The Behavior Of Wave Packet Propagation In 3D Space

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Abstract

We have studied the evolution of light waves while moving through a room air domain, first by measuring the typical dispersion constant to characterize the behavior of a single signal. Second, we propose an efficient experimental method to perform a multiple experiment in single shot.

Introduction Wave theory is the foundation for a broad spectrum of technologies that serve humanity every day. Codes, Error Correction, Distributed Computing, and Grid Computing are several examples of the long list of theories; such that their first assumption is that transferring a piece of information can be done to bounded accuracy.

This article focused on the dispersion due to geometric properties of the space. In the first section, we describe our experimental system, how exactly we have generated the signals and measured them. Then in the second section, we deal with the dispersion measuring, and finally, we propose a new technic to perform much clever measurement.

The solution of the ideal wave equation, prophets that signal lose its amplitude as $\sim \mathcal{O}\left(\frac{1}{r}\right)$; that result got by reduction from the radial wave equation into a single dimension.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\mathbf{r},t) = 0$$
$$\Rightarrow \psi(r,t) = \frac{1}{r}F(r-ct) + \frac{1}{r}G(r+ct)$$

However, as our wish to be self-contained (as possible), we will introduce an alternative way to calculate the excepted measured amplitude. Despite being simple, our way also considers the surface condition, e.g., yields a private solution that matches our system setting.

Figure 1 sets the structure of the system. Suppose that the source of the light rays is settled at A. Because the beams are blocked from "the left," we will think only on the half-sphere surface as legal positions. Then by spherically symmetric, we obtain that the flux over each surface segment is the same.

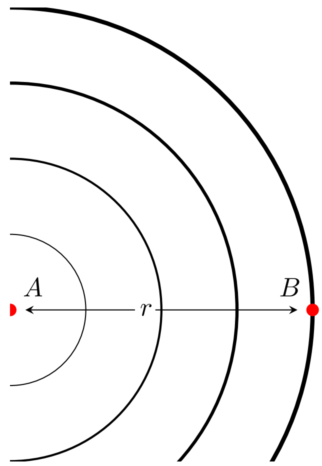

Figure 1: A is the source of the beams, B is the location of our sensor, r is the distance between them. In the first stage of the experiment, we sample the signal from variety ranges (r).

Denote, by r the distance of the packet from A, thus

$$\frac{1}{2}\cdot 4\pi r^2 \cdot |\psi(r)|^2 = Amp$$
$$|\psi(r)| = \frac{1}{\sqrt{2\pi}r}\sqrt{Amp}$$

Therefore, if we can determinate r to accuracy $r+\epsilon$ then we except to bound $\psi(r)$ by: $\frac{1}{\sqrt{2\pi}r}\sqrt{Amp}\left(1\pm\frac{\epsilon}{r}\right)$. Along the paper, we will refer to $\frac{1}{\sqrt{2\pi}}$ as the **Dispersion Constant**, measuring quantity in the error range, will considered as sufficient verification of the theory, while farther result implies incompleteness of the model.

Experiment We have used a standard led diode to output the light, and we have generate a cos signal in the time domain which translated into a $\delta(w)$ function in the frequency domain (e.g, hitting at constant frequency 1_{Hz}). Then we have repeated on the above, changing each time only the distance of the diode from the sensor. The distances were drawn from the following array $[0.35, 0.42, 0.65, 0.8, 0.9, 1.1]_m \pm 0.02_m$. Additionally we have putted the signal over packet at frequency 1_{kHz}.

Eventually, we extracted the amplitude mach to the corespondent frequency. (while assuming that non-neglect noise factors are distributed over separate frequencies).

