NLP Programming Tutorial 7 - Neural Networks

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Prediction Problems

Given x, predict y

Example we will use:

- Given an introductory sentence from Wikipedia
- Predict whether the article is about a person

Given		Predict
Gonso was a Sanron sect priest (754-827) in the late Nara and early Heian periods.	→	Yes!
Shichikuzan Chigogataki Fudomyoo is a historical site located at Magura, Maizuru City, Kyoto Prefecture.	→	No!

- This is binary classification (of course!)

Linear Classifiers

$$y = \text{sign}(\mathbf{w} \cdot \boldsymbol{\varphi}(x)) = \text{sign}\left(\sum_{i=1}^{I} w_i \cdot \varphi_i(x)\right)$$

- x: the input
- **φ(x)**: vector of feature functions $\{\varphi_1(x), \varphi_2(x), \ldots, \varphi_I(x)\}$
- **w**: the weight vector $\{w_1, w_2, \ldots, w_I\}$
- y: the prediction, +1 if “yes”, -1 if “no”
    - (sign(v) is +1 if v >= 0, -1 otherwise)

Example Feature Functions: Unigram Features

- Equal to "number of times a particular word appears"

x = A site , located in Maizuru , Kyoto

$\varphi_{\text{unigram "A"}}(x) = 1 \quad \varphi_{\text{unigram "site"}}(x) = 1 \quad \varphi_{\text{unigram ","}}(x) = 2$

$\varphi_{\text{unigram "located"}}(x) = 1 \quad \varphi_{\text{unigram "in"}}(x) = 1$

$\varphi_{\text{unigram "Maizuru"}}(x) = 1 \quad \varphi_{\text{unigram "Kyoto"}}(x) = 1$

$\varphi_{\text{unigram "the"}}(x) = 0 \quad \varphi_{\text{unigram "temple"}}(x) = 0$

…

The rest are all 0

- For convenience, we use feature names ($\varphi_{\text{unigram "A"}}$) instead of feature indexes (φ_1)

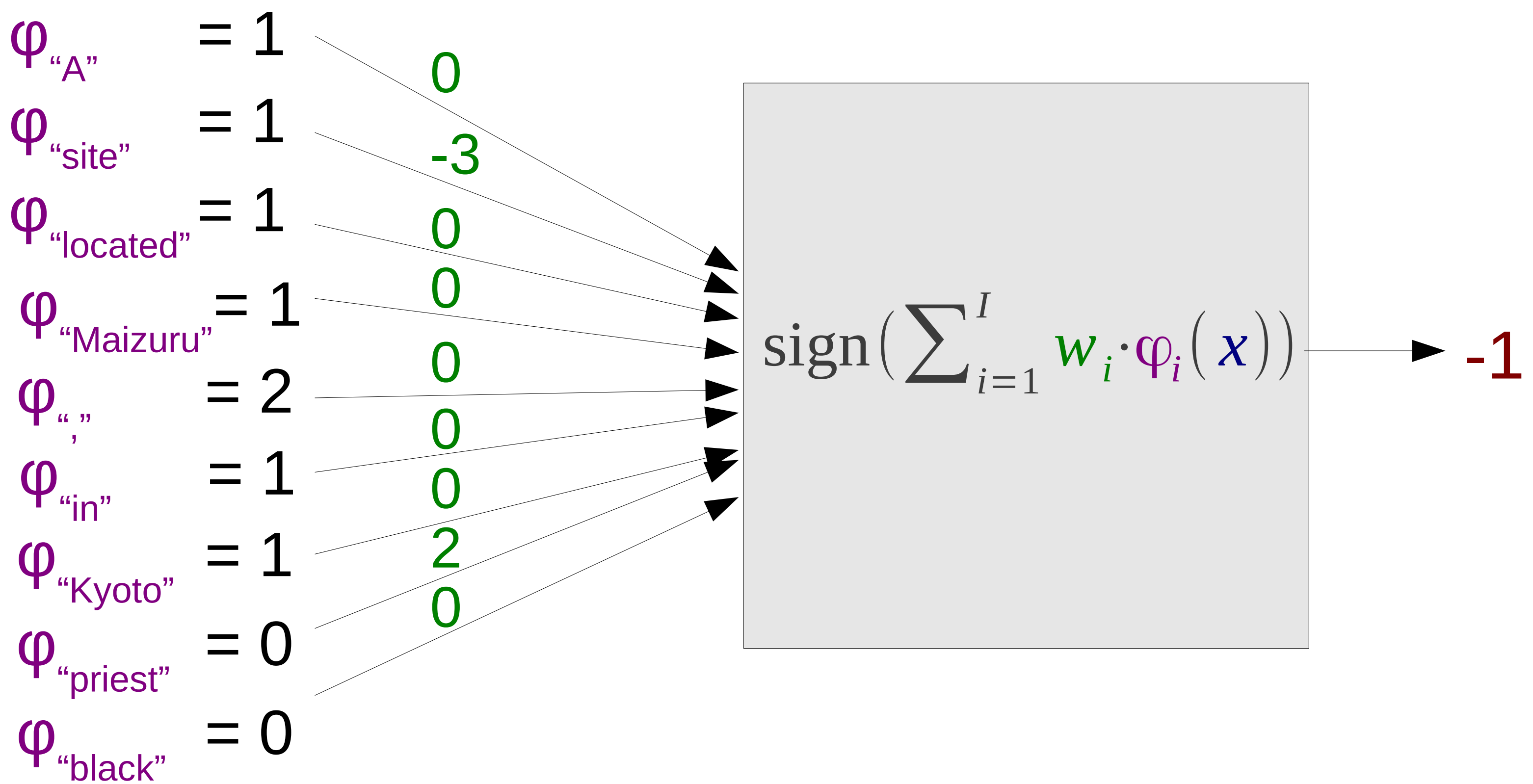
Calculating the Weighted Sum

x = A site , located in Maizuru , Kyoto

$\varphi_{\text{unigram “A”}}(x)$	= 1		$w_{\text{unigram “a”}}$	= 0		0 +
$\varphi_{\text{unigram “site”}}(x)$	= 1		$w_{\text{unigram “site”}}$	= -3		-3 +
$\varphi_{\text{unigram “located”}}(x)$	= 1		$w_{\text{unigram “located”}}$	= 0		0 +
$\varphi_{\text{unigram “Maizuru”}}(x)$	= 1		$w_{\text{unigram “Maizuru”}}$	= 0		0 +
$\varphi_{\text{unigram “,”}}(x)$	= 2	*	$w_{\text{unigram “,”}}$	= 0	=	0 +
$\varphi_{\text{unigram “in”}}(x)$	= 1		$w_{\text{unigram “in”}}$	= 0		0 +
$\varphi_{\text{unigram “Kyoto”}}(x)$	= 1		$w_{\text{unigram “Kyoto”}}$	= 0		0 +
$\varphi_{\text{unigram “priest”}}(x)$	= 0		$w_{\text{unigram “priest”}}$	= 2		0 +
$\varphi_{\text{unigram “black”}}(x)$	= 0		$w_{\text{unigram “black”}}$	= 0		0 +
	…					…
						=
						-3 → No!

