# Online Computation, Ex 3.

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**ex1.** Consider the experts setting with gains: $g_{i,t} \in [0,1]$ is the gain of expert $i$ at step $t$. Hedge updates:

$$P_{i,t+1} = \frac{e^{\eta G_{i,t}}}{\sum_j e^{\eta G_{j,t}}}$$

where $G_{i,t} = \sum_{s \leq t} g_{i,t}$. Prove that the regret of Hedge at time $T$ is $O\left(\sqrt{T \log n}\right)$, for a good choice of the learning rate $\eta$, against the adaptive adversary.

**Solution.** Define the potential $\psi(t) = \sum_j e^{\eta G_{i,t}}$ and notice that by the fact that $e^x$ is positive function we have that $\psi(t) \geq e^{\eta \max_j G_{i,j}}$. In addition:

$$\begin{aligned}
&\psi(t+1) = \sum_j e^{\eta G_{j,t+1}} = \sum_j e^{\eta G_{j,t} + \eta g_{j,t+1}} \\
&\Rightarrow \psi(t) \leq \psi(t+1) \leq \psi(t) e^{\eta} \\
&= \sum_j e^{\eta G_{j,t}} \left(1 + \eta g_{j,t+1}\right) = \psi(t) + \psi(t) \mathbf{E}\left[g_{t+1}\right]
\end{aligned}$$

So after $T$ steps the potential is bounded by $\psi(0) e^{\eta T} \leq n e^{\eta} e^{\eta T}$.

**ex2.** Show a lower bound of $\Omega\left(\sqrt{T}\right)$ in the experts setting on the regret of any online algorithm against the oblivious adversary.

**Solution.** solution.

**ex3.** Consider a system of linear inequalities $Ax \geq b$, where $A \in [0,\infty]^{m \times n}, b \in [0,\infty]^m$, and unknown $x \in [0,\infty]^n$. (we are seeking a non-negative solution). An $\varepsilon$-approximate solution $x \geq 0$ satisfies $Ax \geq b - \varepsilon \mathbf{1}$. Suppose we have an efficient procedure for following problem: Given $p \in [0,1]^m, \sum_{i \in [m]} p_i = 1$, decide if exists $x \geq 0, p^\top Ax \geq p^\top b$. Show how to find an $\varepsilon$-approximate solution to $Ax \geq b$. Analyze the run-time.

**Solution.** solution.

**ex4.** Recall that we showed, for $EXP$ updates, that w.p $1-\delta$

$$RT \leq \beta n T + \gamma T + (1+\beta)\eta + \frac{\ln\left(\delta^{-1} n\right)}{\beta} + \frac{\ln n}{\eta}$$

Infer that for the right choice of $\beta, \gamma, \eta$

$$\mathbf{E}\left[R_T\right] = O\left(\sqrt{Tn \ln n}\right)$$