Bayesian computation project

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Framework implementations and limits[1]

Optimization

- gradient descent
- linear search gd
- Wolfe cond gd
- Newton gd (slow)
- ~~Stochastic gd~~

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[1]more information can be found in the ⍟ repository

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Approximation

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- GVA

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- Laplace
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Sampling

- MH random walk
- MALA
- IS, RS
- ~~Gibbs~~
- ~~MH within Gibbs~~

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Structure

Figure: Hourly wage and features in the USA, May 1985

ED	SOUTH	NONWH	HISP	FE	MARR	MARRFE	EX	EXSQ	UNION	LNWAGE	AGE	MANUF	CONSTR	MANAG	SALES	CLER	SERV	PROF
10	0	0	0	0	1	0	27	729	0	2.1972	43	0	1	0	0	0	0	0
12	0	0	0	0	1	0	20	400	0	1.7047	38	0	0	0	1	0	0	0
12	0	0	0	1	0	0	4	16	0	1.3350	22	0	0	0	1	0	0	0
12	0	0	0	1	1	1	29	841	0	2.3514	47	0	0	0	0	1	0	0
12	0	0	0	0	1	0	40	1600	1	2.7080	58	0	1	0	0	0	0	0

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Purpose

- predict exactly the revenue
- predict if revenue above mean

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Purpose

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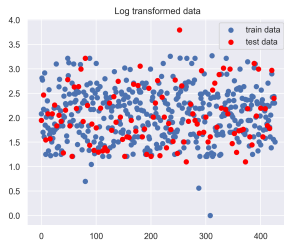
Features dropped due to high correlation

- AGE
- EXSQ

Visualization
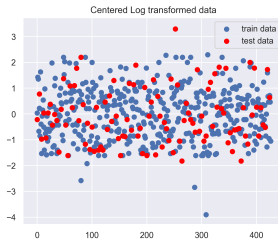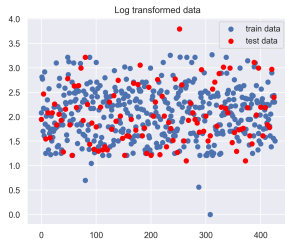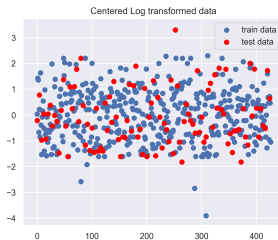
Visualization

Models

3 models implemented:

- Gaussian model

$$Y|\beta, \sigma \sim \mathcal{N}\left(X\beta, \sigma^2\right) \qquad \beta \sim \mathcal{N}_d(\vec{0}, 3^2 I), \ \sigma \sim exp(2)$$

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- Student model

$$Y|\beta, \nu \sim X\beta + t_\nu \qquad \beta \sim \mathcal{N}_d(\vec{0}, 3^2 I), \ \nu \sim \Gamma(2, 4)$$

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$$Y|\beta, \nu \sim X\beta + t_\nu \qquad \beta \sim \mathcal{N}_d(\vec{0}, 3^2 I), \ \nu \sim \Gamma(2, 4)$$

- Logistic regression

$$\mathbb{P}\left(Y = 1 | X, \beta\right) = \frac{e^{X^T\beta}}{1 + e^{X^T\beta}}, \quad \beta \sim \mathcal{N}_d(0, 3^2)$$

MAP (Maximum at posteriori) estimator

$$f(\theta) = -\log(\widetilde{f}(\theta|d))$$

$$\theta_c = \theta - \eta * \nabla_\theta f(\theta)$$

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- Vanilla gradient descent
- Line search gd $(0 < \beta < 1)$
  accept if :

$$f(\theta_c) < f(\theta) - \eta\varepsilon||\nabla_\theta f(\theta)||_2^2$$

  else:

$$\eta = \beta\eta$$

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- Vanilla gradient descent
- Line search gd ($0 < \beta < 1$)
  accept if :

$$f(\theta_c) < f(\theta) - \eta \varepsilon ||\nabla_\theta f(\theta)||_2^2$$

else:

$$\eta = \beta \eta$$

- Wolfe condition ($0 < c1 < c2 < 1$)
  decrease η if

$$f(\theta_c) \geq f(\theta) + c_1 \eta ||\nabla_\theta f(\theta)||_2^2$$

increase η if

$$-\nabla_\theta f(\theta_c)^T \nabla_\theta f(\theta) \geq -c_2 ||\nabla_\theta f(\theta)||_2^2$$

MH with random walk

Theory

1: **for** $i = 1$ to N **do**
2:     draw $\eta \sim \mathcal{N}_d(0, 1)$
3:     $\theta_c = \theta_n + \varepsilon\eta$
4:     $R = f(\theta_c|d)/f(\theta_n|d)$
5:     **if** $U(0, 1) \leq R$ **then**
6:         $\theta_{n+1} = \theta_c$
7:     **else**
8:         $\theta_{n+1} = \theta_n$
9:     **end if**
10: **end for**

Practice

- set ε such that the acceptance rate of the proposal is between 10 and 50 percent.
- compute everything using `expsumlog`
- check visually the chain to determine the burn-in
- test different initialization to detect potential silent failure

MH with Langevin correction (MALA)