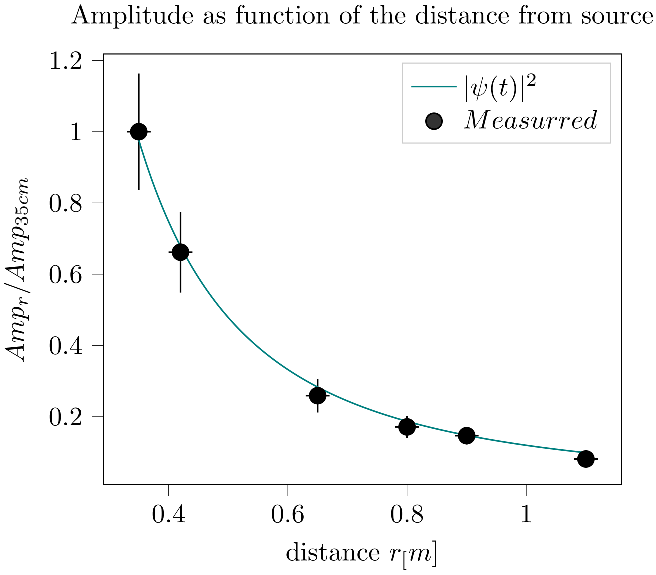

Figure 2: $\hat{y}$ axis is the square of the amplitude divided by the first measured amplitude, $\hat{x}$ is the distance in $[m]$, the black points are the actual measured samples and the blue solid line is the fitted curve. As mentioned above, the error bars taken as $0.02[m]$ in the $\hat{x}$ axis and therefore as $\frac{1}{\sqrt{2\pi r^2}}$ in the $\hat{y}$ axis.

The total **RMS** error of the fitted curve is $4.42 \cdot 10^{-5}$ ("without" units) and the coefficient of the $\frac{1}{r^2}$ term is 0.119 where $\frac{1}{2\pi} = 0.159$. Thus we obtain an accuracy till $1 - \sqrt{2\pi \cdot 0.119} \sim 13.5\%$. Consideration of the fact those results based only over an one-digit number of samples, makes them look impressive. We summarize the first stage of the experiment as successful verification of the theory.

Efficient Measurements While thinking over new research directions, we noticed that, theoretically, the superposition principle allows performing an unlimited number of experiments in one single shot. Assume we want to investigate the dependency of function $f(w_1, w_2)$ over the frequencies w_1, w_2 than, we could put each experiment over other region of the packet, by divide the packet into a sub-packets. The final signal will have the form:

$$\psi(t) = \cos(w_0 t) \sum_{n=1}^{N} e^{-iw_{subpacket}nt} f\left(w_1^{(n)}, w_1^{(n)}\right)$$

As a first trial to test our conjunction, we considered the following problem, what is the minimal frequency shift; which enables to restore a pair of different Gaussians. So, in our case, there is only one interesting dependecis over w_1 (instead, w_1, w_2) and the wave the takes the form:

$$\psi(t) = \cos(w_0 t) \sum_{n=1}^{N} e^{-iw_{sp}nt} \left(G_1\ (t) + e^{-iw_1^{(n)}t} G_2\ (t)\right)$$

where G_1, G_2 are the Gaussians functions, w_{sp} is the sub-packet frequency and $w_1^{(n)}$ are the samples we choose to parameter shift w_1.

In **Figure 3** one can see that "restoring" the signal implies clear traits of the original signal. However, we did not succeed to extract signals equivalent to row of separate transmissions. We emphasise that critical key concept in the correctness of the process, is the relation between the the frequency of the subpackets and the frequencies range of G_2, we believe that if we had more time than we would find a propertied w_{sp} which "isolates" each experiment.

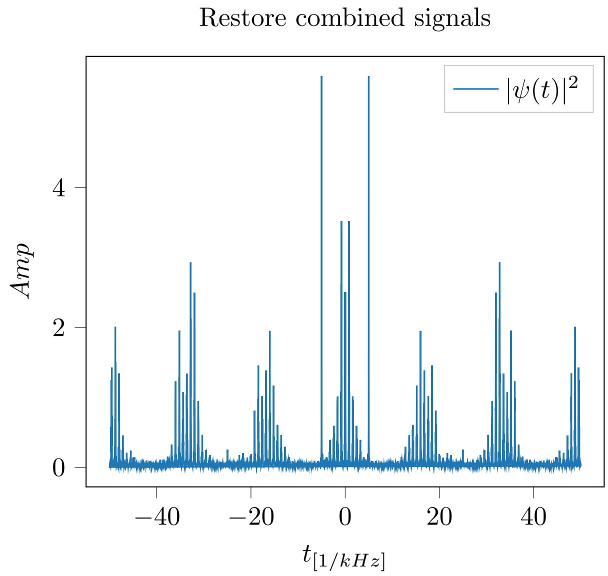

Figure 3: $\hat{y}$ axis is the square amplitude, after performing symmetrtion in Fourier space, and converting back into the time domain, $\hat{x}$ axis is the time in $1/kHz$.

We summarize this section as a nice try that might lead to future research, but not as a solid ground that one can relay on.

Conclusions We conclude that the experimental results we have achieved are match to the theory in the sense of wave propagation and amplitude losing through space progressing. The source code and the samples could be found in that repo: [CP21]. The LaTeX source of that document will be moved there as soon as we will figure out how to integrate the overleaf with **GitHub** repositories.

References

[CP21] M. Choen and D. Ponarovsky. *LABs repository*. 2021. DOI: `https://github.com/dudupo/LABs-diffraction`.