

***DESIGN AND IMPLEMENTATION OF A PARAMETRIC EQUALIZER
USING IIR AND FIR FILTERS***

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Abstract

The main purpose of this project is the development of real time parametric equalizer filters for audio purposes. These will be achieved by establishing a design strategy for implementing FIR and IIR parametric filters. Therefore through the study of the different techniques and strategy designs of both types of filters, an algorithm to design multi-band parametric filters using MATLAB and implemented via the Freescale DSP56321 system is proposed.

The algorithm is based on the construction of an arbitrary frequency response using the frequency sampling method and the bilinear transform for FIR and IIR filters respectively. The problem of minimize the error between the desired and designed frequency response is one of the main cores of the project, therefore the results are discussed in light of this issue, and the implications for future work are shown.

Thus, the project will follow the course of digital filters, graphic and parametric equalizers, and hardware platforms such as the Signal Wizard system. Then conclude with the analysis of the desired, designed and implemented parametric equalizers for both types of filters.

Declaration

No portion of the work referred to in the dissertation has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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To my family

Chapter 0

Reading Guide

The following reading guide focuses on the most important parts of the dissertation and is presented in order to facilitate review.

The first three chapters are introductory, the research is presented and the objectives of the dissertation identified thus leading to the main text. The main background theories are illustrated in Chapter 2, where a brief introduction of the main principles of digital filtering, the z-Transform are given, as well as a short description of the hardware platforms used in the project.

In the following two Chapters the different techniques and strategy designs of FIR and IIR filters are shown. Also the study of parametric equalization and how it is related to the design of multi-band IIR and FIR filters. Thus, Chapter 5 is focused on the frequency sampling method and the design of a FIR filters with an arbitrary frequency response. Also the ways to improve the drawbacks of the method are explained.

Chapter 6 we cover the different aspects related to the design of IIR filters specially the bilinear transform and biquad design methods, and how each method is related to the design of parametric filters.

After establishing a design strategy for implementing the parametric equalizer using FIR and IIR filters, the analytical solutions of the equalizer have been achieved and the methods and procedures are shown in Chapter 7.

First a 20-band off-line semi-parametric equalizer based on IIR filters using the programming language Delphi is designed. Once the off-line graphic equalizer is implemented, a real time high-performance 20-band parametric equalizer based on FIR or IIR filters using Matlab is aimed. The results of both equalizers are shown on Chapter 8 and the respective analysis and discussion on Chapter 9.

Chapter 1

INTRODUCTION

Many musical applications require having the control of the bandwidth, centre frequency, and boost or cut of a of a specified frequency band whereas leaving the rest of the spectrum unaffected. This process is called equalization and is implemented by an equalizer which can be analogue or digital and various types exist, the most common are parametric, semi-parametric, graphic and shelving equalizers. However the main purpose of each one is to alter the frequency response in order to enhance or attenuate certain properties of the audio signal.

The implementation of a parametric equalizer will be carried out in the digital world, by the reason of the advantages of digital filtering over analogue, such as the implementation of high order filters or filtering with an arbitrary frequency design, both impossible in the analogue world. Consequently the knowledge of digital filtering is the most important background theory for the project, as well as the study of the different techniques with their implementation, advantages and disadvantages.

A graphic equalizer is commonly find in basic music software and stereo players, it contains a predetermined number of frequency bands with a fixed bandwidth and centre frequency leaving only the gain to boost or cut a specified range. With a parametric equalizer the user can adjust all the parameters; the centre frequency can be shifted, the bandwidth narrowed or widened, and the gain of each band increased or reduced.

Thus the user can adjust the parameters of the equalizer in order to boost or cut the sounds of certain music instrument. For example to enhance or attenuate certain properties of an audio signal some frequency bands can be boosted or cut, this band can be narrowed as a peak or notch filter or wide as a band-pass or band-stop filter.

For this reason, a parametric equalizer is a more complex type and provides to the user the freedom to modify the whole frequency response into the desired curve, allowing more detailed control of the sound and finer adjustments with less frequency bands needed in comparison with a graphic equalizer.

The design of filters involves the following stages: the specification of the desired properties of the system, the approximation of the specifications using a casual discrete-time system, and the realization of the system (A.V. Oppenheim, 2010).

The first stage depends on the application, in our case the design of a parametric equalizer for audio purposes, the second stage is the design strategy for implementing the equalizer, and the third step depends on the technology, in this case the Signal Wizard system.

In order to establishing a design strategy for implementing the parametric equalizer is of vital importance to understand the different techniques and strategy designs of FIR and IIR filters. Also the study of parametric equalization and how it is related to the design of multi-band IIR and FIR filters.

Once the design strategy is accomplished it means that analytical solutions of the equalizer have been achieved and formulas for the coefficients in terms of the parameters (gain, bandwidth, centre frequency) are obtained.

This is followed by the execution of a 20-band off-line semi-parametric equalizer based on IIR filters using the programming language Delphi. Once the off-line graphic equalizer is implemented, a real time high-performance 20-band parametric equalizer based on FIR or IIR filters using Matlab will be put into practice.

A real-time control of the parametric filters designed in Matlab is carried out. This is achieved after the familiarization and understanding of the parametric equalization using the Signal Wizard software. Finally the comparison of the results obtained with both types of filters can be done, and the conclusions made based on the performance of both equalizers.

Chapter 2

BACKGROUND

A brief introduction of the main principles of digital filtering, the z-Transform will be given below as well as a short description of the hardware platforms used in the project. The rest of the relevant background material will be explained in full detail in chapters 5 and 6, with topics such as the design and implementation of the two different types of digital filters: Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters.

The z-Transform

Before all else an understanding of the z-Transform is needed, this is by the reason that this transformation is extensively used in the different digital filtering theories.

The z-Transform is the counterpart of the Laplace transform for discrete-time signals.

$$Z\{x[n]\} = X(z) = \sum_{n=0}^{\infty} x[n]z^{-n} \quad (2.1)$$

$$z = re^{j\omega} \quad (2.2)$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\omega n} \quad (2.3)$$

It can be seen in equation (2.1) that if z is replaced by $e^{j\omega}$ the equation turns into the DFT equation (equation 2.3) where $\omega = \frac{2\pi k}{N}$. This is the same as limit z in equation (2.2) with $r = 1$, which is why when analyzing the z-Transform using the z-Plane the unit circle is one of the main aspects of the transform. (A.V. Oppenheim, 2010)

The points on the unit circle represent a frequency, and describe discrete signals sampled at certain frequency (Gaydecki, 2004). The Nyquist point is represented by π and is placed at $z = -1$, this mean that the point mapped into the z-plane are not unique.

A main characteristic of the z-Transform is the time-shifting property, which means that a multiplication by z^n advances a signal sample by n intervals or delays it if n is negative.

$$Z\{x[n - a]\} = z^{-a}X(z) \quad (2.4)$$

Digital Filtering Design

The two different types of linear digital filters in the time domain are the FIR and IIR, both can be represented with a difference formula, which can be non-recursive or recursive (FIR and IIR respectively) and the formula contains the distinctive coefficients of the filter. There are several design strategies for each type of filter, where depending on the purposes and needs of the user some have advantages over others.

The non-recursive FIR and recursive IIR discrete-time difference equations are given below respectively. (Gaydecki, 2004)

$$y[n] = \sum_{k=0}^{M-1} h[k]x[n - k] \quad (2.5)$$

$$y[n] = \sum_{k=0}^{\infty} h[k]x[n - k] = \sum_{k=0}^M a[k]x[n - k] - \sum_{k=0}^N b[k]y[n - k] \quad (2.6)$$

Where $h[k]$ represents the impulse response of the system, $a[k]$ and $b[k]$ the feedforward and feedback part of the system (M and N coefficients respectively), $x[n]$ and $y[n]$ the input and output signal correspondingly. Also the infinite nature of an IIR filter can be seen by the infinite duration of the impulse response on equation (2.6) and the opposite for FIR filters (2.5).

Equations (2.7) and (2.8) correspond to a very important parameter at the moment of design digital filters by the reason that provide the transfer function $H(z)$ by dividing the z-transform of $y[n]$ by the z-transform of $x[n]$.

$$Y(z) = H(z)X(z) = Z\{h[n] * x[n]\} \quad (2.7)$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{\infty} h[n]z^{-n} \quad (2.8)$$

In general $H(z)$ can be represented by a polynomial or by the ratio of two polynomials (Gaydecki, 2004).

$$H(z) = \sum_{n=0}^{\infty} h[n]z^{-n} = h[0] + h[1]z^{-1} + \dots + h[M-1]z^{-M-1} \quad (2.9)$$

$$H(z) = \frac{\sum_{m=0}^M a[m]z^{-m}}{1 + \sum_{n=1}^N b[n]z^{-n}} \quad (2.10)$$

Equations (2.9) and (2.10) correspond to a general expression for transfer functions of FIR and IIR filters respectively, also there is a similarity than can be observed between the difference equations (2.5) and (2.6) and the transfer functions (2.9) and (2.10), where the numerator represents the non-recursive and the denominator de recursive part, and this one being inexistent for FIR filters does not appear in the equation (2.9) which denominator is one.

However, time shift, multiplication, and addition are the three operations that DSP processors use to implement digital filters. Equations (2.5) and (2.6) can be easily be represented by these three operations.

Hence the selection of the right technique and right type of filter is of vital importance in the project and both will be explored and studied. The most common strategies for IIR filters are; the bilinear z-transform (which starts with the analogue design of the filter) the pole-zero placement technique, frequency transformations, and biquad filtering. FIR filters design techniques: the window method, the frequency sampling method, and inverse filtering (Gaydecki, 2004).

FIR filters are characterized by stability, linear phase, easy design, they regularly need high filter orders and a substantial amount of computation which can be an issue if a real time application is required. IIR filters require less filter coefficients and less computational load but can produce phase distortion. Also they are sensitive to coefficient quantization

and the effects of word length must be considerate (Shpak, 1992). The merits and disadvantages of FIR and IIR filters can be seen in following table.

| Property | FIR | IIR |
|-------------------------|------|-----------|
| Unconditional stability | Yes | No |
| Phase distortion | No | Yes |
| Easy design | Easy | Difficult |
| Arbitrary response | Easy | Difficult |
| Computational load | High | Low |
| Word length immunity | Good | Poor |
| Analogue equivalent | No | Yes |
| Real-time operation | Yes | Yes |

Table 2.1 FIR and IIR common properties. (Gaydecki, 2004)

Audio DSP Platforms

Real time digital signal processing hardware platforms have traditionally been associated with labour-intensive programming and a steep learning curve to enable their various functions to be realized. Recently, however, a number of “intelligent” DSP products have appeared on the market that only require the user to specify the kind of processing or filter required, and the system proceeds to design the software solution and execute it in real time.

The Signal Wizard system is a platform which using its software system can calculate, implement and execute different types of digital filters. The DSP hardware of the platform can perform offline and real time filtering. It uses the digital signal processor ‘Freescale DSP56321’ capable of processing at 588 MMACS and uses the DSP563xx assembly language programming. It can implement any kind of filter (low-pass, high-pass, multiple band-stop, band-pass, arbitrary filters) and phase distortion can be completely manipulated.

Thus the study of the two types of digital filters is carried out on chapters 5 and 6, where the theoretical background to establish a design strategy is provided. The different methods and procedures are shown on chapter 7. The first approach will be the implementation of a 20-band off-line graphic equalizer based on IIR filters using Delphi

programming language. Then a real time 20-band parametric equalizer based on FIR or IIR filters using Matlab will be implemented. The results of both equalizers are shown on Chapter 8 and the respective analysis and discussion on Chapter 9.

Chapter 3

OBJECTIVES

These are the aims of the project:

- a. Understand the different techniques and strategy designs of FIR and IIR filters.
- b. Study the importance of equalization and the different techniques and strategy designs of high performance parametric equalizers.
- c. Familiarization and understanding of the parametric equalization using the Signal Wizard software.

The objectives of the project:

1. Establish a design strategy for implementing a real-time FIR parametric equalizer.
2. Establish a design strategy for implementing a real-time IIR parametric equalizer.
3. Real-Time control of the parametric equalizers using the Signal Wizard system
4. Compare the results obtained with both types of filters using the Signal Wizard system.

Chapter 5

FINITE IMPULSE RESPONSE FILTERS

The Design of FIR filters involves a polynomial approximation to a desired frequency response; this polynomial is the discrete representation of its impulse response (providing unconditional stability) as we can see in Equations (2.5) and (2.9). The coefficients can be obtained through different methods such as the window method (A.V. Oppenheim, 2010), frequency sampling (Gaydecki, 2004), adaptive method (B. Widrow, 1985) and the optimal method: minimax (J.M.McClellan, 1998) or least-square (C.S. Burrus, 1994) optimization criteria.