The Perceptron

- Think of it as a “machine” to calculate a weighted sum



Perceptron in Numpy

What is Numpy?

- A powerful computation library in Python
- Vector and matrix multiplication is easy
- A part of SciPy (a more extensive scientific computing library)

Example of Numpy (Vectors)

```
import numpy as np

a = np.array( [20,30,40,50] )
b = np.array( [0,1,2,3] )
print(a - b)              # Subtract each element
print(b ** 2)             # Take the power of each element
print(10 * np.tanh(b))    # Hyperbolic tangent * 10 of each element
print(a < 35)             # Check if each element is less than 35
```

Example of Numpy (Matrices)

```
import numpy as np

A = np.array( [[1,1],
               [0,1]] )
B = np.array( [[2,0],
               [3,4]] )

print(A * B)          # elementwise product
print(np.dot(A,B))    # dot product
print(B.T)            # transpose
```

Perceptron Prediction

```
predict_one(w, phi)
    score = 0
    for each name, value in phi          # score = w*φ(x)
        if name exists in w
            score += value * w[name]
    return (1 if score >= 0 else -1)
```

↓ numpy

```
predict_one(w, phi)
    score = np.dot( w, phi )
    return (1 if score[0] >= 0 else -1)
```

Converting Words to IDs

- numpy uses vectors, so we want to convert names into indices

```
ids = defaultdict(lambda: len(ids))  # A trick to convert to IDs

CREATE_FEATURES(x):
    create list phi
    split x into words
    for word in words
        phi[ids["UNI:"+word]] += 1
    return phi
```

Initializing Vectors

- Create a vector as large as the number of features
- With zeros

```
w = np.zeros(len(ids))
```

- Or random between [-0.5,0.5]

```
w = np.random.rand(len(ids)) – 0.5
```

Perceptron Training Pseudo-code

```
# Count the features and initialize the weights
create map ids
for each labeled pair x, y in the data
    create_features(x)
w = np.zeros(len(ids))

# Perform training
for I iterations
    for each labeled pair x, y in the data
        phi = create_features(x)
        y' = predict_one(w, phi)
        if y' != y
            update_weights(w, phi, y)

print w to weight_file
print ids to id_file
```

Perceptron Prediction Code

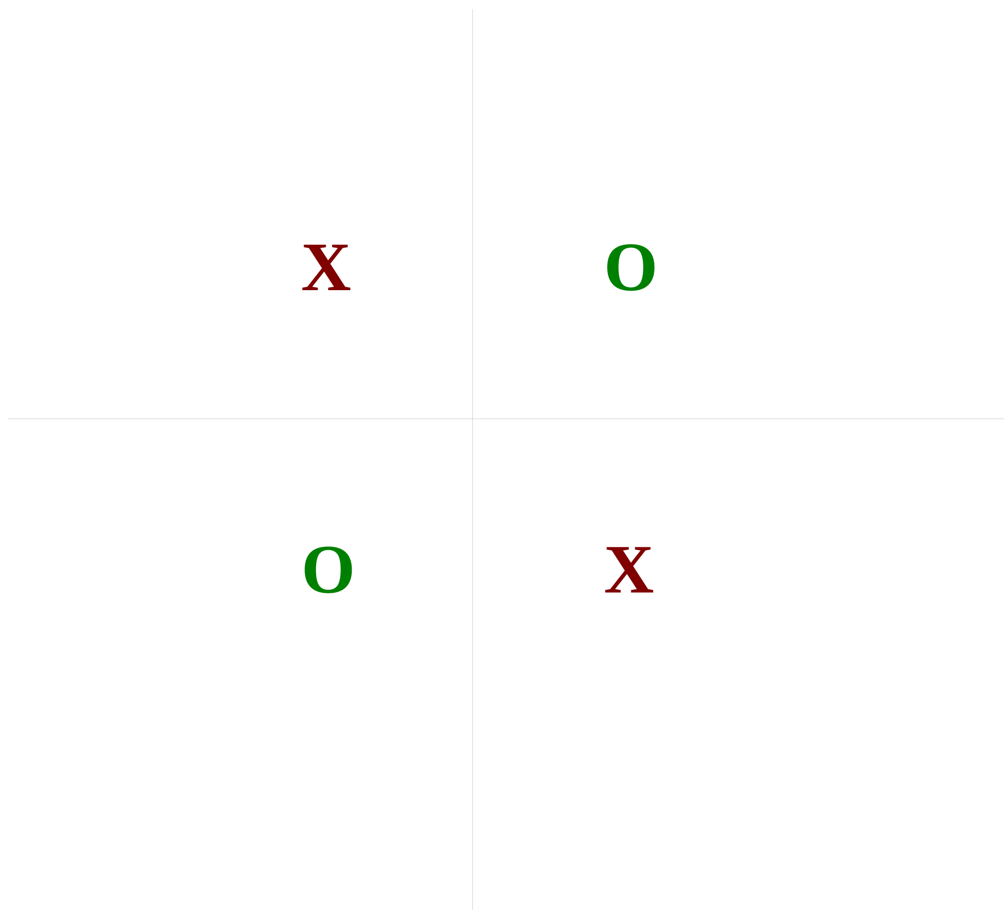
read *ids* **from** *id_file*
read *w* **from** *weights_file*

for each example *x* in the data
    phi = `create_features`(x)
    y' = `predict_one`(w, phi)

