Polytopes.

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1 Basics.

Definition 1 (Convex Polygon). *P will be said a convex polygon if for every $x, y \in P$ we have that any point z that lays on the line between x and y belongs to P.*

1.1 Different Constructions.

Consider two different polytopes $P, Q \subset \mathbb{R}^d$ then we could construct a third polytope by:

1. *Intersection, taking the $P \cap Q \subset \mathbb{R}^d$*
2. *Minkeoski sum, $P+Q = \{p+q : p \in P, q \in Q\} \subset \mathbb{R}^d$*
3. *Product, $P \times Q = \{(p,q) : p \in P, q \in Q\} \subset \mathbb{R}^{2d}$*

$\mathcal{V}$ and $\mathcal{H}$ descriptors of polytopes. *Polytopes can be describe by both a convex hull or inequalities. There is theorem that state that any convex hull has a presentation defined by inequalities system.*

Lemma 1. *A projection of an $\mathcal{H}$-polyhedron is also $\mathcal{H}$-polyhedron.*

Definition 2 (The Cyclic Polytope $\mathcal{C}_d(n)$.). *Let $d, n \in \mathbb{N}$. And let us define the monment curve $x : \mathbb{R} \to \mathbb{R}^d$ as $t_i \mapsto t_i^i$. $\mathcal{C}_d(0)$ is the convex hull of $x(y_1), x(y_2), .., x(y_3)$.*

Definition 3 (Simplicial.). *We will say that polytope P is a Simplicial if all his $d-1$ faces are simplexes.*