

Quantum Information Theory - 67749
Guided Exercise on Recitation, June 12, 2025

1 CSS codes.

1. Prove that the relation $C_X \subset C_Z^\perp$ implies $H_Z H_X^\top = 0$, where H_Z and H_X are the parity check matrices of the codes C_X, C_Z .

[Solution.] H_X^\top is the generator matrix of the subspace spanned by its columns (true for any matrix), namely by H_X rows, which, by definition, are all the vectors perpendicular to codewords in C_X . Thus, H_X^\top is the generator matrix for the code C_X^\perp . Since $C_X^\perp \subset C_Z$, we get the relation $H_Z H_X^\top = 0$.