The window method can be the simplest approach but falls short when the desired frequency response differs from the four basic filter types, and the approximation error is large in comparison to other techniques. The adaptive method is computationally expensive and can be very complex, while the optimization method minimizes the norm of the error between the designed filter response and the desired one. This is beyond the scope of this project due complexity and time available, although is an important part of the future work in this subject.

Therefore, we are going to focus on the frequency sampling method, by the reason that is the one that better matches our specifications in terms of simplicity and results. Since we need to design a parametric equalizer which can be seen as a multiple band filter (peak or notch) where all the parameters of each band are adjusted by the user (center frequency, gain and bandwidth), the best way to approach this requirement is by the design of a FIR filter with an arbitrary frequency response using the frequency sampling method, followed by the ways to improve the possible drawbacks of the method.

The frequency sampling method

The main objective of this method is to obtain the coefficients of the impulse response from a desired frequency response. Which can produce an arbitrary magnitude and phase design, hence this can be useful when designing a multiple band filter.

The desired outcome is to obtain the impulse response, then the design of an N-point FIR filter starts with the desired frequency response; establishing the numerical values of the real and imaginary components. The frequency response can then be evaluated at N-equally spaced points, and then an N-length inverse discrete Fourier transformation and some adjustments are performed in order to generate a time-domain representation of the impulse response.

The following steps describe the method.

1. The design of the desired frequency response setting the numerical values of the real and imaginary terms.
2. The inverse discrete Fourier transform is applied, thus generating a time-domain representation of the impulse response of the filter.
3. The impulse response is windowed in order to obtain a zero tapped representation.

If phase linearity is wanted, the imaginary terms in step one must be set to zero, thus providing a constant phase shift over the entire spectrum. After the inverse discrete Fourier transformation the result is shifted, therefore the impulse response has to be centred. And finally the type of window (Hanning, Kaiser, Rectangular, etc.) is significant, each type will have consequences on the ripples of the pass and stop bands and equally on the transition zone (Gaydecki, 2004).

The impulse response $h[n]$ of an arbitrary frequency response $H[k]$ (real and imaginary coefficients represented by a_k and b_k respectively) can be obtained with the following equation, where $w[n]$ is the window function.

$$h[n] = w[n]F^{-1}\{H[k]\} \quad \begin{cases} H_r[k] = a_k \\ H_i[k] = b_k \end{cases} \quad (5.1)$$

Other important aspects when designing the filter are the sample rate of the signal to be filtered (because the filter is designed up the Nyquist frequency) and the numbers of taps required which will determine the FFT length and the frequency resolution of the filter.

The trade off between the main and side lobes of the resulting filtered signal due windowing is the main issue when selecting the type of window. Several types of windows exist, from Rectangular windows, Hamming windows (Hanning, Hamming), Cosine windows (Blackman), to adjustable windows (Kaiser).

Using *wintool* (a Graphical User Interface (GUI) of Matlab that allows the design and analysis of windows) a comparison of the most common types of windows can be seen in the following table.

| Type of window | Relative sidelobe attenuation (dB) | Mainlobe width (-3dB) * $\frac{\pi}{N} (\frac{rad}{sample})$ | Equivalent Kaiser window β |
|----------------|------------------------------------|--|----------------------------------|
| Rectangular | -13.3 | 0.53 | 0 |
| Hanning | -31.5 | 0.87 | 3.86 |
| Hamming | -42.7 | 0.83 | 4.86 |
| Blackman | -58.1 | 1.02 | 7.04 |

Table 5.1 Comparison of most common types of windows.

In Table 5.1 N is the length of the window. It can be seen the trade-off between sidelobe attenuation and mainlobe width among the different types of windows. Also the adjustable parameter β from the Kaiser window can be established in order to trade sidelobe amplitude for mainlobe width (A.V. Oppenheim, 2010).

Arbitrary frequency response design

The frequency sampling method can be used to design a filter with any type of magnitude or phase behaviour; filters with an unusual frequency response are called arbitrary filters and are used in many applications such as noise cancelation, musical synthesis, biomedical signal processing or sound enhancement.

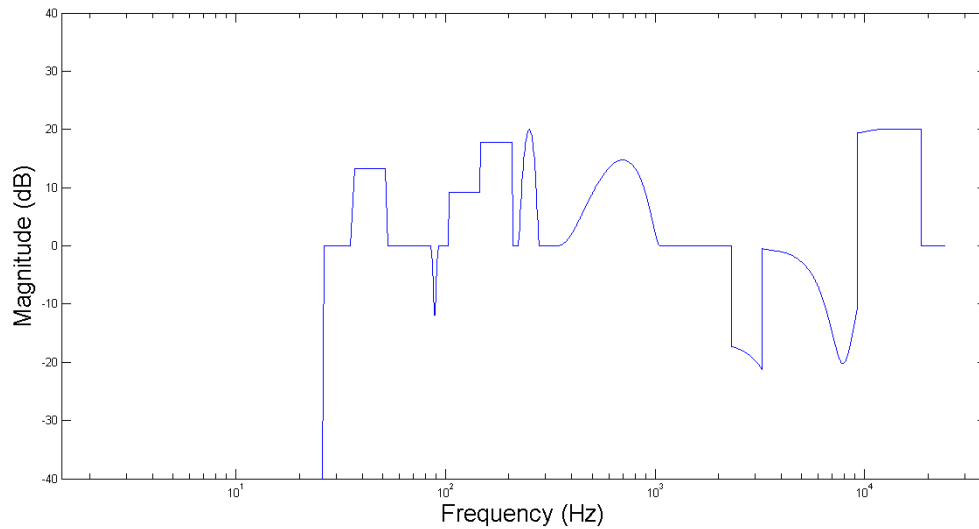


Figure 5.1 Arbitrary frequency response of a filter (16384 taps)

In Figure 5.1 we can see the frequency response of an arbitrary filter using 16384 taps. This filter can be implemented using the frequency sampling method hence obtaining the coefficients of the impulse response in time-domain. This type of filter can be put into practice in an offline or in a real-time application reducing the number of taps.

The main disadvantage when designing an arbitrary filter is the difference between the desired and designed frequency response. By the reason that the obtained impulse response is subject to errors such as lack of smoothness in the frequency response, side lobes in the stop band, Gibbs phenomenon, passband ripples, echoes in the impulse response, or peak errors.

When designing a multi-band filter, the first step is to choose different input-parameters (desired response of each band, edge frequencies, transition zones, etc.). The effects of the choice of these parameters on the frequency response of the resulting filter need to be understood in order to obtain acceptable results, and this can be achieved modifying the values of the design parameters.

The number of bands in the filter is related proportionally with the possible number of ripples in the frequency response of the filter. Depending on the length of the filter these ripples can occur outside the pass-bands or stop bands. Hence the filter response in the 'don't care' or transition regions may contain undesirable errors (L. Rabiner, 1974).

One of the strategies for designing acceptable multiband FIR filters proposed in (L. Rabiner, 1974) is based in the modification of the stopband edge frequencies in order to obtain a monotonic frequency response without ripples in the transition zone. Other strategies are proposed based on optimum filter design which involves error weighting function modification and design of maximal ripple filters only.

Another important aspect when designing the filter with the frequency sampling method is that the desired frequency response will be evaluated at N equally spaced points, thus the resulting frequency will be equal to the desired frequency response just in the samples at the specified frequencies (M. Lightstone, 1994).

By this reason a different approach is introduced in (M. Lightstone, 1994), where the desired frequency response is constructed using different analytical functions for the pass-bands, stop-bands, transition zones and equiripple behaviour. A sub-band division of the filter is performed, by the reason that any FIR filter can be represented by a parallel connection of cascades. This cascades are made by an interpolator and a sub-filter, where each one represents a band in the overall filter (Mitra, 1993).

The filter of length N and L bands now composed of different sub-filters. Each sub-filter has to be sampled at N/L equally spaced points around the unit circle (Mitra, 1993). This new modified frequency-sampling method improves the design of multi-band FIR filters in terms of frequency resolution, and similarity of the results to the desired frequency response.

Additional characteristics to take in count are the maximally flat passband design, where incorporating maximal flatness on the passband in the minimization criterion and minimizing the energy of the error between the desired and designed frequency in the pass-bands and stop-bands. Therefore obtain a smooth frequency response as a result (Hanna, 1996).

Because the constant phase shift of FIR filters some long length filters will introduce delays, when this is undesirable for the application the design of complex filters with non-

linear phase is needed. Such type of filter can be designed using the L_∞ or L_2 criteria (Psarakis, 2003) (Alkhairy, 1993).

The word immunity is also an important part of the FIR filter design, by the reason that coefficients are rounded or truncated after being designed assuming infinite precision. It is important to design the filter with finite precision so specifications meet with the minimum hardware (T. Kah-Howe, 2001). Although FIR filters provide a good perform related to word immunity due the polynomial approximation of the impulse response.

In this chapter we covered the different aspects related to the design of FIR filters specially the frequency sampling method, its applications and how it is related to the design of multi-band or arbitrary frequency response filters. The diverse advantages were seen such as unconditional stability, phase linearity (if desired), easy design of arbitrary filters, word length immunity. Also the main drawbacks of the arbitrary frequency response filter design were discussed and different approaches where shown. Another inconvenient is the computational load due the high number of coefficients needed, and how this limitation affects the resulting filter is discussed in the following chapters.

Chapter 6

INFINITE IMPULSE RESPONSE FILTERS

The design of IIR filters is preferred in applications where the computational cost is the main issue, since IIR filters can be implemented with less memory, at the lowest cost, and with less computational complexity compared with FIR filters. Also when speed is an important factor (e.g. real time operation) IIR filters are more desirable because the number of coefficients is significantly less than its counterpart. Due its recursive nature unconditional stability or phase linearity are not guaranteed, and the design methods are more complex than the FIR.

The bilinear z-transform, the pole-zero placement technique and biquad filtering are the most common strategies for IIR filter design. Parametric or multi-band IIR filters are commonly designed using a cascade of biquad filters where each filter represents a band with different center frequency, bandwidth and gain. Therefore design the filters directly in the frequency domain without the need of an analog counterpart (Reiss, 2011).

A different approach is to start the design from an analog filter, and through the bilinear transformation map the s-space into the z-space using the ‘pre-wrapped’ design variables for the center frequency and bandwidth (R.A. Losada, 2005). Furthermore these designs can be improved into high-order filters such as Chebyshev, Butterworth or elliptic with flatter pass-bands and sharper stop-bands, although higher computational cost (Orfanidis, 2005).

Thus the design of IIR filters can be seen with two main approaches; the direct synthesis in the digital domain, and the conversion of analog filters into digital ones. This chapter is focused in the design of a parametric filter using the bilinear transform, followed by a look

to biquad filtering and the different approaches available in current literature for the design of parametric filters using IIR filters.

The bilinear z-transform

In Chapter 2, the relation between the s-plane and z-plane along with the different characteristics of the z-transform were shown. The bilinear method is based on this transform which is used to map the analog frequency axis into the digital frequency axis. Thus convert an analog filter transfer function into a discrete filter transfer function, followed by obtaining the difference equation and hence the filter coefficients of the impulse response. An important consideration is to design the analog filter with the ‘pre-wrapped’ parameters (Gaydecki, 2004).

The mapping between the s-plane and the z-plane is implemented with the following expression:

$$s = \frac{2}{T} \left[\frac{1 - z^{-1}}{1 + z^{-1}} \right] \quad (6.1)$$

The frequencies in the s-domain and z-domain are related by:

$$\Omega = \frac{2}{T} \tan\left(\frac{\omega T}{2}\right) \quad (6.2)$$

$$\omega = \frac{2}{T} \tan^{-1}\left(\frac{\Omega T}{2}\right) \quad (6.3)$$

Where Ω is the analog frequency, ω is the discrete normalized frequency equal to $2\pi f/f_s$. Because the non-linear relationship between both type of frequencies an adjustment to the design parameters has to be done, thus the analog filter is designed based on a ‘pre-wrapped’ frequency Ω^* .

$$\Omega^* = \frac{2}{T} \tan\left(\frac{\Omega T}{2}\right) \quad (6.4)$$

In summary the bilinear transform is a non-linear one-to-one mapping where by simply applying equation (6.1) to an analog filter transfer function the discrete transfer function

and thereby the difference equation is obtained. It is of our interest the design of a multi-band parametric filter; hence the application of this method to an analog band-pass filter is shown.

In Figure 6.1 we can see the RLC circuit corresponding to a band pass filter and the analog transfer function.

