# Study on the Vehicle Scheduling Problem in Transportation System

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*Abstract*— **The vehicle scheduling problem (VSP) is a classical optimization problem which is part of the operational planning of public transportation systems. This paper analyzes of the optimization models for vehicle scheduling problem in public transport network. The departure time model and departure interval model are formulated in this presented paper. The heuristic algorithms to solve the above models are developed, which optimize the departure time and interval. Finally, the proposed solution methodology is applied to a case study, and the computational results are presented.**

***Keywords-vehicle scheduling problem; public transportation; heuristic; optimization***

## I. INTRODUCTION

This paper analyzes the vehicle scheduling problem (VSP) in public transportation system, mostly presents the optimization models for this transportation problem. The VSP is a classical optimization problem which consists in assigning a set of scheduled trips to a set of vehicles in the public transportation, so VSP is part of the operational planning of public transportation systems [1], also includes network design, allocations of crews and vehicles to routes, and service frequency for each route. This is one of the most complex network problems in operations planning [2]-[4]. There are feasible algorithms for some versions of the problem, i.e., when it minimizes the total schedule delay costs on the network[5]. The schedule for transit vehicles should be chosen to best serve demand. Some passengers transfer from one vehicle to the other at a station [6], and we should force the two vehicles arrive or depart at a station at the same time for the passenger transfers [7], or we minimize the time headway of two vehicles arriving (or departing) at a station. And the time headway of their arriving (or departing) is less as far as possible.

The VSP is considered in public transportation network, and proceed in two steps in this paper. One step is to determine the scheduling that minimizes the total time headway between the arrivals of all vehicles of all routes at a given station. The other step is to determine the departure interval of two near vehicles of the same route in a whole network.

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This paper is organized as follows. Section Ⅱ describes optimization models for VSP, which include the departure time model and departure interval model. Then the heuristic algorithms are developed in order to solve the above models in Section Ⅲ. Section Ⅳ presents the applications of proposed solution methodology to a study case, and computation results are presented. Finally, some concluding remarks and suggestions for further research on VSP are devoted in Section Ⅴ.

## II. MODEL FOR VSP

In this section, we present the optimization model for VSP. We use the following assumptions:

- The time between any given station is constant in a given optimization periods *T*.
- Departure time and arrival time are the same at a given station, so that passenger's transfer time is ignored.
- Vehicle must run from origin to destination by stopping at each station of the route, without any overtaking the vehicle of the same route.

We define the public transportation graph as a directed graph $\boldsymbol{G}=(\boldsymbol{I}, \boldsymbol{J})$. All variables and parameters applied here in the model for VSP are defined in Table 1.

### A. Departure Time Model

Due to reduce the passengers' waiting time and maintain passenger transfers, public transit service is to determine the scheduling that minimizes the time headway between the arrivals of any vehicles of the different routes at a given station in the COO-N. The time headway is formulated as $(x_j+t_{ij})-(x_j+t_{ij})$, for $i\in\boldsymbol{I}, j, k\in\boldsymbol{J}, j\neq k$. Thus we have:

$$P_{ik}=\begin{cases}1\ , & \text{if}\left(x_j+t_{ij}\right)-\left(x_k+t_{ik}\right)=0,\\ 0\ , & \text{otherwise,}\end{cases} \tag{1}$$

$$Y_i=\sum_k P_{ik}+1 \quad \forall i\in\boldsymbol{I}, j,k\in\boldsymbol{J},\ but\ k\neq j. \tag{2}$$

TABLE I. NOTATION

| Variables | Description | Unit |
|---|---|---|
| $\boldsymbol{I}$ | Set of transit stations | |
| $\boldsymbol{J}$ | Set of transit routes | |
| $i$ | Any station among $\boldsymbol{I}$, $i\in\boldsymbol{I}$={1, 2, …, $m$} | |
| $j$, $k$ | Any station among $\boldsymbol{I}$, $j$, $k\in\boldsymbol{J}$={1, 2,…, $n$}, but $k\neq j$ | |
| $x_j$, $x_k$ | Vehicles' departure time of route $j$ and $k$ at their origin stations | |
| $S_j$ | Latest departure time of the first vehicle of route $j$ | |
| $t_{ij}$, $t_{ik}$ | Take the time to arrive station $i$ from origin stations | Min |
| $P_{ik}$ | Number of routes $k$ which could arrival station $i$ with route $j$ at the same time | Bus |
| $Y_i$ | Number of routes which arrival station $i$ at the same time | Bus |
| $R_i$ | Capacity of station $i$ | Bus |
| $h_j$ | Minimum of departure interval of two vehicles of route $j$ | Min |
| $H_j$ | Maximum of departure interval of two vehicles of route $j$ | Min |
| $T_j$ | Bus travel time per trip in route $j$ | Min |
| $B_j$ | Bus operating cost in route $j$ | US$/bus-h |
| $b_j$ | Supplier cost | US$/h |
| $C_j$ | Capacity of bus | pass/bus |
| $Q_j$ | Demand from route $j$ | pass/h |
| $D_j$ | Departure interval of route $j$ | Min/bus |

