## Quantum Information Theory - 67749
## Exercise 2, May 18, 2025

# 1 Submission Guidelines.

- Due date - [11]May 18, 2025.

- Make sure your submission is clear. Unreadable assignments will get zero score.

- Using any Generative AI tool is forbidden but not enforced. Yet, please keep in mind that we might call you for an interview about your assignment.

# 2  AEP.

Recall the converse part of the classical AEP lemma:

**Lemma 2.1.** *Let $P_X$ be a pmf on $\mathcal{X}$ and let $\mathcal{B}^{(n)}$ be a set in $\mathcal{X}^n$ of size at most $2^{n\alpha}$. Then for any $\varepsilon > 0$ and $n$ large enough.*

$$\mathbf{Pr}\left[X^n \in \mathcal{B}^{(n)}\right] = P_X^{\otimes n}\left(\mathcal{B}^{(n)}\right) \leq \varepsilon + 2^{n(\alpha + \varepsilon - H(X))}$$

1. Reformaulate the classical AEP using the density matrices notation.

# 3  Fidelity.

1. When $\rho$ and $\sigma$ commute.

2. Betweem the mixed state and any pure state.

3. Use the construction presented in the proof of Uhlman's to calculate the fidelity between: $p\,|0\rangle\,\langle 0| + (1-p)\,|+\rangle\,\langle +|$ and $(1-p)\,|0\rangle\,\langle 0| + p\,|+\rangle\,\langle +|$.

# 4  Entropy.

Compute the entropy of the following densitiy matrices.

1. The super position over linear subspace. (purestate).

2. The uniform distribution over linear subspace.

# 5  Coding Theorem.

Let $\rho$ be the density matrix: $p\,|\beta_{00}\rangle\,\langle\beta_{00}| + \frac{1}{3}(1-p)\sum_{i\neq 00}|\beta_{ij}\rangle\,\langle\beta_{ij}|$.

# 6  Quantum Teleportation.

Give a quantum circtus that compute the gate $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{8}} \end{bmatrix}$ up to global phase using only pauli, clifford, and measurments.