

# Final Recitation – Information Theory, Application.

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# Introduction

- ▶ Brief overview of the topic
- ▶ Importance and relevance
- ▶ Objectives of the presentation

# Key Points

- ▶ Main point 1
- ▶ Main point 2
- ▶ Main point 3

# Your Title Here

## Claim

Let  $Y$  be a bit given by moving  $X$  through  $\text{BSC}(p)$ , Then there is  $\gamma_p < 1$  such :

$$1 - H(Y) \leq \gamma(1 - H(X))$$

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Denote by  $\delta$  the parameter for which  $X$  distributed as  $\sim \text{Bin}(\frac{1+\delta}{2})$ .  
First notice that:

$$\Pr(Y = 1) = \frac{1+\delta}{2}(1-p) + \frac{1-\delta}{2}p = \frac{1+\delta-2p\delta}{2}$$

So  $Y \sim \text{Bin}(\frac{1+\delta(1-2p)}{2})$ , Or  $\delta \mapsto 1-2p\delta$ .

# Your Title Here

Now expand  $1 - H(X)$  to it's Taylor Seryias at  $\delta$  gives:

$$\begin{aligned}1 - H(X) &= 1 - \frac{1}{2} \left( (1 + \delta) \log \left( \frac{1 + \delta}{2} \right) + (1 - \delta) \log \left( \frac{1 - \delta}{2} \right) \right) \\&= -\frac{1}{2} \left( (1 + \delta) \log \left( \frac{1 + \delta}{2} \right) + (1 - \delta) \log \left( \frac{1 - \delta}{2} \right) \right) \\&= -\frac{1}{2} \cdot (1 + \delta) \sum_{i=1}^{\infty} \frac{(-1)^{n+1} \delta^n}{n} + (1 - \delta) \sum_{i=1}^{\infty} \frac{(-1)^{n+1} (-\delta)^n}{n} \\&= -\frac{1}{2} \cdot \sum_{i=1}^{\infty} 2 \frac{\delta^{2n}}{2n} - \sum_{i=1}^{\infty} 2 \frac{\delta^{2n}}{2n - 1} \\&= \sum_{i=1}^{\infty} \frac{\delta^{2n}}{2n(2n - 1)}\end{aligned}$$

Denote the above by  $K(\delta)$

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Now, observe that:

$$\begin{aligned} 1 - H(Y) &= K(2p\delta) = \sum_{i=1}^{\infty} \frac{(2p\delta)^{2n}}{2n(2n-1)} \\ &\leq (1-2p)^2 K(\delta) = (1-2p)^2 (1 - H(X)) \end{aligned}$$

And notice that since  $p < 1$  we have  $\gamma < 1$ , notice also that inequality is symmetric to  $p \mapsto 1-p$ , in particular the entropy is not increase if either  $p = 0$  or  $p = 1$ .

# Your Title Here

## Claim

Let  $Y = (Y_1, Y_2, \dots, Y_m)$  be a bit given by moving each of  $X_i \in X = (X_1, X_2, \dots, X_m)$  through  $\text{BSC}(p)$ . Then:

$$m - H(Y) \leq (4p^2) (m - H(X))$$



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$$\begin{aligned} m - H(Y_1, Y_2, \dots, Y_m) &= m - \sum_i H(Y_i | Y_1, Y_2, \dots, Y_{i-1}) \\ &\leq m - \sum_i H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i 1 - H(Y_i | X_1, X_2, \dots, X_{i-1}) \\ &\leq \sum_i (1 - p^2) (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &\leq (1 - p^2) \sum_i (1 - H(X_i | X_1, X_2, \dots, X_{i-1})) \\ &= (1 - p^2) (m - H(X)) \end{aligned}$$

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