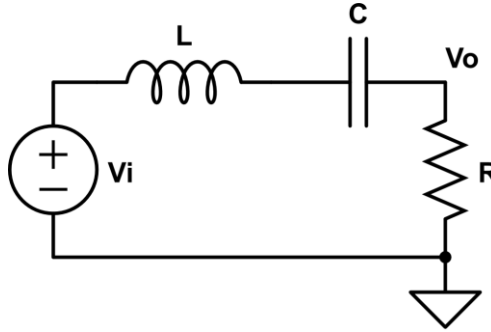


Figure 6.1 Analog band-pass filter.

$$H(s) = \frac{Rs}{Rs + Ls^2 + \frac{1}{C}} \quad (6.5)$$

The discrete transfer function can be found using (6.1) and (6.5), also the difference equation applying the time-shift property of the z-transform from equation (2.4).

$$H(z) = \frac{\alpha(1 - z^{-2})}{\frac{1}{2} - \gamma z^{-1} + \beta z^{-2}} \quad (6.6)$$

$$y[n] = 2\{\alpha[x[n] - x[n - 2]] + \gamma y[n - 1] - \beta y[n - 2]\} \quad (6.7)$$

The coefficients of the filter can be calculated using the following equations (J. Lane, 1991).

$$\beta = \frac{1}{2} \frac{1 - \tan\left(\frac{\theta_0}{2Q}\right)}{1 + \tan\left(\frac{\theta_0}{2Q}\right)} \quad (6.8)$$

$$\gamma = \left(\frac{1}{2} + \beta\right) \cos(\theta_0) \quad (6.9)$$

$$\alpha = \left(\frac{1}{2} - \beta\right) / 2 \quad (6.10)$$

It can be seen that the coefficients only depend on θ_0 and Q which determine the center frequency and bandwidth respectively.

$$\theta_0 = 2\pi(f_0/f_s) \quad (6.11)$$

$$Q = f_0/(f_u - f_l) \quad (6.12)$$

Where f_0 is the center frequency, f_s the sample frequency and $f_u - f_l$ the upper and lower cut-off frequency. Therefore the design of a multi-band parametric filter can be implemented based on this method.

Biquad design strategies

A common design for parametric or multi-band IIR filters is the biquad strategy, where using a cascade of second order biquad sub-filters a high-order filter can be achieved. Thus avoiding the instability related to the normal approach: pole and zero placement of high order filters. Each biquad filter can be used to represent a band with different center frequency, bandwidth and gain whose transfer function can be described by the following generalization.

$$H(z) = c \frac{(z - a_0)(z - a_1)}{(z - b_0)(z - b_1)} \quad (6.13)$$

Where a_0, a_1 and b_0, b_1 are complex conjugate pairs and represent the zeros and poles of the sub-filter respectively. This led to a difference equation with five coefficients.

$$y[n] = c\{x[n] - 2\alpha_0 x[n-1] + \varepsilon_0 x[n-2]\} + 2\alpha_1 y[n-1] - \varepsilon_1 y[n-2] \quad (6.14)$$

The coefficients $\alpha_0, \alpha_1, \varepsilon_0$ and ε_1 are determined by the real and imaginary parts of the complex conjugate pairs in (6.13). There are exactly five coefficients that will satisfy the five design equations of a parametric filter (Reiss, 2011).

$$H(0) = 1$$

$$H(\pi) = 1$$

$$\left. \frac{d|H(\omega)|}{d\omega} \right|_{\omega_c} = 0$$

$$\begin{aligned}
|H(\omega_c)| &= G \\
\omega_u - \omega_l &= B, \text{ where} \\
|H(\omega_u)| &= |H(\omega_l)| = G_B
\end{aligned} \tag{6.15}$$

The gain at DC and at the Nyquist point is set to a unity, there is a maximum or minimum at the center frequency ω_c , and the magnitude of the frequency response at the center frequency is set by G . The bandwidth B is determined by $\omega_u - \omega_l$ the difference between the upper and lower cut-off frequency, and the magnitude of the frequency response at the cut-off frequencies is G_B , normally the -3dB point (Reiss, 2011).

For the design of a parametric filter using IIR filters two type of approaches were presented whose difference equations and relations with the parametric filter constrains were shown. In equation (6.7) the three coefficients are uniquely determined by the bandwidth and center frequency of the designed analog band-pass filter. A boost or cut of the gain can be implemented easily thus turning the filter into a parametric one. An implementation of a 10-band equalizer using this method is presented in (Montgomery, 2001).

As an alternative a direct design in the digital domain was studied, where the five coefficients of equation (6.14) are related to the poles and zeros of each biquad sub-filter and the placement of these is subject to the five constraints in (6.15). In (Reiss, 2011) is shown that there are 32 possible solutions to the parametric filter which some are unstable or with non-minimum phase and only one solution is stable with minimum phase behaviour.

A different option is the design of digital Butterworth and Chebyshev filters without the need of an analog counterpart, using the pole and zero placement technique (A. Fernandez-Vazquez, 2006). In (Bristow-Johnson, 1994) the equivalence of computing various methods of biquad filtering design is presented, where all the methods resulted in the same filter coefficients as long as each approach uses the same definition of bandwidth. Hence the filter performance depends on the implementation and not in the method to obtain the filter coefficients.

In this chapter we covered the different aspects related to the design of IIR filters specially the bilinear transform and biquad design methods, and how each method is related to the design of parametric filters. In comparison with the type of filters showed in Chapter 5, the main advantage of the IIR filters over the FIR is the low computational cost due the few number of coefficients needed. Since the number of coefficients is less the sensitivity to word-length is bigger, however the possible implementation of IIR responses using FIR filters (using the IIR impulse response as a convolution) mitigate this sensitivity, and negated the risk of instability (Gaydecki, 2004).

Chapter 7

METHODS AND PROCEDURE

In this chapter the different methods and procedures taken in the development of this project are shown, and how with the use of different software and techniques; the application of a 20-band real time parametric filter is carried out. The results and proper discussion are presented in the chapters to follow.

The first approach to be conducted is the implementation of a 20-band offline semi-parametric equalizer based on IIR filtering and using Delphi. Followed by the execution of a 20-band real-time parametric equalizer designed using Matlab GUI (Graphical User Interface) and implemented through the Signal Wizard System. The latter can be designed either with FIR or IIR filters.

20-band semi-parametric equalizer (offline)

A semi-parametric equalizer was implemented with a parallel 20 filter bank. Each filter is based on the bilinear transform method described on Chapter 6, where a band-pass filter is designed using the difference equation (6.7). The coefficients can be obtained through (6.8-6.10) depending solely on the center frequency and bandwidth of each band-pass.

After the sampled signal is filtered through the 20 band-pass filters each output is scaled by a gain, which has a range from 0.1 to 10 (i.e. -20dB to 20dB). Then the 20 results are summed and the final output is obtained. The IIR equalizer is explained in Figure 7.1, where $x[n]$ and $y[n]$ is the input and output signal respectively, and $h_i[n]$ and g_i are the corresponding sub-filter frequency response and gain adjustment.

The equalizer was programmed using Delphi, by the reason that is a visual programming system based in Pascal, where DSP algorithms and Wav file manipulation are easily executed.

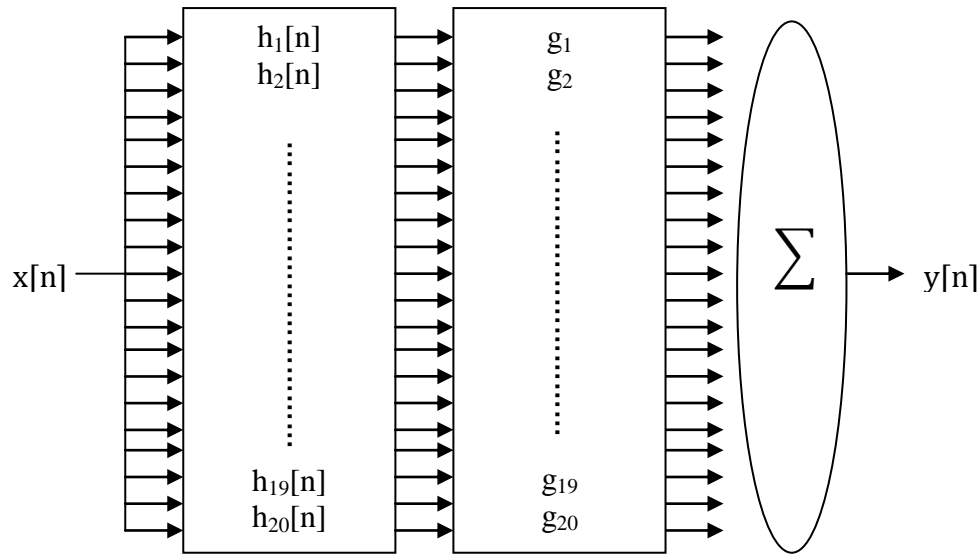


Figure 7.1 Equalizer flow diagram.

The program is based on two units:

Graphic equalizer: This is the main unit of the equalizer which is in charge of the user interface. The gain of each band-pass is controlled through 20 scrollbars whose range is from -20dB to 20dB. The Q factor of all the filters can be adjusted by an Edit box, and the center frequency of the filters is fixed. By the reason that is an offline filter the user has to load a wav file (16 bit – 1 channel), then set the gain and bandwidth of the needed equalization and finally by pressing the 'Filter' button a filtered version of the file is saved.

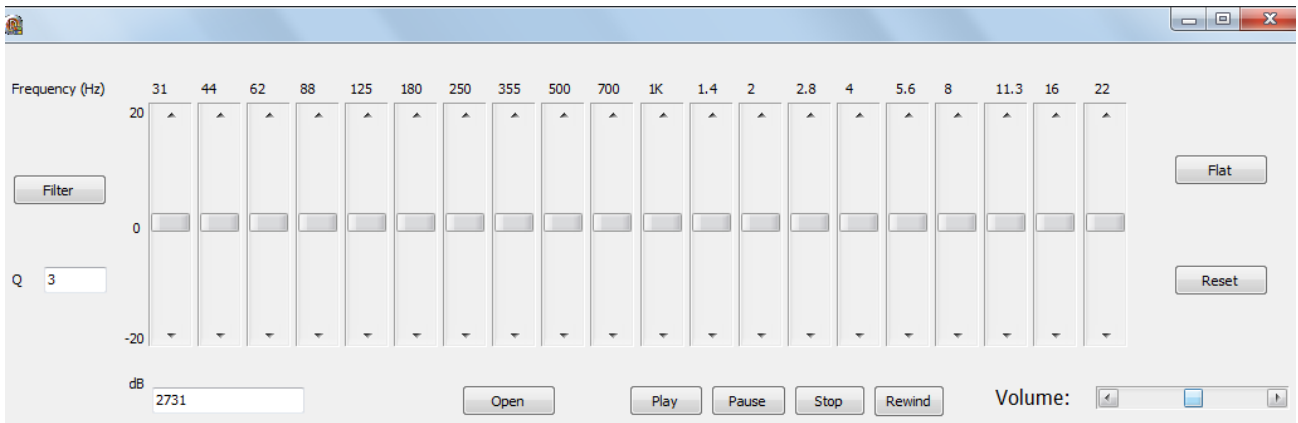


Figure 7.2 Screenshot of the offline semi-parametric equalizer program.

The user can listen to the current wav file through the Media Player controls: Play, Pause, Stop, and Rewind. The 'Flat' button sets all the gains to 0dB, and the 'Reset' button to -20dB and finally a volume control is included.

| | | | | | | | | | |
|------|------|------|------|------|------|------|-------|-------|-------|
| 31 | 44 | 62 | 88 | 125 | 180 | 250 | 350 | 500 | 700 |
| 1000 | 1400 | 2000 | 2800 | 4000 | 5600 | 8000 | 11300 | 16000 | 22000 |

Table 7.1 Center frequencies in Hz of the 20 band-pass filters.

The 20 band-pass filters are executed by the IIR_Filter unit, but this unit implements the gain adjustment and final summation of the 20 filtered outputs, thus creating and saving a new wav file.

IIR Filter: This secondary unit performs an IIR filter based on the difference equation (6.7). Since the equation is the same for the 20 filters, the unit is 're-used' calculating the different coefficients for based on the input parameters, which are the center frequency, Q factor, sample rate of the input wav file and a small_int array which has the numerical values of the wav file. This recursive nature reduces the needed memory space and code length.

Because simplicity the equalizer only is able to filter single channel wav files, and is considered semi-parametric due the fixed center frequency, which cannot be modified by the user, and due the lack of singular bandwidth control. Although an improvement in these matters is very straightforward, and they were performed in the 20 band real-time parametric equalizer. The results were analyzed using the wav processing software Soundtrack.

