Objects With Structures.

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1 Polytopes.

Definition 1 (Convex Polygon)**.** *P will be said a convex polygon if for every $x, y \in P$ we have that any point z that lays on the line between x and y belongs to P.*

1.1 Different Constructions.

Consider two different polytopes $P, Q \subset \mathbb{R}^d$ then we could construct a third polytope by:

1. *Intersection, taking the $P \cap Q \subset \mathbb{R}^d$*
2. *Minkeoski sum, $P+Q = \{p+q : p \in P, q \in Q\} \subset \mathbb{R}^d$*
3. *Product, $P \times Q = \{(p,q) : p \in P, q \in Q\} \subset \mathbb{R}^{2d}$*

$\mathcal{V}$ and $\mathcal{H}$ descriptors of polytopes. *Polytopes can be describe by both a convex hull or inequalities. There is theorem that state that any convex hull has a presentation defined by inequalities system.*

Lemma 1. *A projection of an $\mathcal{H}$-polyhedron is also $\mathcal{H}$-polyhedron.*

Definition 2 (The Cyclic Polytope $\mathcal{C}_d(n)$.)**.** *Let $d, n \in \mathbb{N}$. And let us define the monment curve $x : \mathbb{R} \to \mathbb{R}^d$ as $t_i \mapsto t_i^i$. $\mathcal{C}_d(0)$ is the convex hull of $x(y_1), x(y_2), .., x(y_3)$.*

Lemma 2. *For $S \subset T = \{y_1 < y_2 < y_3 .. y_n\}$ such that $|S| = d$ form a facet if and only if any two different indices $i < j$ we have that the size of the intersection of $y_i, y_{i+1} .. y_j$ with S is even. (That can be used to construct a new LDPC CSS code).*

Definition 3 (Simplicial.)**.** *We will say that polytope P is a Simplicial if all his $d-1$ faces are simplexes.*