

Bayesian computation project

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EPFL

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Table of contents

- 1 Framework
- 2 Data
- 3 Models
- 4 Methods used
 - Laplace approximation
 - Metropolis Hastings and variant
- 5 Comparison
 - Model
 - Method
- 6 Conclusion

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- 2 Data
- 3 Models
- 4 Methods used
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Framework implementations and limits

Optimization

- gradient descent
- linear search gd
- Wolfe cond gd
- Stochastic gd
- Newton gd (slow)

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Approximation

- Laplace
- GVA (unstable)

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Sampling

- MH random walk
- MALA
- IS, RS
- ~~Gibbs~~
- MH within Gibbs

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- 6 Conclusion

Structure

Figure: Hourly wage and features in the USA, May 1985

ED	SOUTH	NONWH	HISP	FE	MARR	MARRFE	EX	EXSQ	UNION	LNWAGE	AGE	MANUF	CONSTR	MANAG	SALES	CLER	SERV	PROF
10	0	0	0	0	1	0	27	729	0	2.1972	43	0	1	0	0	0	0	0
12	0	0	0	0	1	0	20	400	0	1.7047	38	0	0	0	1	0	0	0
12	0	0	0	1	0	0	4	16	0	1.3350	22	0	0	0	1	0	0	0
12	0	0	0	1	1	1	29	841	0	2.3514	47	0	0	0	0	1	0	0
12	0	0	0	0	1	0	40	1600	1	2.7080	58	0	1	0	0	0	0	0

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Purpose

- predict exactly the revenue
- predict if revenue above mean

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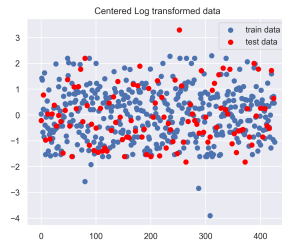
Purpose

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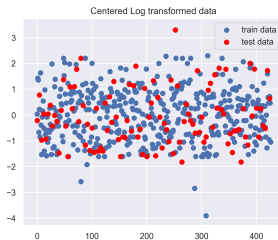
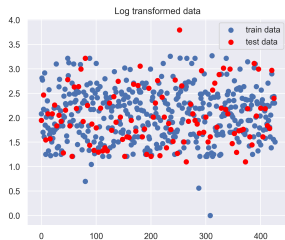
Features dropped due to high correlation

- AGE
- EXSQ

Visualization



Visualization



- 1 Framework
- 2 Data
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Models

3 models implemented:

- Gaussian model

$$Y|\beta, \sigma \sim \mathcal{N}(X\beta, \sigma^2) \quad \beta \sim \mathcal{N}_d(\vec{0}, 3^2 I), \quad \sigma \sim \exp(2)$$

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- Student model

$$Y|\beta, \nu \sim X\beta + t_\nu \quad \beta \sim \mathcal{N}_d(\vec{0}, 3^2 I), \quad \nu \sim \Gamma(2, 4)$$

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- Logistic regression

$$\mathbb{P}(Y = 1|X, \beta) = \frac{e^{X^T \beta}}{1 + e^{X^T \beta}}, \quad \beta \sim \mathcal{N}_d(0, 3^2)$$

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Laplace approximation

Laplace approximation

Fit a Gaussian approximation to the unnormalized posterior:

- mean: $\theta^* = \operatorname{argmax}_{\theta} \tilde{f}(\theta|D = d)$
- covariance matrix: $\Sigma = H_{\psi}(\theta^*)^{-1}$

with $\psi(\theta) = -\log(\tilde{f}(\theta|D = d))$ which will be used in the computations.