Neural Networks

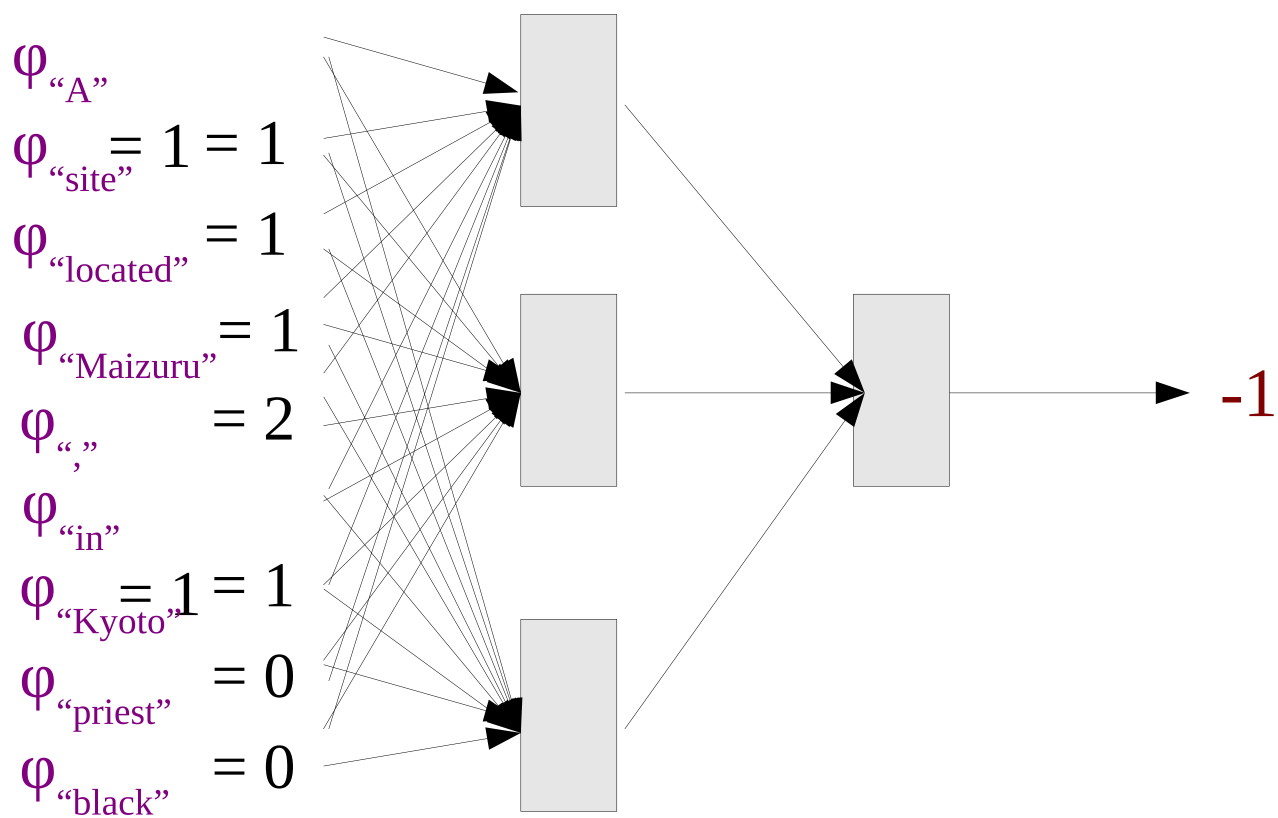
問題：線形分類のみ

- 線形分離不可能な問題に対して高い精度は実現不可



ニューラルネット

- 複数のパーセプトロンをつなげる

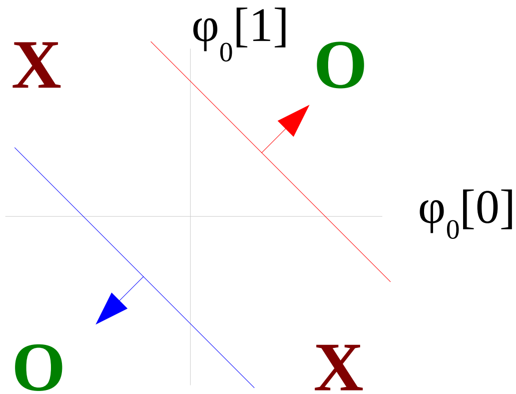


- モチベーション：線形でない関数も表現可能！

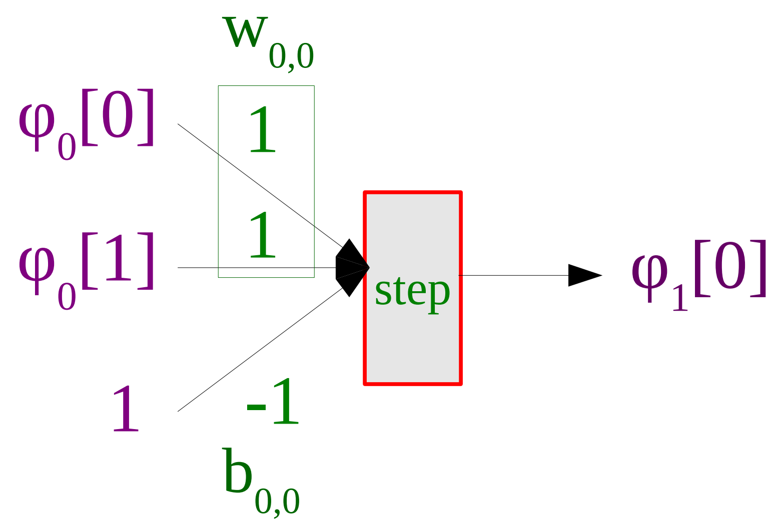