20-band parametric equalizer (real-time)

Following the design of the offline equalizer a real-time parametric equalizer filter was aimed. In this program the user is able to generate an arbitrary frequency response based on a FIR filter, where the frequency response is created via 20 band parametric filters. Also the program can generates an impulse response based on IIR filters, where each band is designed in the same way of the offline equalizer. This frequency or impulse response can be imported to the Signal Wizard System and executed in real time.

The equalizer was programmed using the Matlab GUI (Graphical User Interface) by the reason that Matlab is more orientated to numerical computation and together with the interactive environment from the GUI provides a better tool for the desired requirement. The program can be divided in three main parts: User interface, FIR parametric filters, and IIR parametric filters.

User interface

The user interface was created using the Matlab GUI, which development environment provides a friendly point-and-click layout editor for designing user interfaces. GUIDE (GUI development environment) generates the Matlab code which can be modified in order to create custom applications as in this case the parametric equalizer.

The interface consists of 20 scrollbars in charge of controlling the gain of each band-pass filter (from 20dB to -20dB), 40 edit text boxes for adjusting the 20 center frequencies and 20 Q factors. An additional text box with its corresponding action button 'Q' is present and will set all the bandwidths to the same Q factor. The different filter parameters such as No. of taps, sample rate and type of filter can be adjusted through 3 pop-up menus at the 'Filter Parameters' panel. The frequency or impulse response is generated as an ASCII text file by pressing the 'Filter' button, and finally the 'Reset' button sets all the parameters to the default.

Through the interface the user sets the different parameters for the desired equalizer, together with the given number of taps, sample rate, gains, center frequencies and bandwidths. Then the program creates the frequency response or impulse response if FIR filters or IIR filters are required respectively. This is done by the FIR or IIR parametric filters functions described below.

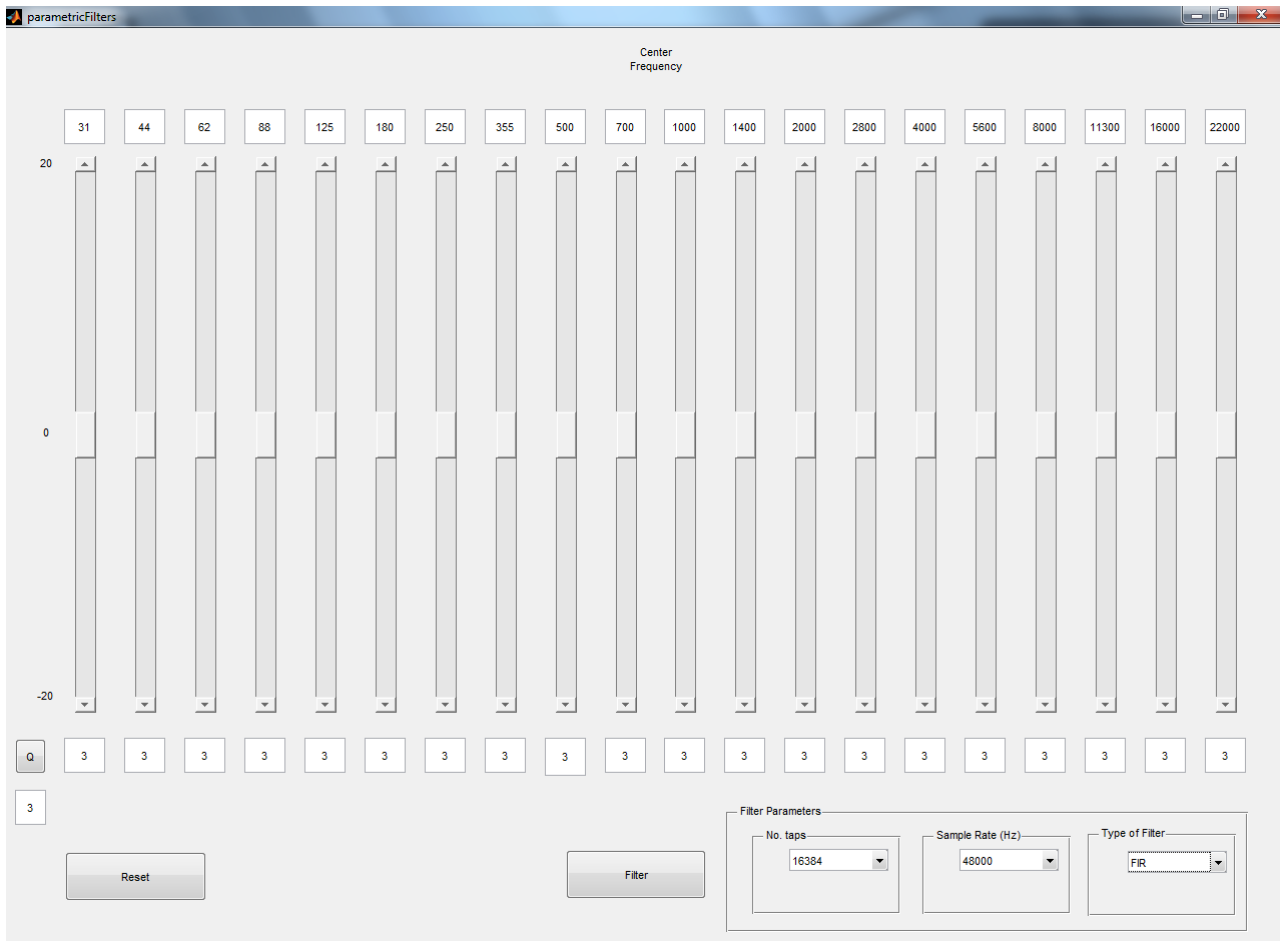


Figure 7.3 Screenshot of the real-time parametric equalizer program.

FIR parametric filters

The filter is calculated using the arbitrary frequency response design described on chapter 5, where based on the frequency sampling method a 20 band parametric filter can be created. This is performed by an algorithm which following the established parameters of each band sets the numerical values of the desired frequency response in a 'brick-wall' way for each band.

The algorithm is very simple and can be divided in the following steps once all the parameters are introduced by the user.

1. Establish the frequency resolution of the filter, which is determined dividing the Nyquist frequency by the number of taps, and whose numerical value is the resulted increment in each frequency bin.
2. Sort in ascending order (based on center frequency) the band-pass filters to be built.

3. Determine the corresponding number of frequency bins for the band-pass filter.

This fixes the bandwidth of the filter and depends on the Q factor and center frequency. Thus to determine the upper and lower frequencies the following equations are used.

$$f_u = f_c + \frac{f_c}{2 * Q} \quad (7.1)$$

$$f_l = f_c - \frac{f_c}{2 * Q} \quad (7.2)$$

Where f_c is the center frequency, f_u and f_l the upper and lower frequencies respectively. The lower and upper frequency bins are obtained by approximating the result of equations (7.1) and (7.2) to the nearest frequency bin.

4. Once the bandwidth of the band-pass filter is represented in frequency bins, the obtained 'brick-wall' filter is multiplied by the corresponding gain G_n .
5. Calculate the upper frequency bin of the transition region (if there is one), i.e. where the next band-pass filter starts.
6. Repeat step 3, 4 and 5 for the following band-pass filter until the 20 bands are represented in the arbitrary frequency response.

Thus a rectangular filter of gain G_n , center frequency f_c and with bandwidth range from f_l to f_u is built for each band.

By default when the Q factor of all the band-pass filters is 3 (i.e. the bandwidth is 33.33% of the center frequency), the bands are equally spaced in the spectrum. This means a totally flat response by the reason that when each 'brick-wall' ends the next band-pass filter will start at the next frequency bin, therefore no transitions regions will occur between bands.

When the bandwidths are increased the bands may overlap creating a region which gain is the sum of the overlapping bands. And when the bandwidths are decreased, the rectangular filter is narrowed and not overlap occurs, the transition region is set to 0dB (i.e. a gain of 1).

The numerical values of the final frequency response are exported into an ASCII text file.

IIR parametric filters

The construction of the parametric equalizer based on IIR filters is performed using the same method applied in the design of the offline equalizer. Where each filter is based on the difference equation (6.7) and the coefficients can be obtained through (6.8-6.10). Thus 20 band-pass filters are designed, scaled by a gain, and finally summed together (Figure 7.1) in order to obtain a frequency response whose impulse response is easily found.

In chapter 6 we discussed the advantages of FIR implementations of IIR filters, where rather than use the IIR difference equation, the impulse response is used as a FIR convolution. This can produce results which the advantages of both type of filters.

Once all the parameters are set by the user the impulse response can be obtained through the following steps.

1. Calculate the coefficients of the difference equation for the respective band-pass filter.
2. Obtain the frequency response of the band-pass filter.
3. Multiply the frequency response by the corresponding gain.
4. Sum the resulting response to the overall frequency response of the parametric equalizer.
5. Repeat from 1 to 4 for each individual band-pass filter.
6. Obtain the impulse response

The coefficients in step 1 are calculated using equations (6.8-6.10). The frequency response and the coefficients are obtained through MATLAB and the impulse response is exported into an ASCII text file.

Finally the parametric equalizer filter is imported to the Signal Wizard System as a frequency or impulse response. The obtained results are shown in the following chapter.

Chapter 8

RESULTS

After the implementation of the offline semi-parametric and real-time parametric equalizers described on last chapter different types of tests were conducted to obtain the results presented in this chapter. Without losing generality the tests were focused on the behaviour of a single band of the equalizer, these ones applied to white noise signals and single sine waves. In the first equalizer the results were obtained and analysed using Soundtrack, which is a wave processing software. The second equalizer was executed through the Signal Wizard platform and the results obtained with a digital oscilloscope (Tektronix).

20-band offline semi-parametric equalizer

Using the equalizer created in Delphi (Figure7.2) a filtered wav file is generated. The adjustable parameters are the different gains of the 20 bands, the Q factor which is the same for all the IIR sub-filters and the center frequencies which are fixed (Table 7.1). The signal to be filtered has a flat spectrum (white noise) and the applied filter corresponds to a band-pass with center frequency 4000 Hz, gain 20 dB and Q factor 1, 3 and 10. Also a notch filter with center frequency 4000 Hz, gain -20 dB and Q factor 4 was executed. And finally the different coefficients of the IIR sub-filters are shown.

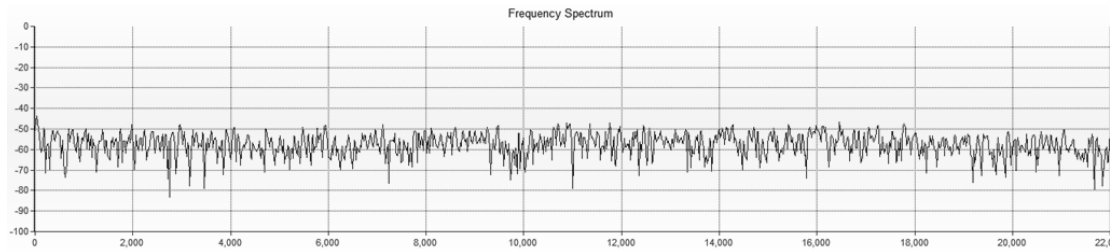


Figure 8.1 Frequency spectrum of the input signal (white noise).

In the figures below the frequency spectrums of the output signal are shown. The white noise signal was filtered through a band-pass filter with center frequency 4000 Hz, gain 20 dB and Q factor 1, 3 and 10.