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Optimization routines

- Vanilla gradient descent
- Stochastic gradient descent
- Line search backtracking gradient descent
- Wolfe condition checking gradient

MH with random walk

Theory

```
1: for  $i = 1$  to  $N$  do
2:   draw  $\eta \sim \mathcal{N}_d(0, 1)$ 
3:    $\theta_c = \theta_n + \varepsilon \eta$ 
4:    $R = f(\theta_c|d)/f(\theta_n|d)$ 
5:   if  $U(0, 1) \leq R$  then
6:      $\theta_{n+1} = \theta_c$ 
7:   else
8:      $\theta_{n+1} = \theta_n$ 
9:   end if
10: end for
```

Practice

- set ε such that the acceptance rate of the proposal is between 10 and 50 percent.
- compute everything using `expsumlog`
- check visually the chain to determine the burn-in
- test different initialization to detect potential silent failure

MH with Langevin correction (MALA)

Theory

As for the random walk MH algorithm except for

- the proposal:

$$\theta_c = \theta_n + \tau \nabla \log f(\theta_n | d) + \sqrt{2\tau} \eta$$

- the acceptance ratio:

$$R = \frac{f(\theta_c | d) q(\theta_n | \theta_c)}{f(\theta_n | d) q(\theta_c | \theta_n)}$$

Practice

- as before but be more careful with tuning the step size τ

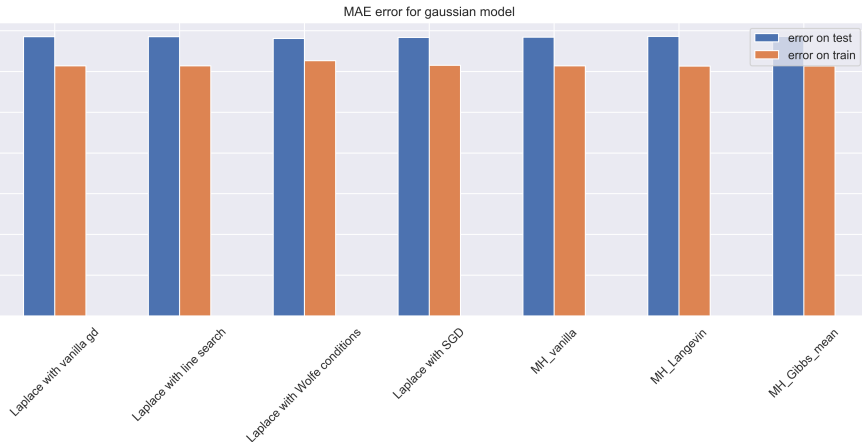
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$$q(x, x') \propto \exp \left(\frac{\|x' - x - \tau \nabla \log f(x | d)\|_2^2}{-4\tau} \right)$$

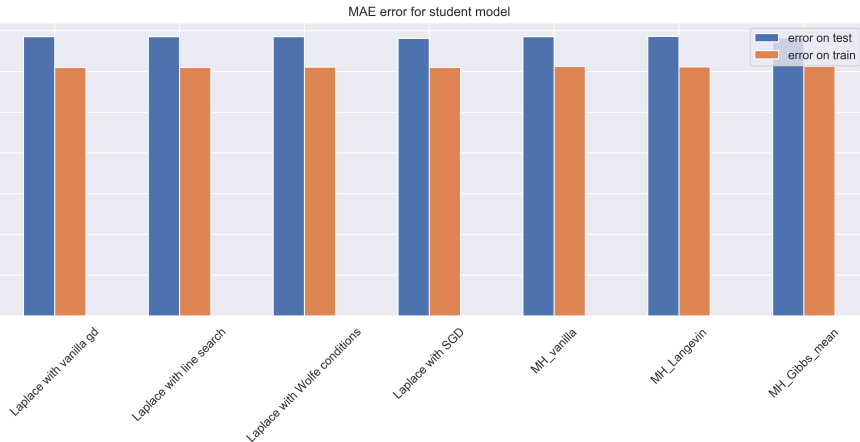
- biggest challenge: implement computation of gradient in efficient manner