例：

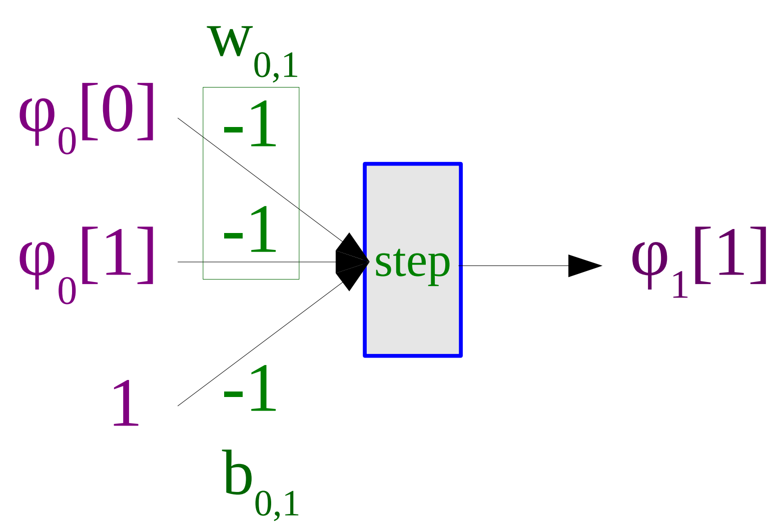
- ２つの分類器を作成

$\varphi_0(x_1) = \{-1, 1\}$ $\varphi_0(x_2) = \{1, 1\}$



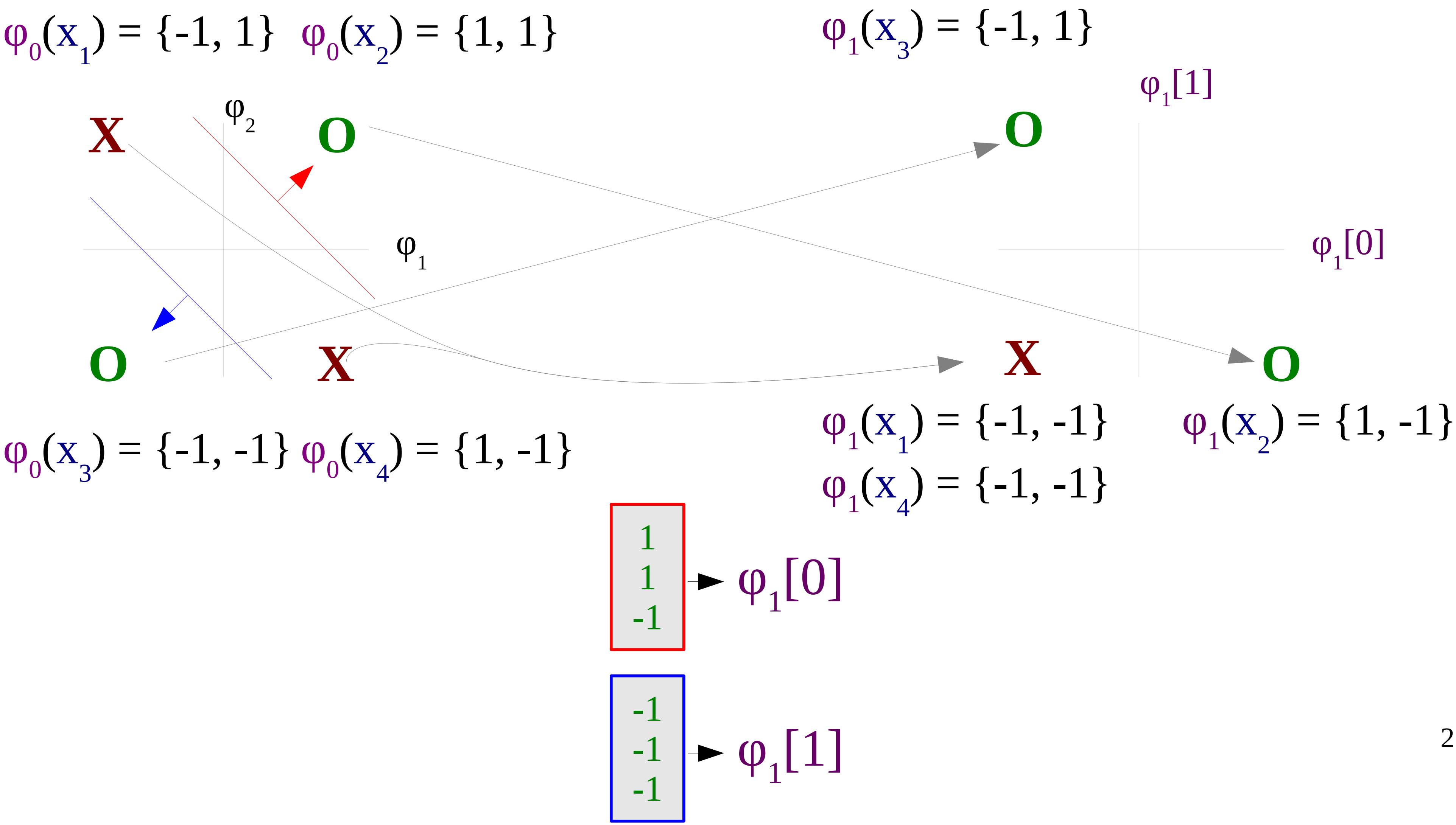
$\varphi_0(x_3) = \{-1, -1\}$ $\varphi_0(x_4) = \{1, -1\}$





