Final Recitation – Information Theory, Application.

David Ponarovsky

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Introduction

- Brief overview of the topic
- Importance and relevance
- Objectives of the presentation

Key Points

- Main point 1
- Main point 2
- Main point 3

Your Title Here

Claim

Let Y be a bit given by moving X trough $\mathrm{BSC}(p)$. Then:

$$1 - H(Y) \leq \left(1 - p^2\right)(1 - H(X))$$

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Denote by δ the parameter for which X distributed as $\sim \mathrm{Bin}(\frac{1+\delta}{2})$. First notice that:

$$\mathbf{Pr}\left(Y = 1\right) = \frac{1+\delta}{2}(1-p) + \frac{1-\delta}{2}p = \frac{1-2\delta p}{2}$$

So $Y \sim \mathrm{Bin}(\frac{1-2\delta p}{2})$, Or $\delta \mapsto -2p\delta$.

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Now expand $1 - H(X)$ to it's Taylor Seryias at δ gives:

$$
\begin{aligned}
1 - H(X) &= 1 - \frac{1}{2}\left((1+\delta)\log\left(\frac{1+\delta}{2}\right) + (1-\delta)\log\left(\frac{1-\delta}{2}\right)\right) \\
&= -\frac{1}{2}\left((1+\delta)\log\left(\frac{1+\delta}{2}\right) + (1-\delta)\log\left(\frac{1-\delta}{2}\right)\right) \\
&= -\frac{1}{2} \cdot (1+\delta)\sum_{i=1}^{\infty}\frac{(-1)^{n+1}\delta^n}{n} + (1-\delta)\sum_{i=1}^{\infty}\frac{(-1)^{n+1}(-\delta)^n}{n}
\end{aligned}
$$

Denote the above by $K(\delta)$

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Now, observes that:

$$1 - H(Y) = K(2p\delta) = \sum_{i=1}^{\infty} \frac{(2p\delta)^{2n}}{2n(2n-1)}$$

$$\leq 2p^2 K(\delta) = 2p^2(1 - H(X))$$

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Claim

Let $Y = (Y_1, Y_2, .., Y_m)$ be a bit given by moving each of $X_i \in X = (X_1, X_2, .., X_m)$ trough BSC(p). Then:

$$m - H(Y) \leq \left(1 - p^2\right)\left(m - H(X)\right)$$

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$$
\begin{aligned}
m - H\left(Y_1, Y_2, .., Y_m\right) &= m - \sum_i H\left(Y_i \middle| Y_1, Y_2, .., Y_{i-1}\right) \\
&\leq m - \sum_i H\left(Y_i | X_1, X_2, .., X_{i-1}\right) \\
&\leq \sum_i 1 - H\left(Y_i | X_1, X_2, .., X_{i-1}\right) \\
&\leq \sum_i \left(1 - p^2\right)\left(1 - H\left(X_i | X_1, X_2, .., X_{i-1}\right)\right) \\
&\leq \left(1 - p^2\right) \sum_i \left(1 - H\left(X_i | X_1, X_2, .., X_{i-1}\right)\right) \\
&= \left(1 - p^2\right)\left(m - H\left(X\right)\right)
\end{aligned}
$$

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