Submission Guidelines.

Due date - [11].

Make sure your submission is clear. Unreadable assignments will get zero score.

Using any Generative AI tool is forbidden but not enforced. Yet, please keep in mind that we might call you for an inte

Tensors Products.

Show that $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ whenever the dimensions are compatible. Use the ket-bra notation. Then s

Let v_A, v_B be the vectors of eigenvalues of the matrices A, B respectively. Show that $v_A \otimes v_B$ is the vector of eigenvalue

Unambiguous Discrimination. Recall the distinguishing task, we are promised to be given either ψ_1 or ψ_2 and have

[**Not Mandatory**] In your words. Point out exactly what we skipped in the proof, and repeat the success probabilit

Write E_3 explicitly in the $\{\psi_1, \psi_1^\perp\}$ basis. The answer should depend on a and θ.

Show that $a = 1 + \cos\theta$ gives the optimal measurement.

Density Matrices.

Show that the density matrices of the mixed state resulting from picking uniformly random a quantum state from an or

Let ρ be the density matrix of the **EPR**. Divide the qubits into two local systems A and B.

Compute the mixed state over A given by first applying $THSZXSH$ on A and then tracing out B.

Diagonalize Noisy State. Let A_+ and A_- be two orthogonal pure states over single qubit, i.e $A_+|A_- = 0$ and let ρ

Solve it for the general case. (Hint: What does the orthogonality condition guarantee?).
[**Bonus:**] Let A_1, A_2, A_3, A_4 be four orthogonal states in $\mathcal{H}_4$. Suppose now that ρ is supported on two qubits. And fo

Quantum Circuits. Prove equivalence for the following circuit pairs.

H Z H = Z
X

H H = X
-1 X

[Not Mandatory] Measuring a Tensor Product.
Consider the task of distinguishing between the two quantum states

Given with symmetric prior probabilities. The goal is to distinguish between them with minimal error probability. This
Show that for $N = 1$, the Helstrom bound for the minimal error probability p_e (achieved by the optimal measurement)

Interpret the error probability p_e as the posterior probability. That is, after one optimal measurement, the posterior dis
Show that for $N = 1$ with a non-uniform prior $(q, 1 - q)$, the optimal error probability satisfies:

For $N = 2$, use parts (2) and (3) to show that performing the optimal measurement on the second copy—using the post

Compute the inner product $\psi_0\psi_0|\psi_1\psi_1$ in terms of $\psi_0|\psi_1$. Conclude from this that the two-stage measurement strategy
Extend the above reasoning by induction to any N, and describe the structure of the optimal measurement strategy.