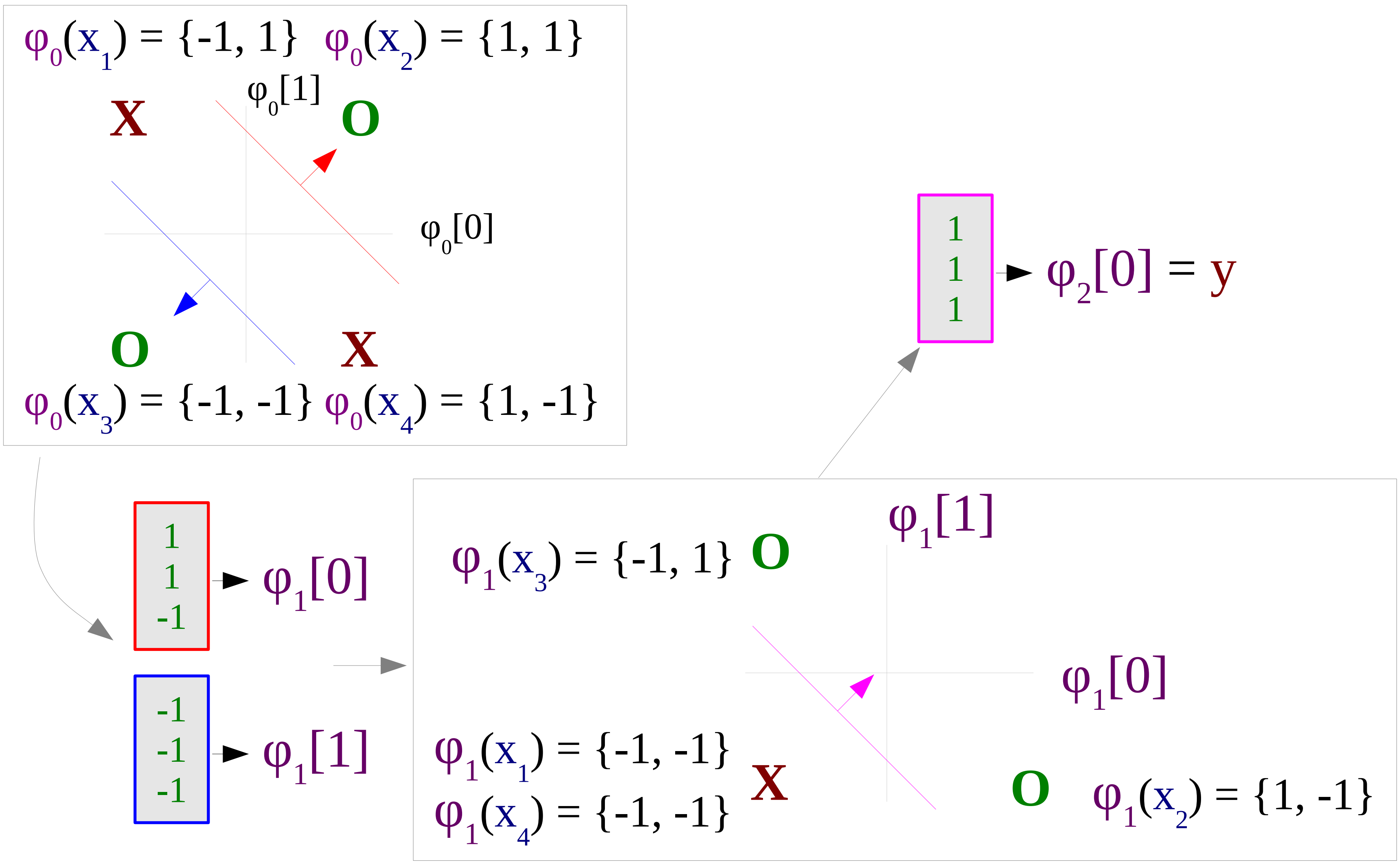
例：

- 分類器は新しい素性空間へマッピング



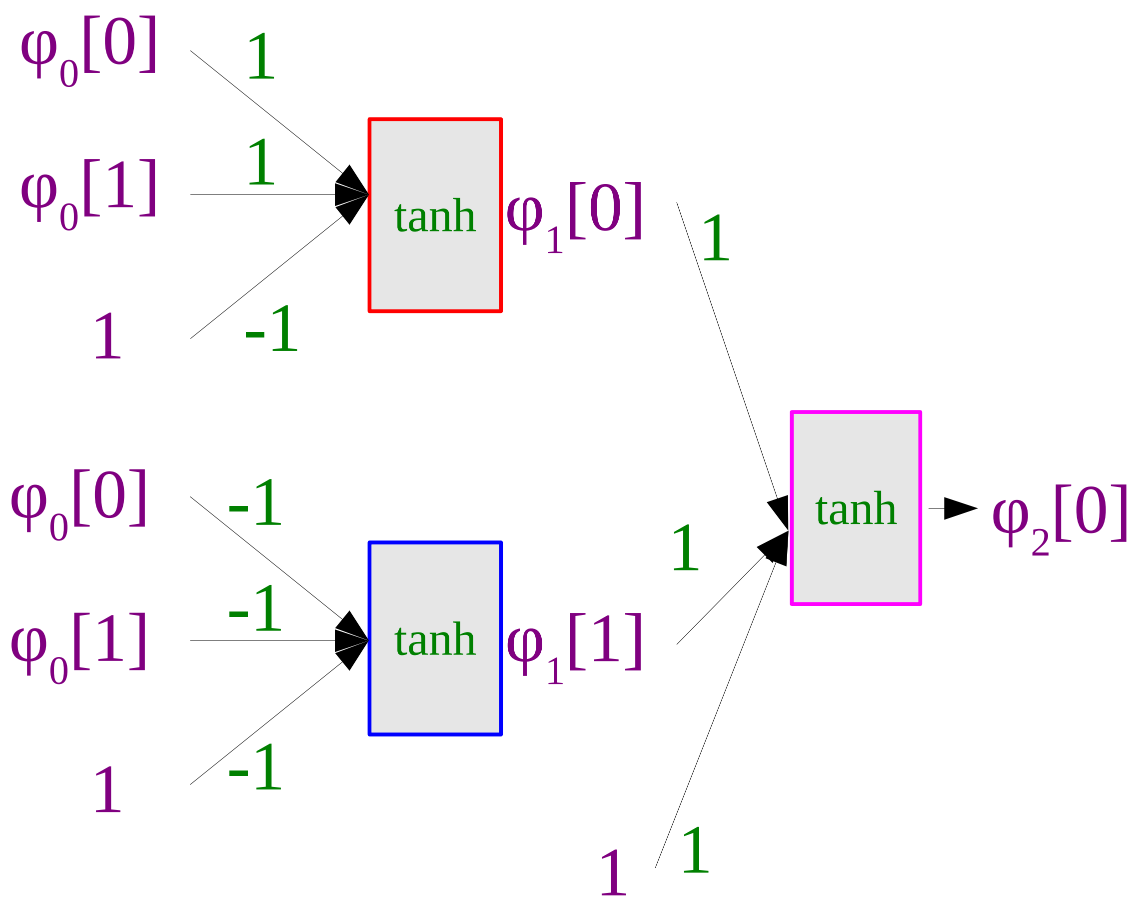
例：

- 新しい空間で、事例が分類可能に！



例：

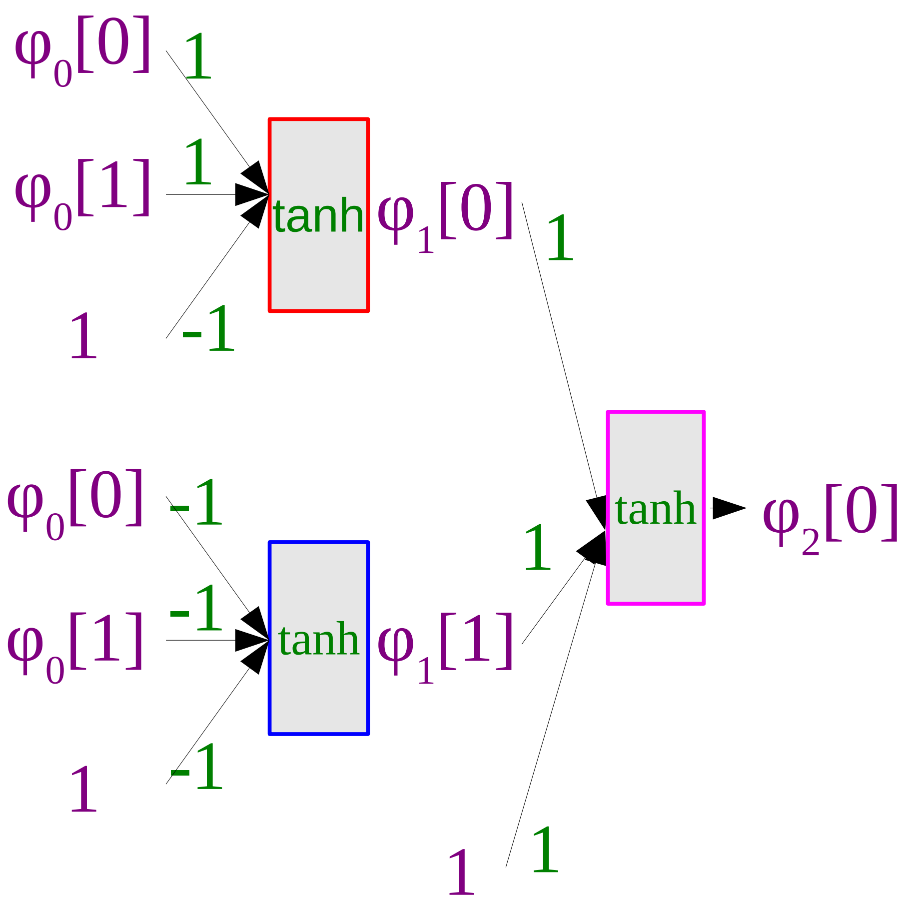
- 最終的なニューラルネット



2層ニューラルネットの例(ベクトル編)

入力

φ_0 = np.array([1, -1])



一層目の計算

$w_{0,0}$ = np.array([1, 1])
$b_{0,0}$ = np.array([-1])
$w_{0,1}$ = np.array([-1, -1])
$b_{0,1}$ = np.array([-1])
φ_1 = np.zeros(2)
$\varphi_1[0]$ = np.tanh($\varphi_0 w_{0,0}$ + $b_{0,0}$)[0]
$\varphi_1[1]$ = np.tanh($\varphi_0 w_{0,1}$ + $b_{0,1}$)[0]

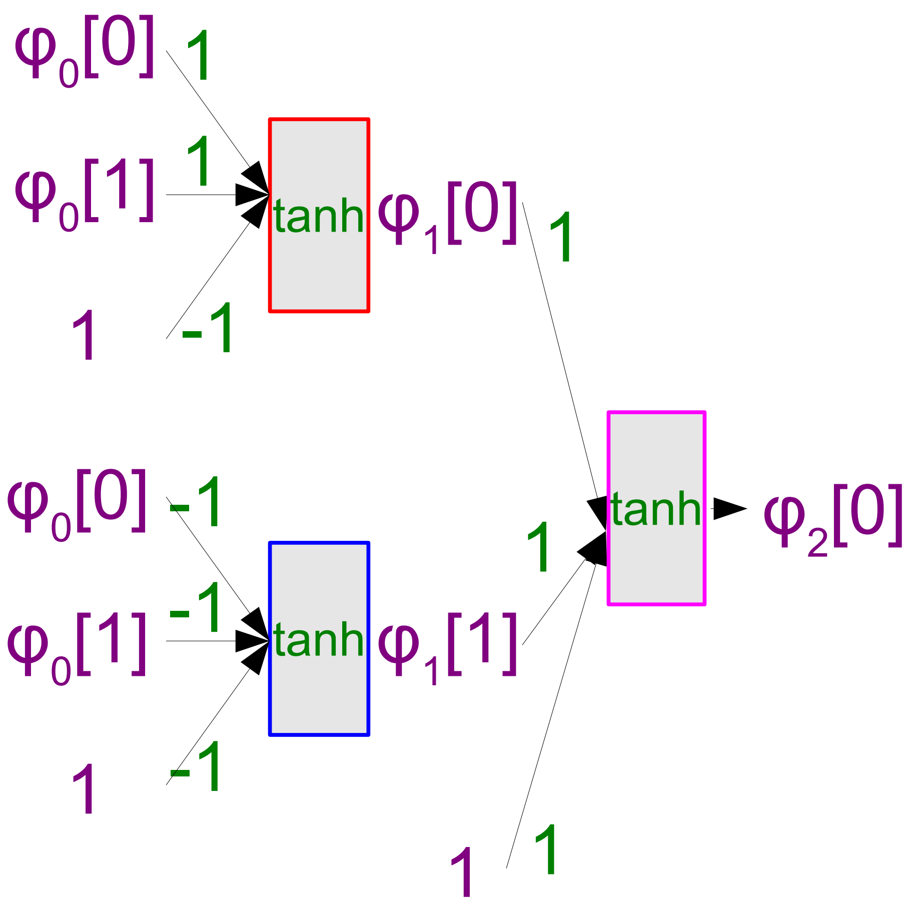
2層目の計算

$w_{1,0}$ = np.array([1, 1])
$b_{1,0}$ = np.array([-1])
φ_2 = np.zeros(1)
$\varphi_2[0]$ = np.tanh($\varphi_1 w_{1,0}$ + $b_{1,0}$)[0]

2 層ニューラルネットの例 (行列編)

入力

φ0 = np.array([1, -1])



一層目の計算

w0 = np.array([[1, 1], [-1,-1]])

b0 = np.array([-1, -1])

φ1 = np.tanh(np.dot(w0, φ0) + b0)

２層目の計算

w1 = np.array([[1, 1]])

b1 = np.array([-1])

φ2 = np.tanh(np.dot(w1, φ1) + b1)

ニューラルネットの伝搬コード

```
forward_nn(network, φ0)
    φ= [ φ0 ] # 各層の値
    for each layer i in 0 .. len(network)-1:
        w, b = network[i]
        # 前の層の値に基づいて値を計算
        φ[i] = np.dot( w, φ[i-1] ) + b
    return φ # 各層の結果を返す
```

tanh を用いたパーセプトロン学習

- エラー関数：二乗誤差

$$\mathrm{err} = (y' - y)^2 / 2$$

正解　　システム出力

- エラーの勾配 :