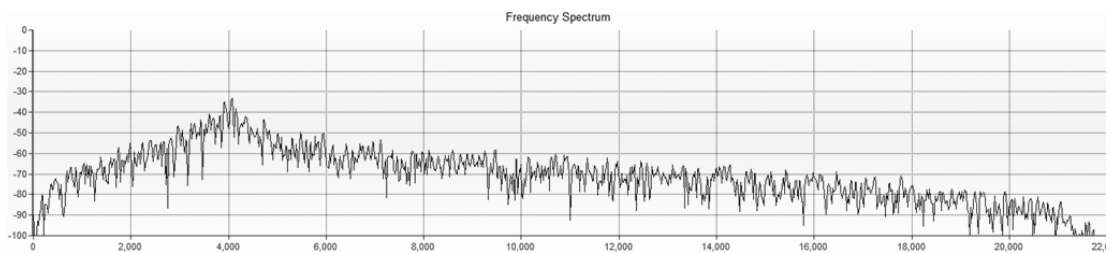


Figure 8.2 Filtered signal spectrum. Parameters: Center frequency = 4000Hz, Gain = 20dB, Q = 10

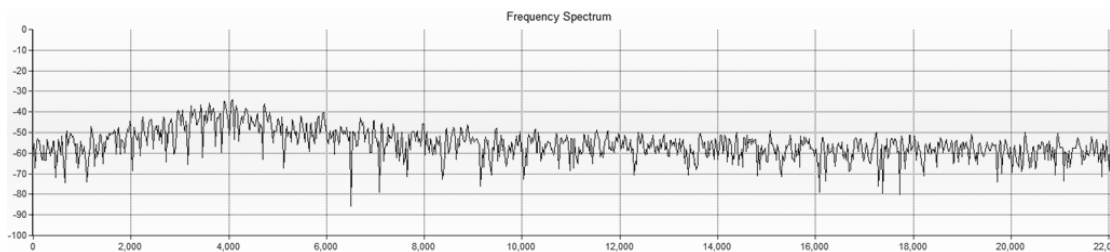


Figure 8.3 Filtered signal spectrum. Parameters: Center frequency = 4000Hz, Gain = 20dB, Q = 3

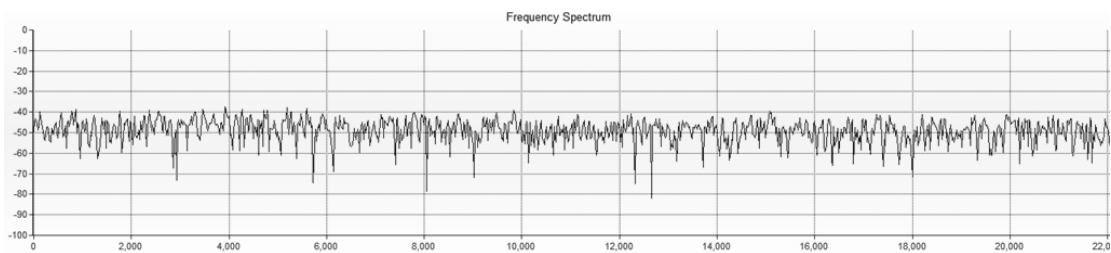


Figure 8.4 Filtered signal spectrum. Parameters: Center frequency = 4000Hz, Gain = 20dB, Q = 1

Following a notch filter with center frequency 4000 Hz, gain -20 dB and Q factor 4 was executed.

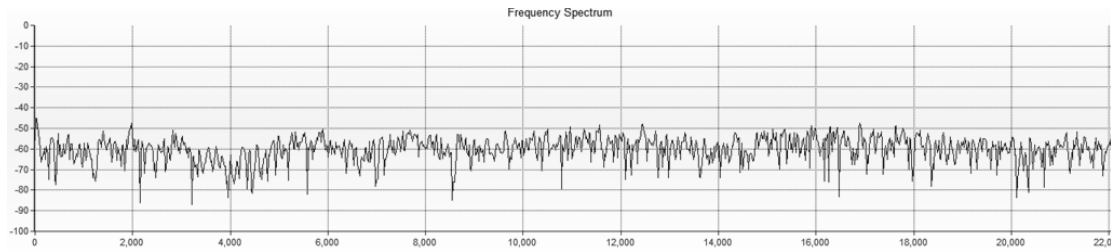


Figure 8.5 Filtered signal spectrum. Parameters: Center frequency = 4000Hz, Gain = -20dB, Q = 4

The coefficients (6.8-6.10) of the 20 IIR band-pass filters are shown in table 8.1. These were calculated with a Q factor of 3.

Coefficients\Center Frequency (Hz)

| | 31 | 44 | 62 | 88 | 125 | 180 | 250 | 350 | 500 |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| β | 0.499324 | 0.499041 | 0.498649 | 0.498084 | 0.49728 | 0.496088 | 0.494575 | 0.492314 | 0.489209 |
| γ | 0.999316 | 0.999024 | 0.998616 | 0.998018 | 0.997147 | 0.995812 | 0.994043 | 0.991243 | 0.987091 |
| α | 0.000338 | 0.00048 | 0.000675 | 0.000958 | 0.00136 | 0.001956 | 0.002712 | 0.003843 | 0.005396 |

| | 1000 | 1400 | 2000 | 2800 | 4000 | 5600 | 8000 | 11300 | 16000 |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| β | 0.478646 | 0.470353 | 0.458166 | 0.442363 | 0.41955 | 0.390643 | 0.350104 | 0.298948 | 0.233154 |
| γ | 0.970273 | 0.954104 | 0.925517 | 0.879771 | 0.796354 | 0.661877 | 0.425052 | 0.073105 | -0.36658 |
| α | 0.010677 | 0.014823 | 0.020917 | 0.028819 | 0.040225 | 0.054679 | 0.074948 | 0.100526 | 0.133423 |

Table 8.1 IIR band-pass filter coefficients. Q = 3.

20-band parametric equalizer

After the execution of the real-time parametric equalizer using FIR or IIR filters the frequency or impulse response of the different band-pass filters was exported to the Signal Wizard 2.5. In order to observe the behaviour of FIR and IIR filters each test was designed with both types of filters, and the signal to be filtered was a white noise or sine wave signal depending on what parameter is being tested.

Therefore band-pass, peak and band-stop IIR and FIR filters with different bandwidth, center frequency and gain were designed. Thus the results correspond to the desired frequency response of the filters, the obtained one, and the spectrum of the filtered signal. Experimental values for the different parameters such as bandwidth, center frequency and boost or cut gain were measured.

Three band-pass filters were designed with 16384 No. of taps, sample rate of 48000Hz and the following parameters.

| | | | |
|-----------------------|------|------|------|
| Center Frequency (Hz) | 4000 | 4000 | 4000 |
| Gain | 10 | 10 | 10 |
| Q | 1 | 3 | 10 |

Table 8.2 Design parameters for the band-pass filters.

The desired frequency response is obtained through the Matlab program, and the resulted frequency response implemented via the Signal Wizard System. The band-pass filters were implemented using 1024 No. of taps, Hanning windows and gain compensation due clipping problems.

The desired and obtained frequency responses using FIR and IIR filters can be seen in figure 8.6 and 8.7 respectively, where the x-axis represents the frequency in Hz (log) and the y-axis the magnitude in dB.

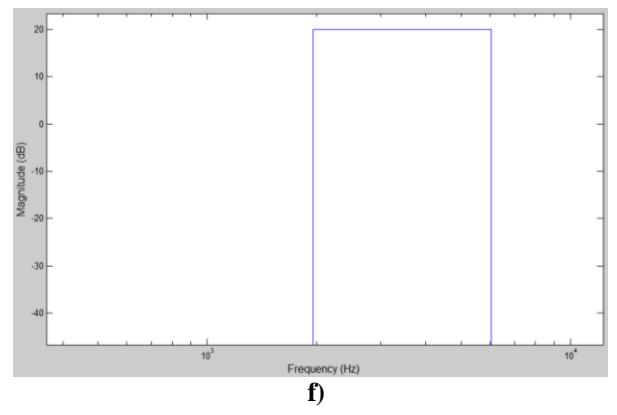
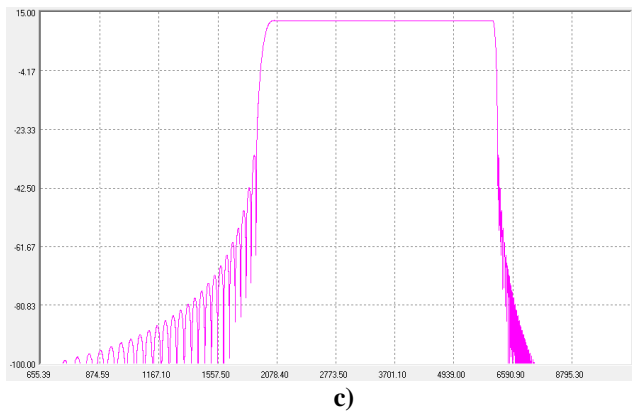
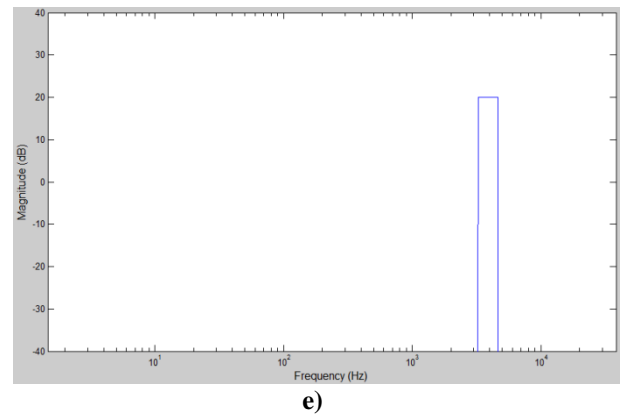
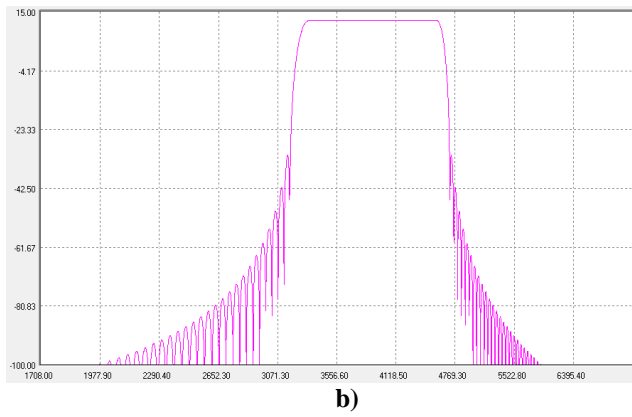
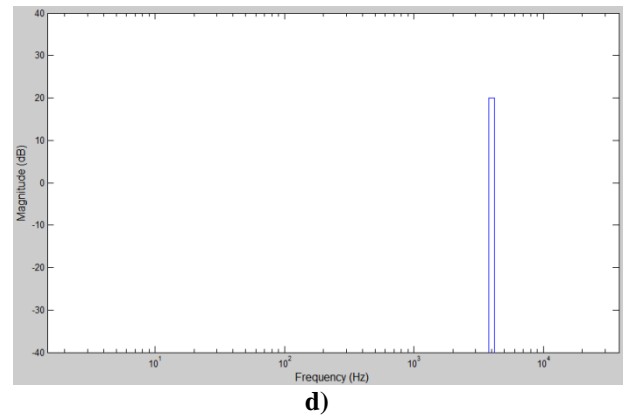
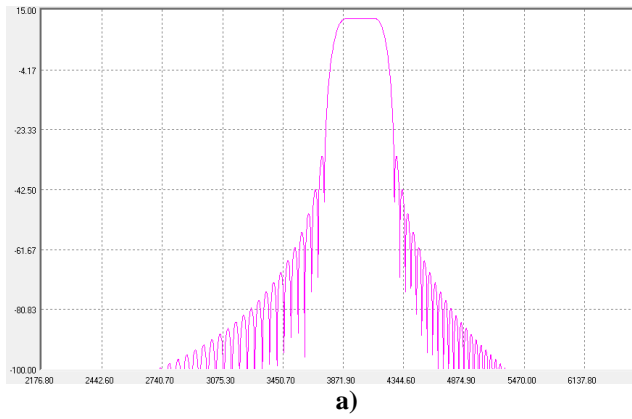
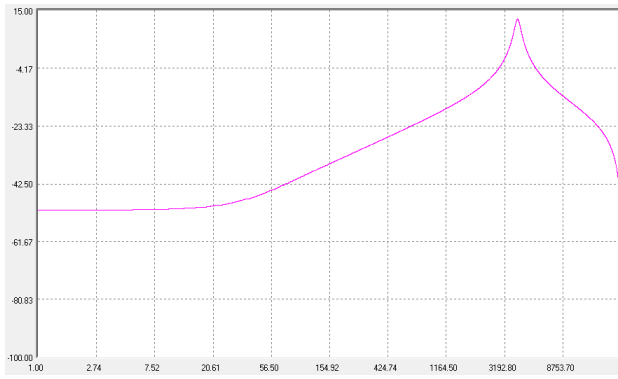
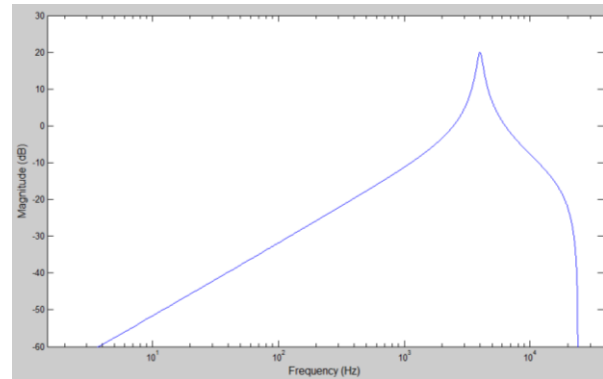


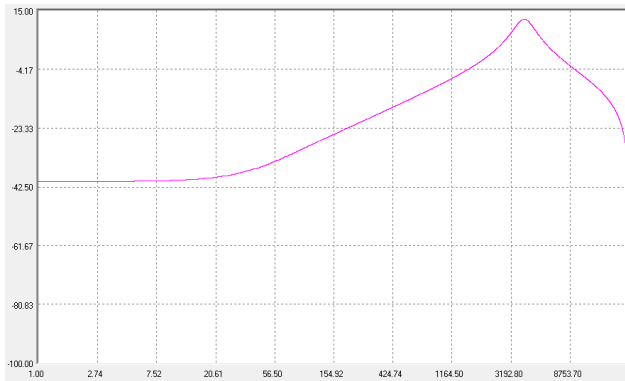
Figure 8.6 Obtained FIR frequency response. 1024 Taps, Hanning window, Center frequency = 4000Hz, Gain = 12dB. **a)** $Q = 10$. **b)** $Q = 3$. **c)** $Q = 1$. Desired FIR frequency response. 16384 Taps, Center frequency = 4000Hz, Gain = 20dB. **d)** $Q = 10$. **e)** $Q = 3$. **f)** $Q = 1$.



