

Simple LTC Good LDPC Codes

David Ponnarovsky

January 30, 2023

Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC testable code.

ex1. Find a simple description of the work-function algorithm in the case of uniform metric space.

ex2. Consider the following 3-point metric space, $w(a, b) = 1$ and $w(\cdot, c) = M$. The initial configuration is $\{b, c\}$ (2 servers). Show that randomized competitive ratio, for some value of M is $> H_2 = 1 + \frac{1}{2}$.

ex3. Show that randomized marking algorithm cannot be c -competitive against the adaptive online adversary, for $c = o(k)$.

ex4 - Ski Rental. At each step, the adversary decides either continue or stop. Stop terminate the game. If it continues, the online algorithm decides, either rent or buy. Rent costs 1 Buy costs $M > 1$. Design a primal-dual randomized online ski-rental algorithm with better than 2 competitive ratio.

ex5. Prove Yao's minimax principle.

And by the fact that inequality of expectation between random variables follows an existence of atomic event on which the inequality holds, we obtain that there must exist at least a single σ such that:

$$\mathbf{E} \left[\mathbf{E} \left[c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}} \right] : \sigma \sim \tilde{\sigma} \right] \geq c \cdot \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

And that ends the proof.

$$\forall \text{rand. } \tilde{\text{alg}} \exists \sigma$$

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] \geq c \cdot c_{\text{base}}(\sigma)$$

$$\Leftrightarrow \exists \text{rand. } \tilde{\sigma} \forall \text{alg}$$

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] \geq c \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

Solution. First direction, assume through contradiction that there exists a deterministic algorithm such that for all distributions $\tilde{\sigma}$:

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] < c \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

And that holds, in particular, for distribution $\tilde{\sigma}$ which supported by a single σ . Hence, by the fact that any deterministic algorithm is also a randomized algorithm, set it to be $\tilde{\text{alg}}$ and that immediately yields a contradiction. It left to show the second direction, By the monotonic property of random variables we have that for any distribution $\tilde{\sigma}$:

$$\begin{aligned} & \mathbf{E} \left[\mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] : \text{alg} \sim \tilde{\text{alg}} \right] \\ & \geq c \cdot \mathbf{E} \left[\mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}] : \text{alg} \sim \tilde{\text{alg}} \right] \\ & \mathbf{E} \left[\mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \geq c \cdot \mathbf{E} \left[\mathbf{E} [c_{\text{base}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \mathbf{E} \left[\mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \geq c \cdot \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}] \end{aligned}$$