$$\mathrm{err}' = \delta = y' - y$$

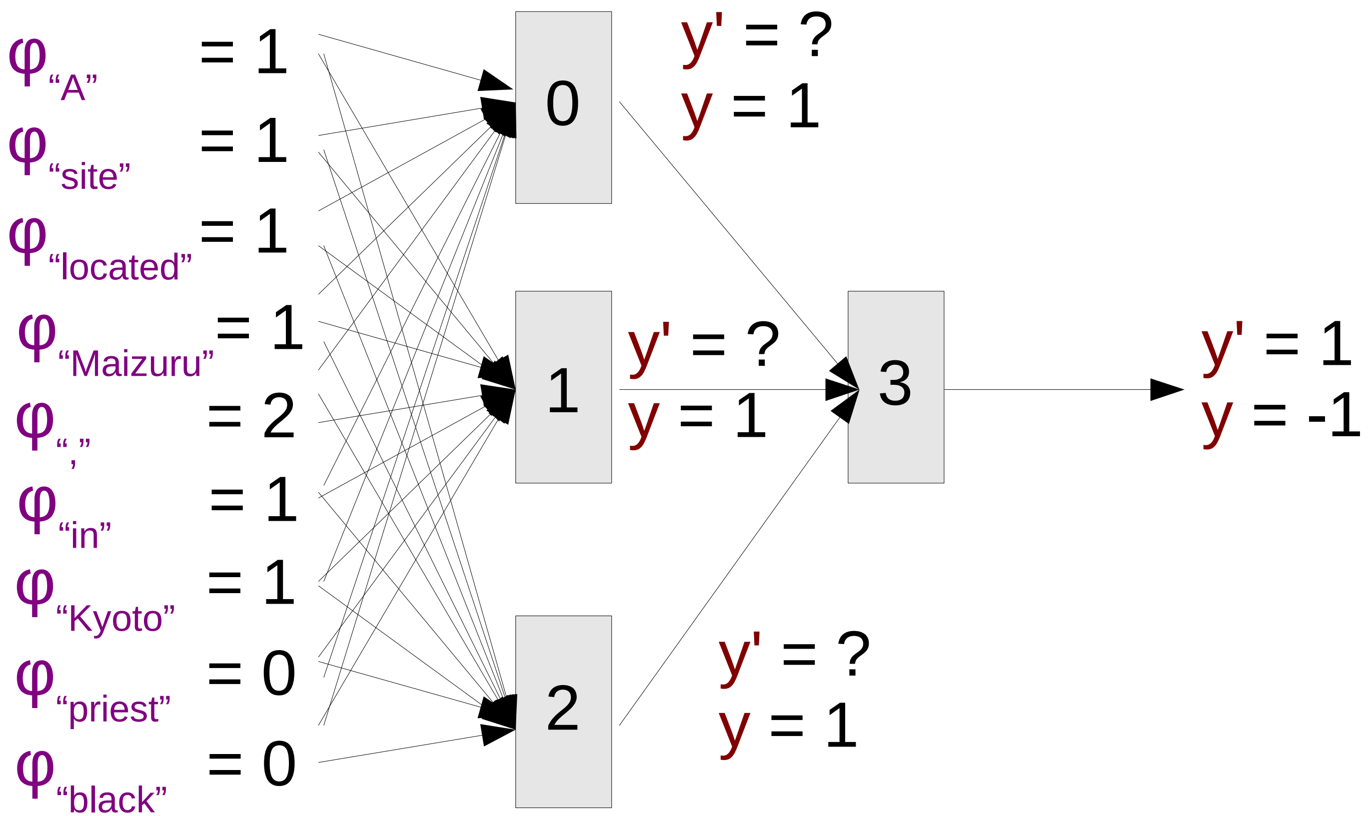
- 各重みを更新 :

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \lambda \cdot \delta \cdot \varphi(x)$$

