

# Simple LTC Good LDPC Codes

David Ponnarovsky

January 30, 2023

## Abstract

We propose a new simple construction based on Tanner Codes, which yields a good LDPC testable code.

**ex1.** Find a simple description of the work-function algorithm in the case of uniform metric space.

**ex2.** Consider the following 3-point metric space,  $w(a, b) = 1$  and  $w(\cdot, c) = M$ . The initial configuration is  $\{b, c\}$  (2 servers). Show that randomized competitive ratio, for some value of  $M$  is  $> H_2 = 1 + \frac{1}{2}$ .

**Solution.** Define the following distribution:

$$\tilde{\sigma} = \begin{cases} (ab)^{\frac{M}{3}} & \text{w.p. } \frac{1}{2} \\ (ab)^{\frac{M^{100}}{3}} & \text{w.p. } \frac{1}{2} \end{cases}$$

Using Yao's principle, it's enough to show that any deterministic algorithm is  $H_2$  competitive in expectation against the that specific distribution. First notice that knowing what is the exactly drawn  $\sigma$  fix an optimal strategy which is one of the following: moving alternately the server which initialized at  $a$  between  $a, b$  points, or choosing first the server that located in  $c$  into  $a$  in the second scenario. Putting down we obtain that:

$$\mathbf{E}[c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}] = \frac{1}{2} \left( \frac{M}{3} + M \right) = \frac{5}{6} M$$

Meanwhile, by the fact that reading any prefix of requests series at length less than  $\frac{M}{3}$  doesn't expose any information about the drawn input which wasn't known at the initialized moment, it follows by undistinguishable arguments that the best an randomized algorithm can do is to guess.

**ex3.** Show that randomized marking algorithm cannot be  $c$ -competitive against the adaptive online adversary, for  $c = o(k)$ .

**Solution.** Assume by contradiction that there is a constant  $c > 1$ , and a randomized algorithm which is an  $c$ -competitive in the adaptive online setting. According to theorem that shown at class, If there exists an  $\alpha$  competitive alg for an online problem in the non-adaptive setting and in addition there exists a  $\beta$  competitive algorithm for the same problem against adaptive online adversary, then it holds that there exists an algorithm which is  $\alpha\beta$  competitive against an offline adaptive adversary. Combine the fact that randomized can't help against such adversary we obtain that the deterministic competitive ratio is lower than  $\alpha\beta$ . As we know that a  $k$ -lowerbound for the deterministic regime and also a  $\log k$  solution using randomization against a non-adaptive adversary, we obtains that

$$\begin{aligned} \alpha\beta &\geq k \\ \Rightarrow \frac{k}{c} \log k &\geq k \end{aligned}$$

But for any  $k \leq \log 2^c$  we obtain the opposite direction. Which mean that there is a range of valid  $k$  that obtains a better ratio than the lower bound. And that is a contradiction.

**ex4 - Ski Rental.** At each step, the adversary decides either continue or stop. Stop terminate the game. If it continues, the online algorithm decides, either rent or buy. Rent costs 1 Buy costs  $M > 1$ . Design a primal-dual randomized online ski-rental algorithm with better than 2 competitive ratio.

**Solution.** Let's start by formulate an integer LP for the Ski-Rental problem. Denote by  $m$  the days number, associate a variable  $x$  indicating whether or not the algorithm decide to buy. Also let's associate with each day a variable  $\xi_j$  which indicate if the algorithm pays for rent. In each turn the solution must satisfy the restrictions  $\xi_j + x \geq 1$ . The cost which we would like to minimize is  $M \cdot x + \sum_j \xi_j$ . So, in overall we get that out LP is:

$$\begin{aligned} \min & Mx + \sum_j \xi_j \\ \text{s.t. } & x + \xi_j \geq 1 \Leftrightarrow \\ & \begin{bmatrix} 1 & 1 & 0 & 0 & \cdot \\ 1 & 0 & 1 & 0 & \cdot \\ 1 & 0 & 0 & 1 & \cdot \\ 1 & 0 & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x \\ \xi_1 \\ \xi_2 \\ \xi_3 \\ \cdot \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \cdot \end{bmatrix} \end{aligned}$$

So the dual program is

$$\begin{aligned} \max & \sum_j z_j \\ \text{subject to} & \\ & \begin{bmatrix} 1 & 1 & 1 & 1 & \cdot \\ 1 & 0 & 0 & 0 & \cdot \\ 0 & 1 & 0 & 0 & \cdot \\ 0 & 0 & 1 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x \\ z_1 \\ z_2 \\ z_3 \\ \cdot \end{bmatrix} \leq \begin{bmatrix} M \\ 1 \\ 1 \\ 1 \\ \cdot \end{bmatrix} \end{aligned}$$

So in total.

---

### Algorithm 1: Ski-Rental

---

```

1 for each new day  $j$  do
2   if  $x < 1$  then
3      $\xi_j \leftarrow 1 - x$ 
4      $x \leftarrow \left(1 + \frac{1}{M}\right)x + \frac{1}{(c-1)M}$ 
5      $z_j \leftarrow 1$ 
6   end
7 end
```

---

**ex5.** Prove Yao's minimax principle.

$$\forall \text{rand. } \tilde{\text{alg}} \exists \sigma$$

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] \geq c \cdot c_{\text{base}}(\sigma)$$

$$\Leftrightarrow \exists \text{rand. } \tilde{\sigma} \forall \text{alg}$$

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] \geq c \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

**Solution.** First direction, assume through contradiction that there exists a deterministic algorithm such that for all distributions  $\tilde{\sigma}$  :

$$\mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] < c \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

And that holds, in particular, for distribution  $\tilde{\sigma}$  which supported by a single  $\sigma$ . Hence, by the fact that any deterministic algorithm is also a randomized algorithm, set it to be  $\tilde{\text{alg}}$  and that immediately yields a contradiction. It left to show the second direction, By the monotonic property of random variables we have that for any distribution  $\tilde{\sigma}$ :

$$\begin{aligned} & \mathbf{E} \left[ \mathbf{E} [c_{\text{alg}}(\sigma) : \sigma \sim \tilde{\sigma}] : \text{alg} \sim \tilde{\text{alg}} \right] \\ & \geq c \cdot \mathbf{E} \left[ \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}] : \text{alg} \sim \tilde{\text{alg}} \right] \\ & \mathbf{E} \left[ \mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \geq c \cdot \mathbf{E} \left[ \mathbf{E} [c_{\text{base}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \mathbf{E} \left[ \mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \\ & \geq c \cdot \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}] \end{aligned}$$

And by the fact that inequality of expectation between random variables follows an existence of atomic event on which the inequality holds, we obtain that there must exist at least a single  $\sigma$  such that:

$$\mathbf{E} \left[ \mathbf{E} [c_{\text{alg}}(\sigma) : \text{alg} \sim \tilde{\text{alg}}] : \sigma \sim \tilde{\sigma} \right] \geq c \cdot \mathbf{E} [c_{\text{base}}(\sigma) : \sigma \sim \tilde{\sigma}]$$

And that ends the proof.