Theory

As for the random walk MH algorithm except for

- the proposal:

$$\theta_c = \theta_n + \tau \nabla \log f(\theta_n | d) + \sqrt{2\tau} \eta$$

- the acceptance ratio:

$$R = \frac{f(\theta_c | d) q(\theta_n | \theta_c)}{f(\theta_n | d) q(\theta_c | \theta_n)}$$

Practice

- as before but be more careful with tuning the step size τ

-
$$q(x, x') \propto$$
$$\exp\left(\frac{||x' - x - \tau \nabla \log f(x|d)||_2^2}{-4\tau} \right)$$

- biggest challenge: implement computation of gradient in efficient manner

Comparison of accuracy on test set

Gaussian MAE

	error on test	error on train
gd	0.685706	0.613765
line_search_gd	0.685706	0.613765
Wolfe_cond_gd	0.738525	0.692682
MH_vanilla_mean	0.687246	0.614395
MH_vanilla_median	0.688279	0.614266
MH_Langevin_mean	0.687873	0.613893
MH_Langevin_median	0.688071	0.614006

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Student MAE

	error on test	error on train
gd	0.685292	0.609907
line_search_gd	0.685159	0.611209
Wolfe_cond_gd	0.685284	0.609814
MH_vanilla_mean	0.687430	0.611630
MH_vanilla_median	0.685471	0.612177
MH_Langevin_mean	0.685110	0.611450
MH_Langevin_median	0.684638	0.611450

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Logistic error rate

	error on test	error on train
gd	0.448598	0.271663
line_search_gd	0.467290	0.269321
Wolfe_cond_gd	0.467290	0.269321
MH_vanilla_mean	0.467290	0.274005
MH_vanilla_median	0.467290	0.271663
MH_Langevin_mean	0.467290	0.271663
MH_Langevin_median	0.467290	0.274005

Comparison in term of total time
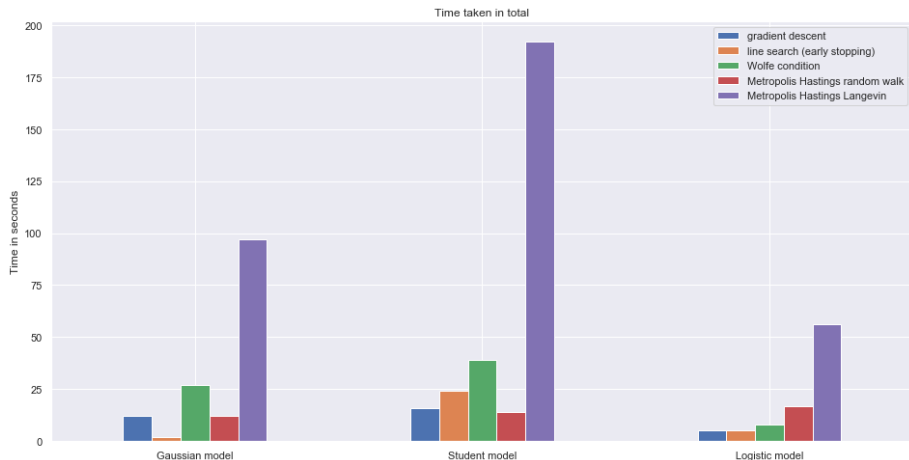
Comparison in term of time per iteration

Time taken per iterations
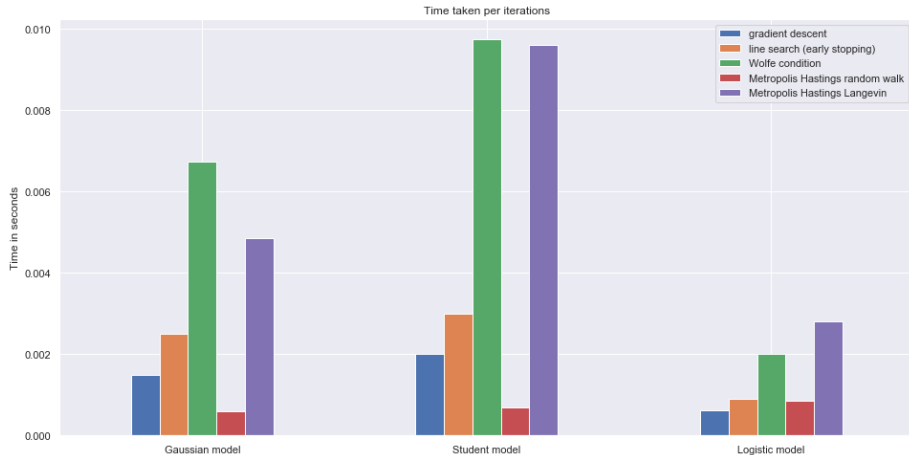
Legend:
- gradient descent
- line search (early stopping)
- Wolfe condition
- Metropolis Hastings random walk
- Metropolis Hastings Langevin

Conclusion

Modelization

- Simpler methods and models performed the best
- Relationship highly non-linear

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- Tuning of hyper-parameters
- Feature engineering

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Improvement

- Tuning of hyper-parameters
- Feature engineering
- Gamma model
- Classification in multiple ordered classes
- More robust and faster module to use more advanced techniques

References

- 📕 Guillaume Dehaene
  Lecture Notes, Bayesian computation MATH-435, 2019.
- 🗄 E. R. Berndt
  The practice of econometrics : classic and contemporary. Addison-Wesley Pub. Co., 1991
- 🎧 GitHub repository
  https://github.com/dufourc1/Bayesian computation