- λ は学習率

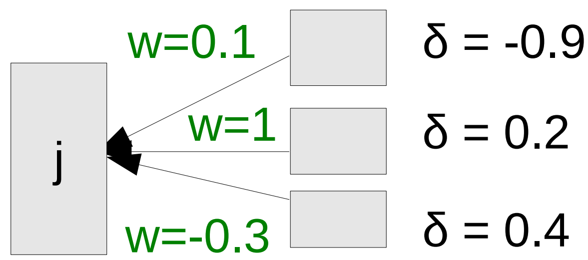
問題：正解は分からない！

- NN では出力層のみで正解が与えられる



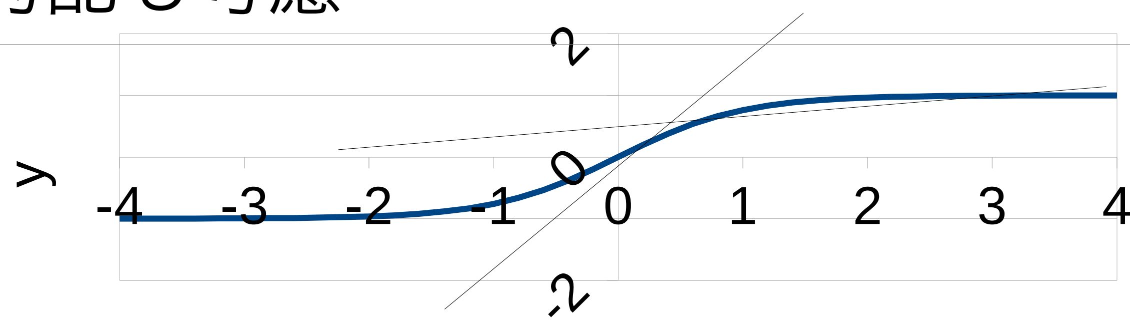
解決策：逆伝搬法

- 出力層からエラーを後ろへ伝搬



$$\sum_i \delta_i w_{j,i}$$

- tanh の勾配も考慮



$$d \tanh(\varphi(x) * w) = 1 - \tanh(\varphi(x) * w)^2 = 1 - y_j^2$$

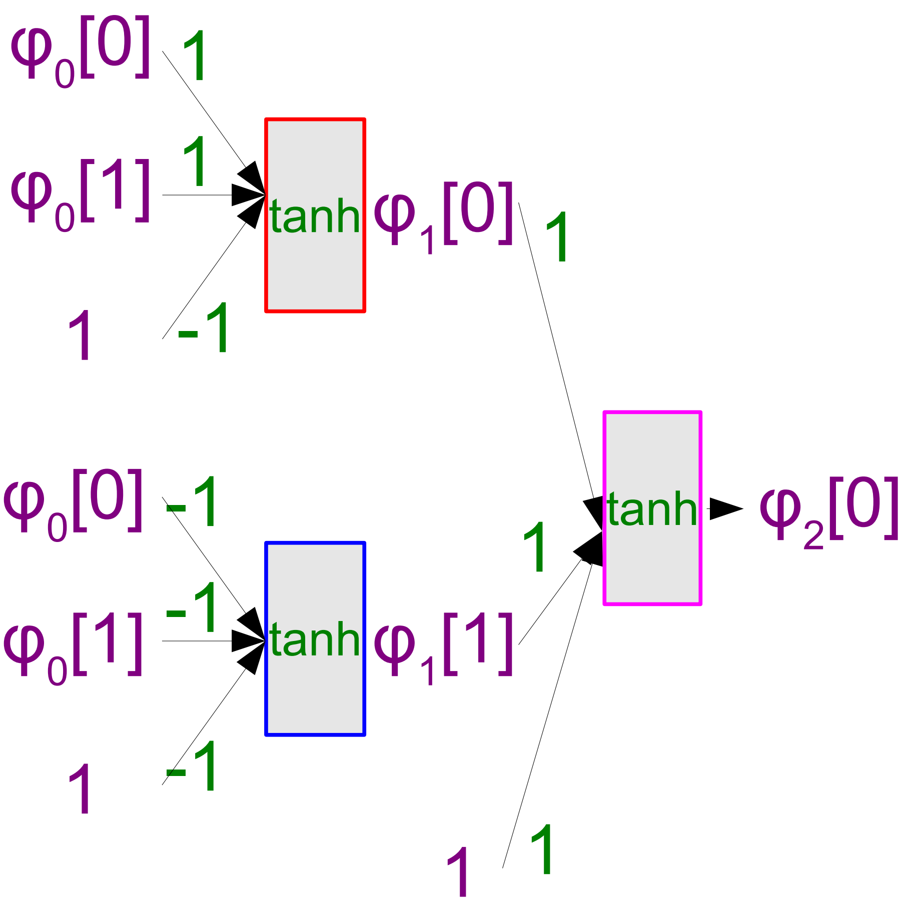
- 合わせて：

$$\delta_j = (1 - y_j^2) \sum_i \delta_i w_{j,i}$$

逆伝搬の例（行列編）

出力層のエラー

$\boldsymbol{\delta}_2$ = np.array([y'-y])



１層目の計算

$\boldsymbol{\delta'}_2 = \boldsymbol{\delta}_2 * (1-\boldsymbol{\varphi}_2^2)$

$\boldsymbol{\delta}_1$ = np.dot($\boldsymbol{\delta'}_2$, $\mathbf{w}_1$)

０層目の計算

$\boldsymbol{\delta'}_1 = \boldsymbol{\delta}_1 * (1-\boldsymbol{\varphi}_1^2)$

$\boldsymbol{\delta}_0$ = np.dot($\boldsymbol{\delta'}_1$, $\mathbf{w}_0$)

逆伝搬のコード

```
backward_nn(net, φ, y′)
    J = len(net)
    create array δ = [ 0, 0, …, np.array([y′ – φ[J][0]]) ] # length J+1
    create array δ′ = [ 0, 0, …, 0 ]
    for i in J-1 .. 0:
        δ′[i+1] = δ[i+1] * (1 – φ[i+1]^2)
        w, b = net[i]
        δ[i] = np.dot(δ′[i+1], w)
    return δ′
```

重み更新

- 最後に、重みを更新
- 重み ***w*** の勾配、次の **δ'** と、前の ***φ*** の外積で求める

$$-\mathrm{derr}/\mathrm{d}\mathbf{w}_i = \mathrm{np.outer}(\ \boldsymbol{\delta}'_{i+1},\ \boldsymbol{\varphi}_i\ )$$

- 学習率をかけ、重みを更新

$$\mathbf{w}_i \mathrel{+}= \lambda * -\mathrm{derr}/\mathrm{d}\mathbf{w}_i$$

- バイアス項は単純に **δ'** と同等

$$-\mathrm{derr}/\mathrm{d}\mathbf{b}_i = \boldsymbol{\delta}'_{i+1}$$

$$\mathbf{b}_i \mathrel{+}= \lambda * -\mathrm{derr}/\mathrm{d}\mathbf{b}_i$$

重み更新のコード

```
update_weights(net, φ, δ', λ)
    for i in 0 .. len(net)-1:
        w, b = net[i]
        w += λ * np.outer( δ'[i+1], φ[i] )
        b += λ * δ'[i+1]
```

学習の全体像

```
# 素性を作り、ネットワークをランダムな値で初期化
create map ids, array feat_lab
for each labeled pair x, y in the data
    add (create_features(x), y ) to feat_lab
initialize net randomly

# 学習を行う
for I iterations
    for each labeled pair φ0, y in the feat_lab
        φ= forward_nn(net, φ0)
        δ'= backward_nn(net, φ, y)
        update_weights(net, φ, δ', λ)

print net to weight_file
print ids to id_file
```

ニューラルネット学習のこつ

学習の安定化

- ニューラルネットはパラメータが多い→学習が不安定
- 重みの初期値：
  - ランダム、 -0.1~0.1 の間の一様分布など
- 学習率：
  - 0.1 から始めることが多い
  - エラーが前イタレーションに比較して増加した場合は学習率を下げる (*= 0.9 や *= 0.5)
- 隠れ層の大きさ：
  - だいたい色々試して一番精度の良いものを選択

テスト

- **手軽：**エラーの値をプリントし、イタレーションごとにだいたい減ることを確認
- **本気：**有限差分法で勾配を確認

アイデア：

重み更新の際、重み w_i の勾配を計算： derr/dw_i

つまり、この重みを少しだけ（ω だけ）揺らせば

wi = x の場合 err = y	なら	wi = x + ω の場合 err = y + ω * derr/dw_i	が成り立つはず！

有限差分法で、 wi を揺らしてみて、上記が（1e-6 など、一定の誤差内）成り立たなければ、勾配計算のバグがあると判断

詳細： http://cs231n.github.io/neural-networks-3/

演習課題

演習課題 (1)

- 実装
  - train-nn: NN を学習するプログラム
  - test-nn: NN を用いて予測するプログラム
- テスト
  - 入力 : test/03-train-input.txt
  - 学習 1 回、隠れ層 1 つ , 隠れ層のノード 2 つ
  - 更新を手で確認

演習課題 (2)

- 学習 `data/titles-en-train.labeled`
- 予測 `data/titles-en-test.word`
- 評価
  - script/grade-prediction.py data-en/titles-en-test.labeled your_answer
- 比較
  - 単純なパーセプトロン、 SVM 、ロジスティック回帰
  - ノード数、初期学習率、ランダムな値の初期レンジ
- チャレンジ
  - 複数の隠れ層を使ったネットの実装
  - エラー増加の場合、学習率を減らす手法を実装

Thank You!