle\langle\beta_{00}| + \frac{1}{3}(1-p)\sum_{i\neq 00}|\beta_{ij}\rangle\langle\beta_{ij}|$.

6 Quantum Teleportation.

Give a quantum circuit that compute the single qubit gate $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{8}} \end{bmatrix}$ up to global phase using only pauli, clifford, and measurements.