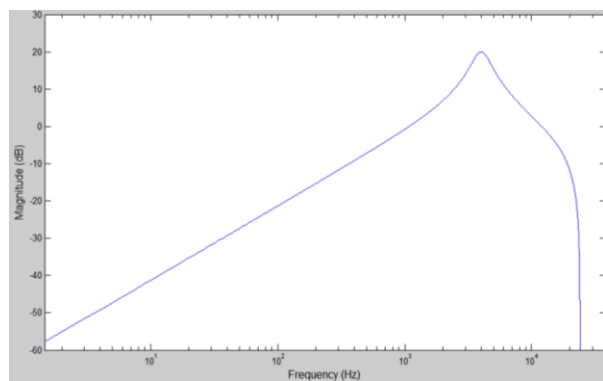
a)



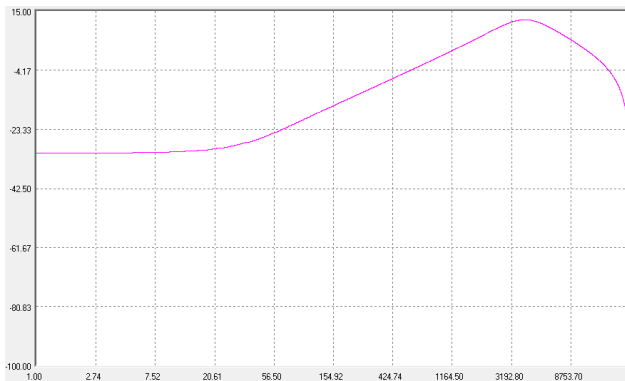
d)



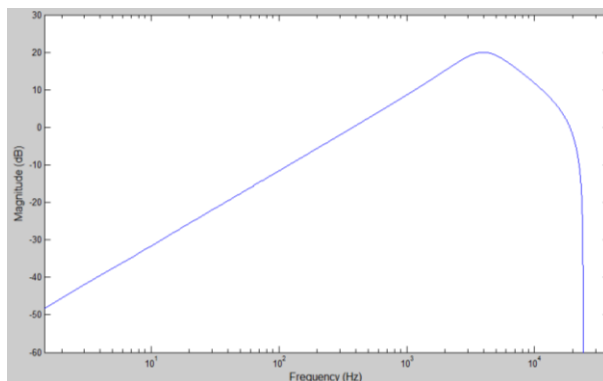
b)



e)



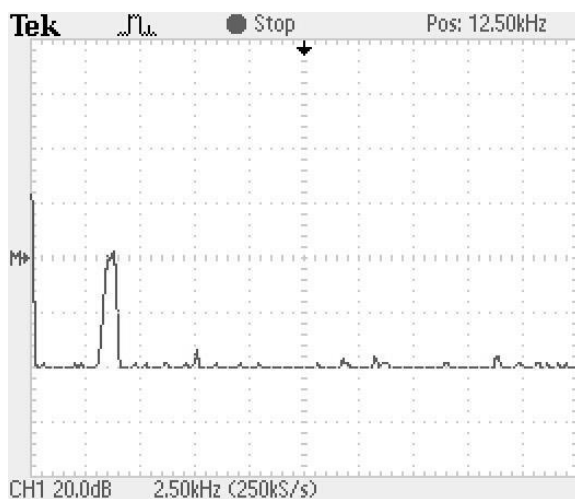
c)



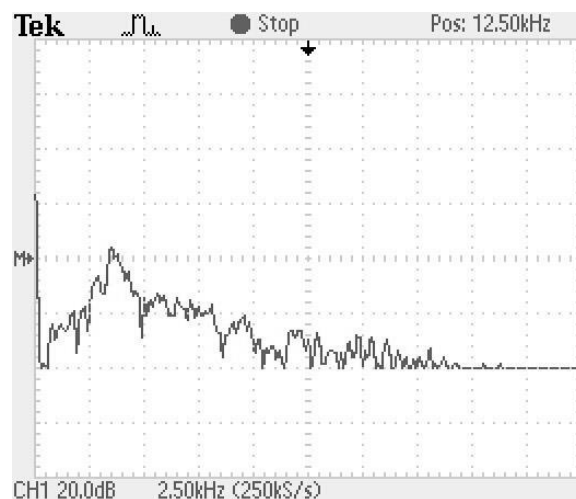
f)

Figure 8.7 Obtained IIR frequency response. 1024 Taps, Hanning window, Gain = 12dB. **a)** $Q = 10$, Center frequency = 4001Hz. **b)** $Q = 3$ Center frequency = 3991Hz. **c)** $Q = 1$, Center frequency = 3991Hz. Desired IIR frequency response. 16384 Taps, Center frequency = 4000Hz, Gain = 20dB. **d)** $Q = 10$. **e)** $Q = 3$. **f)** $Q = 1$.

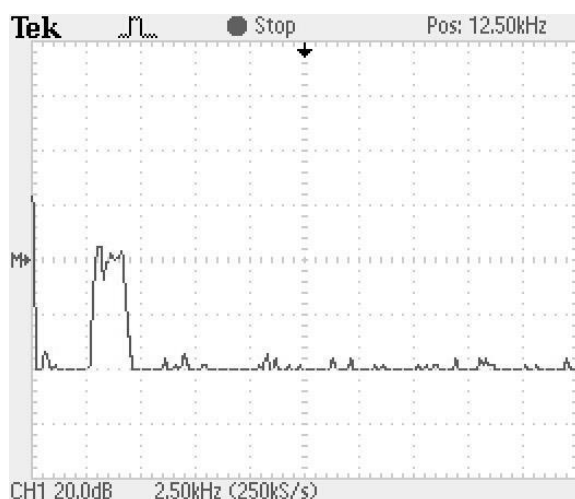
The filters from figures 8.6 (a,b,c) and 8.7 (a,b,c) were executed via the Signal Wizard System to filter a white noise signal. The following figures show the FFT of the filtered signals taken from a digital oscilloscope, where the x-axis represents the frequency in Hz (2.5 kHz linear division) and the y-axis the magnitude in dB (20dB division).



a)



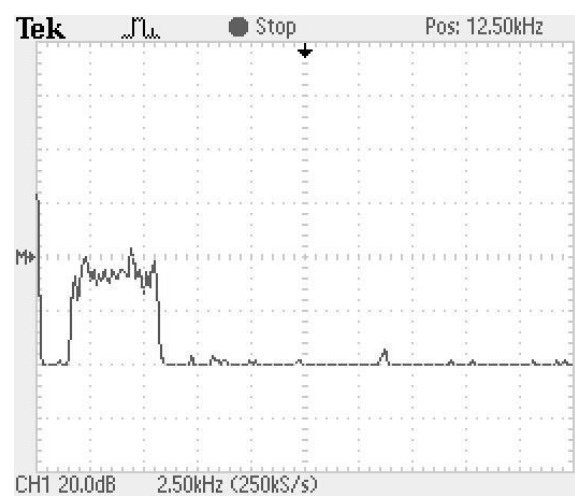
d)



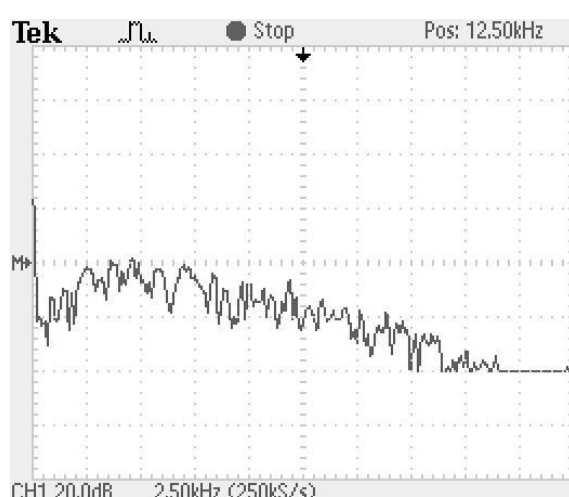
b)



e)



c)



f)

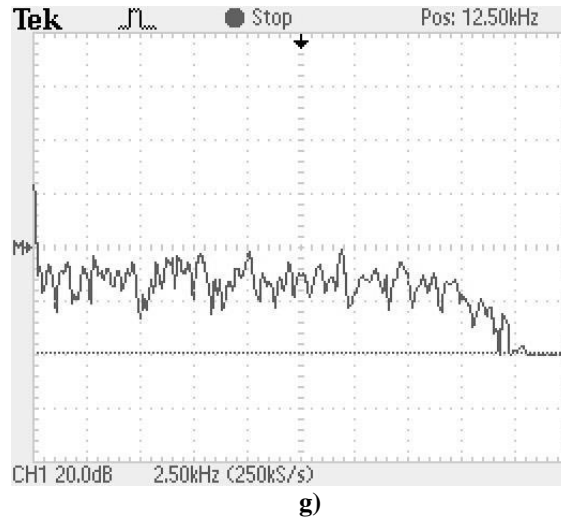


Figure 8.8 Obtained FFT using Hanning windows after filtering a white noise signal. x-axis magnitude in dB (20dB per division), y-axis linear frequency (2.5 kHz per division). Using: **a)** FIR filter 8.6a. **b)** FIR filter 8.6b. **c)** FIR filter 8.6c. **d)** IIR filter 8.7a. **e)** IIR filter 8.7b. **f)** IIR filter 8.7c. Also in **g)** FFT of the white noise signal.

The measured parameters such as bandwidth, real Q factor, center frequency, and gain of the desired frequency response via Matlab, the designed frequency response implemented through the Signal Wizard and the experimental FFT (oscilloscope) after filtering a white noise signal can be seen in the following table.

| Filter | Frequency spectrum | Bandwidth (Hz) | | | Center frequency (Hz) | gain (dB) |
|----------|--------------------|-----------------|-----------------|------|-----------------------|-----------|
| | | lower frequency | upper frequency | Q | | |
| FIR 8.6a | Matlab | 3812 | 4189 | 10.6 | 4000 | 20 |
| | Signal Wizard | 3830 | 4172 | 11.7 | 4000 | 15 |
| | Oscilloscope | 3500 | 4100 | 6.3 | 3800 | 15.2 |
| IIR 8.7a | Matlab | 3806 | 4204 | 10.1 | 4000 | 20 |
| | Signal Wizard | 3804 | 4203 | 10.0 | 4001 | 15 |
| | Oscilloscope | 3500 | 4100 | 6.5 | 3900 | 16 |
| FIR 8.6b | Matlab | 3243 | 4630 | 2.9 | 4000 | 20 |
| | Signal Wizard | 3263 | 4600 | 3.0 | 4000 | 15 |
| | Oscilloscope | 2900 | 4500 | 2.4 | 3800 | 16.8 |
| IIR 8.7b | Matlab | 3385 | 4715 | 3.0 | 4000 | 20 |
| | Signal Wizard | 3368 | 4728 | 2.9 | 3991 | 15 |
| | Oscilloscope | 3500 | 4400 | 4.3 | 3900 | 17.6 |
| FIR 8.6c | Matlab | 1954 | 6045 | 1.0 | 4000 | 20 |
| | Signal Wizard | 1976 | 6037 | 1.0 | 4000 | 15 |
| | Oscilloscope | 1800 | 5900 | 1.0 | 4000 | 14.4 |
| IIR 8.7c | Matlab | 2432 | 6420 | 1.0 | 4000 | 20 |
| | Signal Wizard | 2427 | 6479 | 1.0 | 3991 | 15 |
| | Oscilloscope | 2500 | 6600 | 1.1 | 4600 | 14.4 |

Table 8.3 Measured parameters (bandwidth, real Q factor, center frequency, and gain) of the desired (Matlab), designed (Signal Wizard) and implemented (oscilloscope) band-pass filters (8.6a-b-c and 8.7a-b-c).

