Exercise answers - Chapter 3 - Growth of functions

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Exercise 3.1-1

Defining $h(n) = \max(f(n), g(n))$. Since f and g are assymptotically, non-negative it holds that, $f(n) + g(n) \leq 2h(n)$ and $f(n) + g(n) \geq h(n)$ for all $n > n_0$, for some n_0. So, we have,

$$\frac{1}{2}(f(n) + g(n)) \leq h(n) \leq f(n) + g(n) \quad \forall n > n_0$$

which ensures that $h(n)$ is a member of $\Theta(f(n)+g(n))$, with constants $c_1 = 1/2$ and $c_2 = 1$.

Exercise 3.1-2

For $f(n) = (n+a)^b$ to be a member of the set $\Theta(n^b)$, there must exist constants c_1 and c_2 such that,

$$c_1 n^b \leq (n+a)^b \leq c_2 n^b \tag{1}$$

for all n greater than some lower bound n_0.

First we notice that for some value of n, $n + a$ will become positive, and f will we monotonically growing in n, from that point on. We also notice that $(n + a)^b \to n^b$ as $n \to \infty$. So (1) will hold for some n_0, with $c_1 = 1 - \epsilon$ and $c_2 = 1 + \epsilon$ for some positive ϵ, hence $(n + a)^b = \Theta(n^b)$.