Simple LTC Good LDPC Codes

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Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC testable code.

ex1. Find a simple description of the work-function algorithm in the case of uniform metric space.

ex2. Consider the following 3-point metric space, $w(a,b) = 1$ and $w(\cdot, c) = M$. The initial configuration is $\{b, c\}$ (2 servers). Show that randomized competitive ratio, for some value of M is $> H_2 = 1 + \frac{1}{2}$.

ex3. Show that randomized marking algortihm cannot be c-competitive against the adaptive online adversary, for $c = o(k)$.

ex4 - Ski Rental. At each step, the adversary decides etiher continue or stop. Stop terminate the game. If it continues, the online algorithm decides, etiher rent or buy. Rent costs 1 Buy costs $M > 1$. Deisgn a primal-dual randomized online ski-rental algorithm with better than 2 competitive ratio.

ex5. Prove Yao's minimax principle.

$$\begin{aligned}
&\forall \text{rand. } \tilde{\text{alg}}\ \exists\ \sigma \\
&\quad \mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] \geq c \cdot c_{\text{base}}(\sigma) \\
&\Leftrightarrow \exists \text{ rand. } \tilde{\sigma} \forall \text{alg} \\
&\quad \mathbf{E}\left[c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}\right] \geq c\mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]
\end{aligned}$$

Solution. First direction, assume through contriduction that there exists an determenstic algorithm such that for all distrubtons $\tilde{\sigma}$:

$$\mathbf{E}\left[c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}\right] < c\mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]$$

And that holds, in paritcular, for distribution $\tilde{\sigma}$ which suportted by a single σ. Hence, by the fact that any deterministic algortihm is also a randomized algortithm, set it to be $\tilde{\text{alg}}$ and that imdetly yeilds a contrudioction. It left to show the seconed direaction, By the monotonic preporty of random variables we have that for any distribution $\tilde{\sigma}$:

$$\mathbf{E}\left[\mathbf{E}\left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}\right] : \sigma \sim \tilde{\sigma}\right] \geq c \cdot \mathbf{E}\left[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}\right]$$