A band-stop and peak filter with parameters shown in table 8.4 were designed and implemented. The band-stop filter was executed to filter a white noise signal whereas the peak filter a sine wave summed with white noise. The frequency of the sine signal was 6432 Hz. Both were designed using 16384 taps and 48 kHz as the sample rate.

| | Band-stop filter | Peak filter |
|-----------------------|------------------|-------------|
| Center Frequency (Hz) | 8000 | 6432 |
| Gain | 0 | 20 |
| Q | 3 | 1000 |

Table 8.4 Design parameters for the band-stop and peak filters.

The filters were implemented via the Signal Wizard using 1024 taps and Hanning windows. The desired and obtained frequency responses using FIR and IIR filters are shown in the following figures, where the x-axis represents the frequency in Hz (log) and the y-axis the magnitude in dB. Following by the resulting experimental FFT where the x-axis represents the frequency in Hz (2.5 kHz linear division) and the y-axis the magnitude in dB (20dB division).

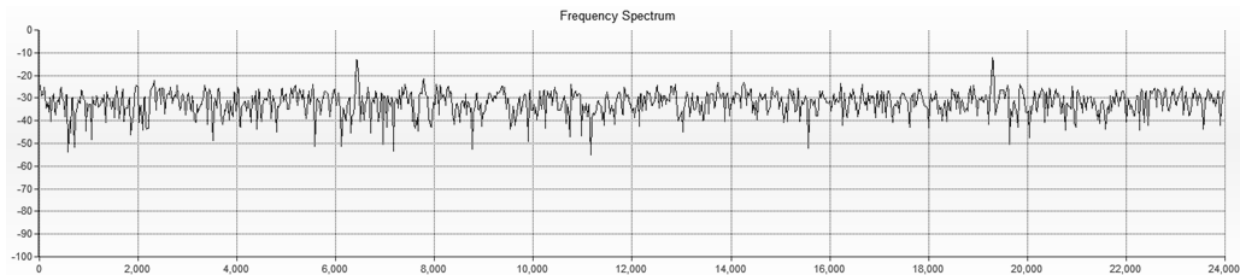


Figure 8.9 Frequency spectrum of a sine wave with frequency 6432 Hz summed with white noise.

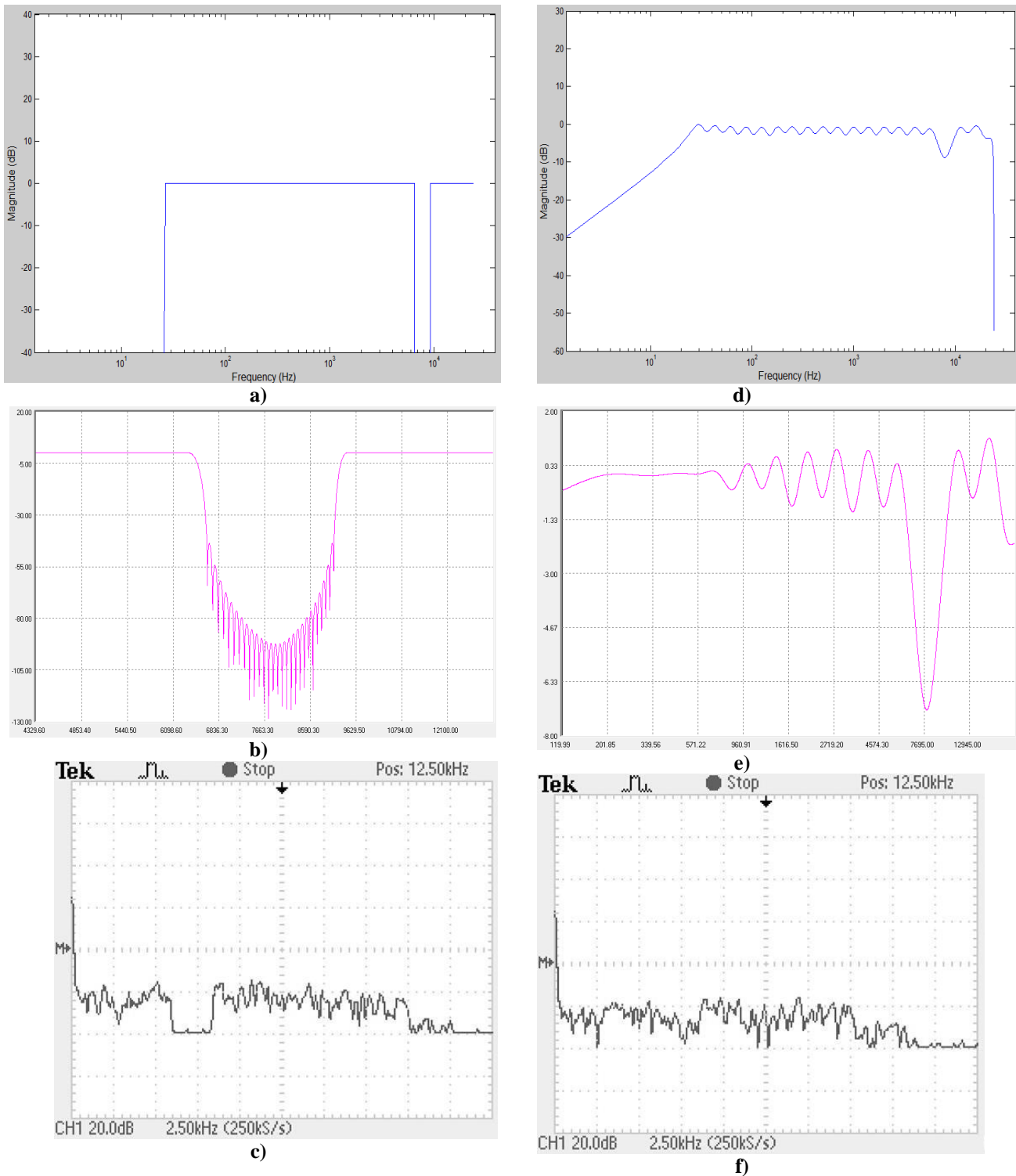
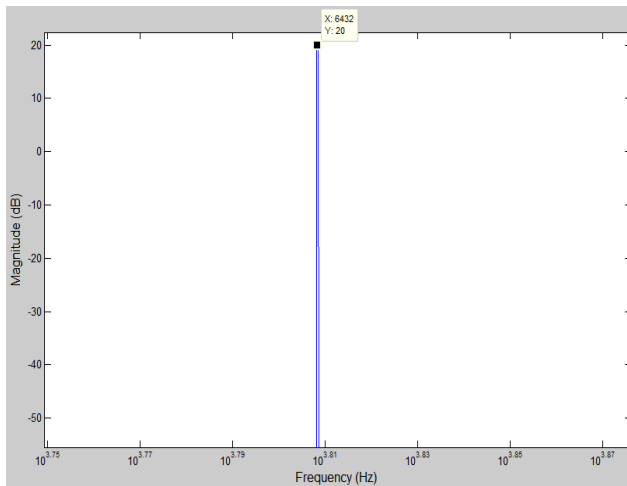
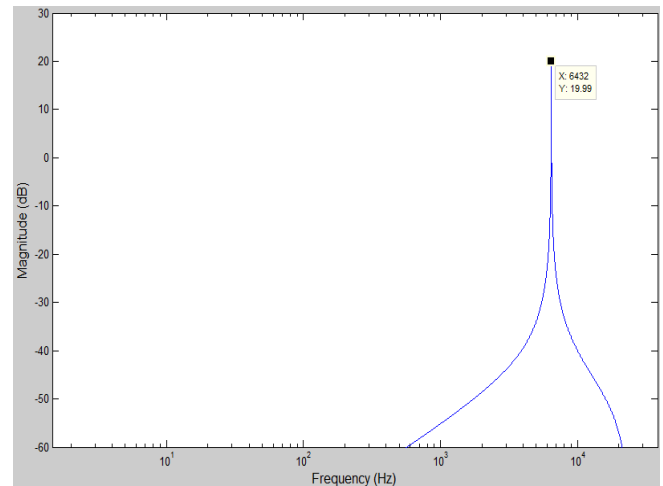


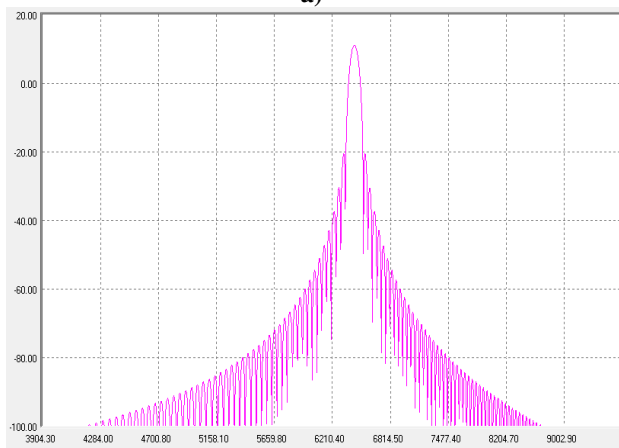
Figure 8.10 Band-stop frequency responses for FIR and IIR filters. FIR: **a)** Desired (Matlab), 16384 taps, Center frequency = 8 kHz, $Q = 3$, Gain = 0 **b)** Designed (Signal Wizard), 1024 taps, Hanning window. **c)** Resulting FFT after filtering a white noise signal (Figure 8.8). IIR: **d)** Desired (Matlab), 16384 taps, Center frequency = 8 kHz, $Q = 3$, Gain = 0 **e)** Designed (Signal Wizard), 1024 taps, Hanning window. **f)** Resulting FFT after filtering a white noise signal (Figure 8.8).



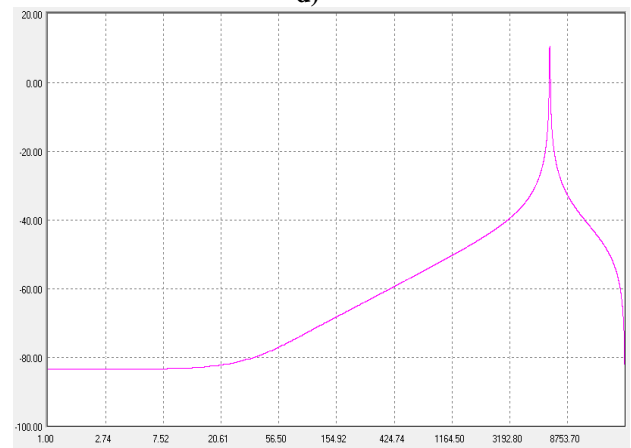
a)



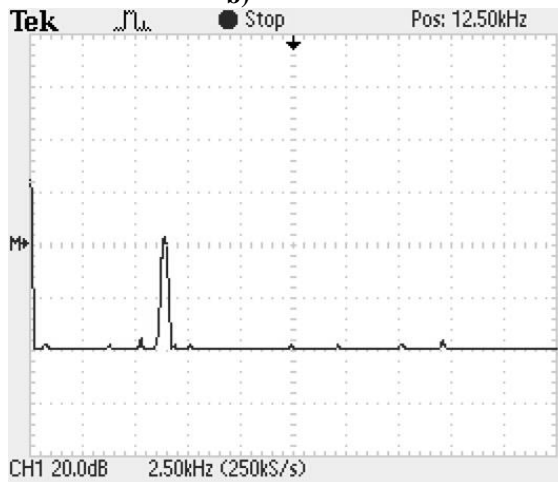
d)



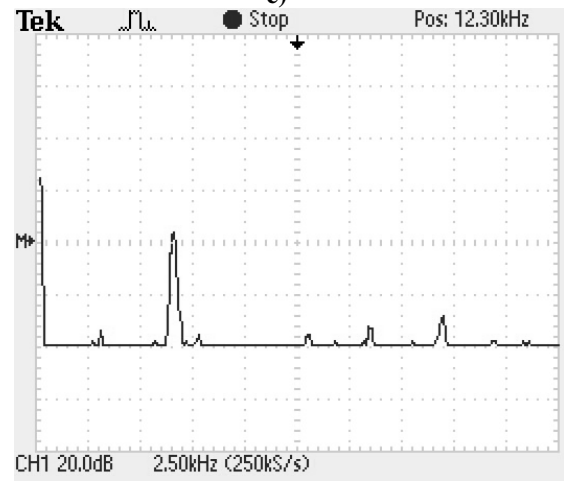
b)



e)



c)



f)

