Simple LTC Good LDPC Codes

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC testable code.

ex1. Find a simple description of the work-function algorithm in the case of uniform metric space.

ex2. Consider the following 3-point metric space, $w(a,b) = 1$ and $w(\cdot, c) = M$. The initial configuration is $\{b, c\}$ (2 servers). Show that randomized competitive ratio, for some value of M is $> H_2 = 1 + \frac{1}{2}$.

ex3. Show that randomized marking algortihm cannot be c-competitive against the adaptive online adversary, for $c = o(k)$.

ex4 - Ski Rental. At each step, the adversary decides etiher continue or stop. Stop terminate the game. If it continues, the online algorithm decides, etiher rent or buy. Rent costs 1 Buy costs $M > 1$. Deisgn a primal-dual randomized online ski-rental algorithm with better than 2 competitive ratio.

ex5. Prove Yao's minimax principle.

$$\begin{aligned}
&\forall \text{rand. } \tilde{\text{alg}}\ \exists\ \sigma \\
&\quad \mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] \geq c \cdot c_{\text{base}}(\sigma) \\
&\Leftrightarrow \exists \text{ rand. } \tilde{\sigma} \forall \text{alg} \\
&\quad \mathbf{E}\left[c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}\right] \geq c\mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]
\end{aligned}$$

Solution. First direction, assume through contriduction that there exists an determenstic algorithm such that for all distrubtons $\tilde{\sigma}$:

$$\mathbf{E}\left[c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}\right] < c\mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]$$

And that holds, in paritcular, for distribution $\tilde{\sigma}$ which supportted by a single σ. Hence, by the fact that any deterministic algortihm is also a randomized algortithm, set it to be $\tilde{\text{alg}}$ and that imdetly yeilds a contrudioction. It left to show the seconed direaction, By the monotonic preporty of random variables we have that for any distribution $\tilde{\sigma}$:

$$\begin{aligned}
&\mathbf{E}\left[\mathbf{E}\left[c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}\right] : \text{alg} \sim \tilde{\text{alg}}\right] \geq c \cdot \mathbf{E}\left[\mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right] : \text{alg} \sim \tilde{\text{alg}}\right] \\
&\mathbf{E}\left[\mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] : \sigma \sim \tilde{\sigma}\right] \geq c \cdot \mathbf{E}\left[\mathbf{E}\left[c_{\text{base}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] : \sigma \sim \tilde{\sigma}\right] \\
&\mathbf{E}\left[\mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] : \sigma \sim \tilde{\sigma}\right] \geq c \cdot \mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]
\end{aligned}$$

And by the fact that inequlity of excpection between random varibles follows an existness of atomic event on which the inequality holds, we obtain that there is must exists at least a sinlge σ such that:

$$\mathbf{E}\left[\mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] : \sigma \sim \tilde{\sigma}\right] \geq c \cdot \mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]$$

And that ends the proof.