

# Final Recitation – Information Theory, Application.

David Ponarovsky

June 19, 2025

# Introduction

- ▶ Brief overview of the topic
- ▶ Importance and relevance
- ▶ Objectives of the presentation

# Key Points

- ▶ Main point 1
- ▶ Main point 2
- ▶ Main point 3

# Your Title Here

## Claim

Let  $Y$  be a bit given by moving  $X$  through  $\text{BSC}(p)$ . Then:

$$1 - H(Y) \leq (1 - p^2) (1 - H(X))$$

# Your Title Here

Denote by  $\delta$  the parameter for which  $X$  distributed as  $\sim \text{Bin}(\frac{1+\delta}{2})$ .  
First notice that:

$$\Pr(Y = 1) = \frac{1+\delta}{2}(1-p) + \frac{1-\delta}{2}p = \frac{1-2\delta p}{2}$$

So  $Y \sim \text{Bin}(\frac{1-2\delta p}{2})$ , Or  $\delta \mapsto -2p\delta$ .

# Your Title Here

Now expand  $1 - H(X)$  to it's Taylor Seryias at  $\delta$  gives:

Denote the above by  $K(\delta)$

# Your Title Here

Now, observes that:

$$\begin{aligned} 1 - H(Y) &= K(2p\delta) = \sum_{i=1}^{\infty} \frac{(2p\delta)^{2n}}{2n(2n-1)} \\ &\leq 2p^2 K(\delta) = 2p^2(1 - H(X)) \end{aligned}$$

# Your Title Here

## Claim

Let  $Y = (Y_1, Y_2, \dots, Y_m)$  be a bit given by moving each of  $X_i \in X = (X_1, X_2, \dots, X_m)$  through  $\text{BSC}(p)$ . Then:

$$m - H(Y) \leq (1 - p^2) (m - H(X))$$



# Your Title Here

$$\begin{aligned} m - H(Y_1, Y_2, \dots, Y_m) &= m - \sum_i H(Y_i | Y_1, Y_2, \dots, Y_{i-1}) \\ &\leq m - \sum_i H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i 1 - H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i (1 - p^2) (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &\leq (1 - p^2) \sum_i (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &= (1 - p^2) (m - H(X)) \end{aligned}$$

# Your Title Here

# Your Title Here

# Your Title Here