Figure 8.11 Peak filter frequency responses for FIR and IIR filters. FIR: **a)** Desired (Matlab), 16384 taps, Center frequency = 6432 Hz, $Q = 1000$, Gain = 10 **b)** Designed (Signal Wizard), 1024 taps, Hanning window. **c)** Resulting FFT after filtering a sine wave summed with white noise (Figure 8.9). IIR: **d)** Desired (Matlab), 16384 taps, Center frequency = 6432 Hz, $Q = 1000$, Gain = 10 **e)** Designed (Signal Wizard), 1024 taps, Hanning window. **f)** Resulting FFT after filtering signal from Figure 8.9

The measured parameters of the FIR and IIR band-stop and peak filters are shown in the following table. These were obtained through Matlab, Signal Wizard and a digital oscilloscope after filtering the signals from Figure 8.8g and 8.9 with the band-stop and peak filters respectively.

| Filter | Frequency spectrum | Bandwidth (Hz) | | | Center frequency (Hz) | gain (dB) |
|---------------------|--------------------|-----------------|-----------------|--------|-----------------------|-----------|
| | | lower frequency | upper frequency | Q | | |
| FIR Band-stop 8.10b | Matlab | 6483 | 9262 | 2.9 | 8000 | $-\infty$ |
| | Signal Wizard | 6444 | 9306 | 2.8 | 8008 | -123 |
| | Oscilloscope | 6000 | 8500 | 3.2 | 8000 | -20 |
| IIR Band-stop 8.10e | Matlab | 6875 | 9136 | 3.5 | 7923 | -8.8 |
| | Signal Wizard | 6878 | 9130 | 3.5 | 7915 | -7.3 |
| | Oscilloscope | 7100 | 8100 | 7.7 | 7700 | -8 |
| FIR peak 8.11b | Matlab | 6429 | 6435 | 1072 | 6432 | 20 |
| | Signal Wizard | 6400 | 6468 | 94.6 | 6432 | 10 |
| | Oscilloscope | 6300 | 6500 | 32.0 | 6400 | 20 |
| IIR peak 8.11e | Matlab | 6429 | 6435 | 1072.0 | 6432 | 20 |
| | Signal Wizard | 6392 | 6471 | 81.4 | 6432 | 10 |
| | Oscilloscope | 6300 | 6500 | 32.0 | 6400 | 20 |

Table 8.3 Measured parameters (bandwidth, real Q factor, center frequency, and gain) of the desired (Matlab), designed (Signal Wizard) and implemented (oscilloscope) band-stop and peak filters: (8.10b-e) and (8.11b-e) respectively.

Chapter 9

DISCUSSION

The different results obtained from the semi-parametric and parametric equalizer were presented on the last chapter, therefore the proper analysis, main findings and implications are discussed and shown throughout this chapter. In this manner the aims and objectives are evaluated hand in hand with the limitations and future work of the project.

20-band offline semi-parametric equalizer

The equalizer implemented through Delphi was tested filtering a flat spectrum signal (Figure 8.1). First three band-pass filters with different bandwidths were used to filter the signal. It can be seen on figures 8.2-3-4 the evidence of the different Q factors and how they affect the filtered signal.

The filter with $Q = 10$ achieved better results in terms of actually filter and boosting the signal at the center frequency, where a gain of 20dB was obtained. The second filter ($Q = 3$) reached similar results where the filter was less selective due its wider bandwidth, although at the specific center frequency the boost was realized. Finally with a larger bandwidth ($Q = 1$) the results were not satisfactory, by the reasons that the whole spectrum was boost in the same way and not selectiveness at the center frequency was attained.

Analysing the results the consequences of choosing the right Q factor are evident, because in this type of equalizer the band-pass filters are based on IIR filters and each band has effect over the whole spectrum. Thus for an audio application the equalizer with $Q = 10$ in each band will provide a result narrower than and will not generate a flat response when

all the bands are at the same gain. In a similar manner the equalizer with $Q = 1$ will result in a flat response for all the possible gain configurations. Therefore the equalizer with $Q = 3$ provides a better solution related to the trade-off between selectiveness and center frequency boost or cut of each band.

After testing the gain boost and the effect of different Q factors for the IIR band-pass filters a band-stop filter (Figure 8.5) was implemented. In the resulting spectrum the problem of the selectiveness is evident, by the reason that only a gain of -10dB was achieved. This with a Q factor of 4, thus the trade-off among band selectiveness and gain performance is again manifest.

Due the bands of the equalizer are based on IIR filters the conclusion about the performance of the semi-parametric equalizer performance is that provides acceptable results when the objective is to boost an specific frequency band, but not offers a high performance cutting certain bands. This is because each band generates an effect over the whole spectrum, and when this matter is improved by reducing the bandwidth the equalizer becomes a series of peak filters not suitable for audio applications.

Therefore the equalizer has the benefits of IIR filtering such as few coefficients (Table 8.1), but also the drawbacks like the difficulty at designing arbitrary frequency responses. For this reason the improvement was done by designing a completely parametric equalizer whose results were shown and the proper analysis and comparison with its counterpart (a parametric FIR equalizer) is below.

20-band parametric equalizer

The parametric equalizer implemented through Matlab and executed experimentally via the Signal Wizard System was tested with the design of different band-pass, band-stop and peak filters. Form the results we can analyse the difference between desired and designed filters, also the performance by testing the final implementation with a DSP platform. This was realized with both types of filters in order to make the proper comparison.

First three FIR band-pass filters were designed (Figure 8.6), where we can see the difference between the desired and designed frequency response. This difference is because the number of taps was reduced from 16384 to 1024 in order to implement the filter in real-time, and also by the reason of the rectangular 'brick-wall' shape of the band-pass. The effect of this latter is reduced by the selection of Hanning windows at the Signal Wizard System, accordingly the first side lobes appeared at -40dB from the main lobe. Apart from these side lobes, whose magnitude is small compared to the main lobe magnitude, and the change of shape due the new number of taps; the filter conversion from desired to designed in the Signal Wizard System can be described as satisfactory.

The same filters were designed for IIR filters (Figure 8.7). It can be seen that the conversion from desired to designed in Signal Wizard is performed in a very fitting way, where the implemented filters are almost identical to the desired ones. This is due the desired IIR filter does not have a rectangular 'brick-wall' shape and its transition zones are characteristically smooth. Also for this reason the number of taps does not affect the designed filter. In both types of filters the gain was reduced from 20dB to 15dB due clipping problems.

The 6 filters (8.6a-b-c 8.7a-b-c) were executed experimentally to filter a white noise signal on the Signal Wizard and the performance was shown on figure 8.8. It can be observed that the results are as expected, where the three FIR filters produced a rectangular band-pass and the IIR filters a smooth changing band-pass.

The different parameters such as bandwidth, Q factor, center frequency and gain were measured and consigned in Table 8.3; this was done for the designed, desired and implemented band-pass filters. The obtained parameters from the desired filters are very accurate, where the attained gain and center frequency was exactly the desired one, and the Q factor was more precise for the IIR filters. Although the obtained difference for the FIR filters is not considerable, therefore it can be conclude that the desired parameters in the Matlab program match seamlessly the ones entered by the user.

From the data measured in Signal Wizard it can be seen that the parameters still very precise and close to the desired ones. The center frequency is exactly the same for the FIR

filters, albeit show a small deviation for the IIR filters. The Q factor is very accurate for IIR filters, but for FIR filters presents a small error when the bandwidth is narrow ($Q=10$).

When the filters were tested experimentally both type of filters delivered reasonable results in regards the desired parameters. Although an error ($\sim 3.75\%$) was obtained for the center frequencies its gain was very close to the designed 15dB. Yet the biggest deviation is related to the Q factor, where a Q factor of 10 was not achieved being 6.5 the closest, also for the desired $Q = 3$ the obtained factor was 2.4 and 4.3 for FIR and IIR respectively. These errors are present as well as in FIR filters or IIR. It can be conclude that in terms of gain and center frequency the performance is very satisfactory, and acceptable for the Q factor.

A band-stop filter was implemented in order to see how much both types of filters can decrease the gain in a specific frequency band. The resulting performance was very different for FIR and IIR filters. First a similarity among the desired and designed frequency response of the filters was achieved again. For the IIR band-stop filter the main difference was related the center frequency, and both type of filters accomplished acceptable results in regards the Q factor. The main drawback of the IIR filter was that only a gain of -8dB was reached, whereas the FIR filter totally cut the frequency band (-20dB). This is due the lack of selectiveness of the IIR equalizer where all the bands affect the whole spectrum.

Another important aspect to take into account is that the FIR band-stop (8.10b) shows a totally flat response in the bands which gain was set to 0dB, whereas the response of the IIR (8.10e) is not totally flat and the effect of each band is visible. This for an audio equalizer is of big importance because a flat response in some bands is always necessary, and with a non-flat response some frequencies will be boost or cut.

Finally a high-selective peak filter was realized in order to test the accuracy of the program when designing specific center frequencies. The selected center frequency was 6432 Hz and the results were shown figure 8.11 and the measured parameters on table 8.3. Both types of filters were successful at separating the sine wave with center frequency 6432Hz from the added white noise. Nevertheless the Q factor of 1000 was not achieved, and

experimentally a Q factor of 32 was obtained. This is because the decreasing in the number of taps. Yet the obtained center frequency for both types of filters was highly accurate.

Also the frequency resolution represents an important factor in this type of equalizer, for the FIR at 1024 taps and 48 kHz sample rate is 23.4 Hz. This means that at low frequencies the center frequency is not largely adjustable and is strongly conditioned by multiples of this number. This does not occur with the IIR filters, but since its implementation in the program is based on the convolution of the impulse response as a FIR filter the same problem applies in this case.

From the results it can be conclude that the best performance was obtained with FIR filters, where the band-pass and band-stop filters achieved better results than the IIR. Nevertheless a drawback was evident and was the difference between the desired and designed frequency response, yet this difference does not exist with the IIR filters. This aspect could be improved by increasing the number of taps, which is possible with offline filtering, then producing frequency responses that match the desired one.

In real-time filtering a possible solution to the abrupt changes is mentioned in (M. Lightstone, 1994), where the desired frequency response is constructed using different analytical functions for the pass-bands, stop-bands, and transition zones. Also as a future work a new modified frequency sampling method (Mitra, 1993) can be applied. Then an improvement of the design in terms of frequency resolution, and similarity of the results to the desired frequency response can be achieved.

Another implication of the FIR results is the possible improvement by incorporating maximal flatness design (Hanna, 1996) on the passband. This minimizes the energy of the error between the desired and designed frequency in the pass-bands and stop-bands. And therefore a smooth frequency response is obtained. Finally another strategy for designing multiband FIR filters is proposed in (L. Rabiner, 1974), where the result is a monotonic frequency response without ripples in the transition zone.

The main drawback of the IIR filters was evident in the poor performance of the band-stop filter. This can be improved by designing the parametric filters directly in the digital

domain, where each band is represented by a biquad sub-filter. This biquad filters can be designed through an optimum pole and zero placement as shown in (Reiss, 2011). With this improvement the implementation of the IIR parametric filters can be done without the need of performing the impulse response as a FIR convolution. Thus generating a real IIR parametric filter and not a IIR-to-FIR conversion.

As a future work the obtained parametric equalizer can be compared to parametric filters implemented through different DSP platforms such as the miniDSP system or the Cirrus Logic audio-system-on-a-chip just to name a few.

The design and implementation of a parametric equalizer was achieved. A strategy for implementing FIR and IIR parametric filters was established and an algorithm to design multi-band parametric filters using Matlab and executed via the Signal Wizard system was accomplished. The algorithm can generate an arbitrary frequency or impulse response based on band-pass, band-stop, peak and notch filters and uses the frequency sampling method and the bilinear transform for FIR and IIR filters respectively.

It can be conclude that the parametric equalizer was accomplished through the algorithm executed via the Matlab program. With this program the user can adjust the parameters of the equalizer in order to boost or cut the sounds of certain music instrument. For example in order to enhance or attenuate certain properties of an audio signal the user can modify all the parameters of the frequency bands. This can be done for both types of filters and can be exported to any DSP platform as an ASCII text file.

The generated frequency or impulse response was exported to the Signal Wizard software, where the differences between the desired and designed frequency response were analysed. It was concluded that this error is bigger for FIR filters than for IIR filters, however the FIR parametric filters provide better results experimentally in regards the desired parameters such as gain, center frequency and bandwidth. The final implementation through the Signal Wizard hardware exhibited satisfactory results, where white noise signals and sine waves were filtered in real-time. The frequency spectrum of these filtered signals showed acceptable similarity to the desired frequency response. Also the limitations and possible future work were discussed, thus concluding this project.

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