

# Polytopes.

David Ponarovsky

March 12, 2023

## 1 Basics.

**Definition 1** (Convex Polygon).  *$P$  will be said a convex polygon if for every  $x, y \in P$  we have that any point  $z$  that lays on the line between  $x$  and  $y$  belongs to  $P$ .*

### 1.1 Different Constructions.

*Consider two different polytopes  $P, Q \subset \mathbb{R}^d$  then we could construct a third polytope by:*

- 1. Intersection, taking the  $P \cap Q \subset \mathbb{R}^d$*
- 2. Minkeoski sum,  $P+Q = \{p+q : p \in P, q \in Q\} \subset \mathbb{R}^d$*
- 3. Product,  $P \times Q = \{(p, q) : p \in P, q \in Q\} \subset \mathbb{R}^{2d}$*

**$\mathcal{V}$  and  $\mathcal{H}$  descriptors of polytopes.** *Polytopes can be describe by both a convex hull or inequalities. There is theorem that state that any convex hull has a presentation defined by inequalities system.*

**Lemma 1.** *A projection of an  $\mathcal{H}$ -polyhedron is also  $\mathcal{H}$ -polyhedron.*