Online Computation, Ex 3.

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ex1. Consider the experts setting with gains: $g_{i,t} \in [0,1]$ is the gain of expert i at step t. Hedge updates:

$$P_{i,t+1} = \frac{e^{\eta G_{i,t}}}{\sum_j e^{\eta G_{j,t}}}$$

where $G_{i,t} = \sum_{s \leq t} g_{i,t}$. Prove that the regret of Hedge at time T is $O\left(\sqrt{T \log n}\right)$, for a good choice of the learning rate η, against the adaptive adversary.

Solution. Define the potential $\psi(t) = \sum_j e^{\eta G_{i,t}}$ and notice that:

$$\psi(t+1) = \sum_j e^{\eta G_{j,t+1}} = \sum_i \frac{e^{\eta G_{i,t}}}{\sum_j e^{\eta G_{j,t}}}$$
$$= \frac{\sum_i e^{\eta G_{i,t}}}{\sum_j e^{\eta G_{j,t}}}$$

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ex2. Show a lower bound of $\Omega\left(\sqrt{T}\right)$ in the experts setting on the regret of any online algorithm against the oblivious adversary.

Solution. solution.

ex3. Consider a system of linear inequalities $Ax \geq b$, where $A \in [0,\infty]^{m \times n}, b \in [0,\infty]^m$, and unknown $x \in [0,\infty]^n$. (we are seeking a non-negative solution). An ε-approximate solution $x \geq 0$ satisfies $Ax \geq b - \varepsilon \mathbf{1}$. Suppose we have an efficient procedure for following problem: Given $p \in [0,1]^m, \sum_{i \in [m]} p_i = 1$, decide if exists $x \geq 0, p^\top Ax \geq p^\top b$. Show how to find an ε-approximate solution to $Ax \geq b$. Analyze the run-time.

Solution. solution.

ex4. Recall that we showed, for EXP updates, that w.p $1-\delta$

$$RT \leq \beta n T + \gamma T + (1+\beta)\eta + \frac{\ln\left(\delta^{-1} n\right)}{\beta} + \frac{\ln n}{\eta}$$

Infer that for the right choice of β, γ, η

$$\mathbf{E}\left[R_T\right] = O\left(\sqrt{Tn \ln n}\right)$$