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- 2 Data
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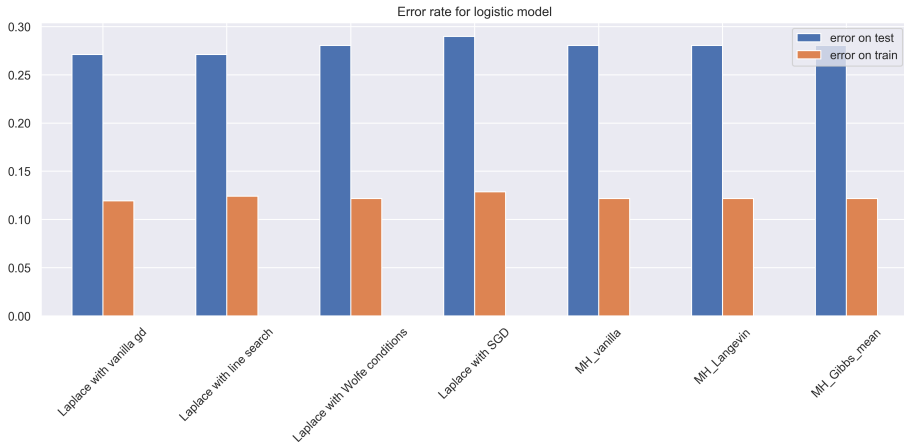
Accuracy of the Gaussian model



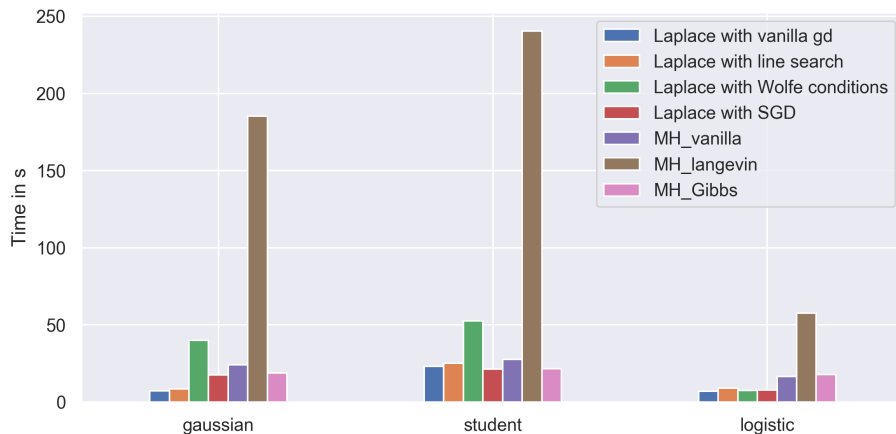
Accuracy of the Student model



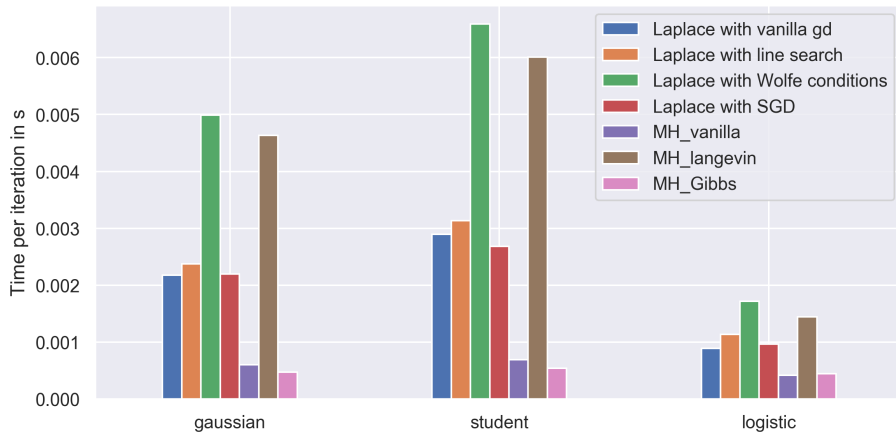
Accuracy of the Logistic model



Comparison in term of total time



Comparison in term of time per iteration



- 1 Framework
- 2 Data
- 3 Models
- 4 Methods used
- 5 Comparison
- 6 Conclusion**

Conclusion

Modelization

- Simpler methods and models performed the best
- Relationship highly non-linear

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Improvement

- Tuning of hyper-parameters
- Feature engineering

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- Gamma model
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Improvement

- Tuning of hyper-parameters
- Feature engineering
- Gamma model
- Classification in multiple ordered classes
- More robust and faster module to use more advanced techniques

References

-  E. R. Berndt
The practice of econometrics : classic and contemporary. Addison-Wesley Pub. Co., 1991
-  GitHub repository
https://github.com/dufourc1/Bayesian_computation