

<p>Quantum Information Theory - 67749</p> <p>Exercise 2, May 18, 2025</p>
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## 1 Submission Guidelines.

- Due date - May 29, 2025.
- Make sure your submission is clear. Unreadable assignments will get zero score.
- Using any Generative AI tool is forbidden but not enforced. Yet, please keep in mind that we might call you for an interview about your assignment.

## 2 AEP.

Recall the converse part of the classical AEP lemma:

**Lemma 2.1.** *Let  $P_X$  be a pmf on  $\mathcal{X}$  and let  $\mathcal{B}^{(n)}$  be a set in  $\mathcal{X}^n$  of size at most  $2^{n\alpha}$ . Then for any  $\varepsilon > 0$  and  $n$  large enough.*

$$\Pr [X^n \in \mathcal{B}^{(n)}] = P_X^{\otimes n}(\mathcal{B}^{(n)}) \leq \varepsilon + 2^{n(\alpha + \varepsilon - H(X))}$$

1. Reformulate the classical AEP using the density matrices notation.

## 3 Fidelity.

1. When  $\rho$  and  $\sigma$  commute.
2. Between the mixed state and any pure state.
3. Use the construction presented in the proof of Uhlman's to calculate the fidelity between:  $p|0\rangle\langle 0| + (1-p)|+\rangle\langle +|$  and  $(1-p)|0\rangle\langle 0| + p|+\rangle\langle +|$ .

## 4 Entropy.

Compute the entropy of the following density matrices.

1. The super position over linear subspace. (purestate).
2. The uniform distribution over linear subspace.

## 5 Coding Theorem.

Let  $\rho$  be the density matrix:  $p|\beta_{00}\rangle\langle\beta_{00}| + \frac{1}{3}(1-p)\sum_{i\neq 00}|\beta_{ij}\rangle\langle\beta_{ij}|$ .

## 6 Quantum Teleportation.

Give a quantum circuit that compute the gate  $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{8}} \end{bmatrix}$  up to global phase using only pauli, clifford, and measurements.