# Quantum Information Theory - 67749
# Guided Exercise on Recitation, June 12, 2025

## 1 CSS codes.

1. Prove that the relation $C_X \subset C_Z^{\perp}$ implies $H_Z H_X^{\top} = 0$, where $H_Z$ and $H_X$ are the parity check matrices of the codes $C_X, C_Z$.

   **[Solution.]** $H_X^{\top}$ is the generator matrix of the subspace spanned by its columns (True for any matrix), namely by $H_X$ rows, which, by definition, are all the vectors perpendicular to codewords in $C_X$. Thus, $H_X^{\top}$ is the generator matrix for the code $C_X^{\perp}$. Since $C_X^{\perp} \subset C_Z$, we get the relation $H_Z H_X^{\top} = 0$.