The VSP can be formulated as follows:

$$\min\sum_{i=1}\sum_{\forall j,k\in\boldsymbol{J},j<k}\left|\left(x_j+t_{ij}\right)-\left(x_k+t_{ik}\right)\right| \cdot \tag{3}$$

Also, it can be formulated like this:

$$\min\sum_{i=1}\sum_{\forall j,k\in\boldsymbol{J},j<k}\left|\left(x_j+t_{ij}+\frac{D_k}{2}\right)-\left(x_k+t_{ik}+\frac{D_j}{2}\right)\right|, \tag{4}$$

where the $D_k$=$D_j$, (4) is the same to (3).

$$\text{s.t}\begin{cases}x_j, x_k\le S_j,\\ Y_i\le R_i,\\ \forall i\in\boldsymbol{I}, j,k\in\boldsymbol{J},\ but\ k\neq j.\end{cases}$$

The aim is to minimize total time headway between the arrivals of all vehicles of all routes at a given station in (3). The other aim is to minimize total time headway between the arrivals of all vehicles of all routes, which let one bus arrival the station at the time of middle interval of others route's near two bus in (4). The constraints say that the departure time of the first vehicle of route must be less or equal to the latest departure time, and the capacity of station must be greater or equal to the number of routes which arrival station at the same time.

### *B. Departure Interval Model*

A given optimization period $T$ divides into several departure intervals. The bound of $D_j$ is [$h_j$, $H_j$]. The minimum interval $h_j$ is to ensure that the supplier cost will not exceed the operating cost, and the maximum interval $H_j$ is to ensure the service capacity satisfy the demand. $h_j$ and $H_j$ can be formulated as follows:

$$h_j=\frac{2T_jB_j}{b_j}, \tag{5}$$

$$H_j=\frac{60\times C_j}{Q_j}. \tag{6}$$

Thus we have:

$$\boldsymbol{D}_j=\left[\frac{2T_jB_j}{b_j},\frac{60\times C_j}{Q_j}\right].\ \forall j\in\boldsymbol{J} \tag{7}$$

We let all the intervals of any route into one intersection $\boldsymbol{D}$, thus we have:

$$\boldsymbol{D}=\boldsymbol{D}_1\cap\cdots\cap\boldsymbol{D}_j\cap\cdots\cap\boldsymbol{D}_k\cap\cdots\cap\boldsymbol{D}_n\quad k\neq j\,\cdot \tag{8}$$

Let $d$ be the minimum of $\boldsymbol{D}$. That is, $d$ is the departure interval of the public transportation network.

If $\boldsymbol{D}$ is null set, this present model can not get the departure interval of the whole network. Therefore the network is divided into several subsets to maintain as many as routes to have a same interval.

## III. HEURISTIC ALGORITHMS

Define $t_{ij}\in\boldsymbol{T}$ and $x_j\in\boldsymbol{X}$=($x_1$, $x_2$,…, $x_n$): $i\in\boldsymbol{I}$, $j\in\boldsymbol{J}$. If the vehicle of route $j$ does not stop at station $i$, set $t_{ij}$:= $+\infty$. Also we define a matrix to denote the real arrival time, as follows:

$$\mathbf{W}=\begin{pmatrix}x_1+t_{11} & x_2+t_{12} & \cdots & x_n+t_{1n}\\ x_1+t_{21} & \ddots & & \vdots\\ \vdots & & \ddots & x_n+t_{m-1,n}\\ x_1+t_{m1} & \cdots & x_{n-1}+t_{m,n-1} & x_n+t_{mn}\end{pmatrix}.$$

Thus the heuristic algorithms to the above models as follows:

Step 1. Generate the initial solution: $\boldsymbol{X}^0$=0, $\boldsymbol{D}^0$=0, and hence $\boldsymbol{W}^0$=$\boldsymbol{T}^0$.

Step 2. Generate the mean $E$ and variance of each row in matrix $\boldsymbol{W}$. The row with maximum variance is the iterative direction. Let the iterative step be $[E_i^{a-1}-(x_i+t_{ij})^{a-1}]/2$, for $a$=$a$+1.

Step 3. Improve $\boldsymbol{W}$ by step. i.e., let $(x_i+t_{ij})^a$ := $(x_i+t_{ij})^{a-1}$+ $[E_i^{a-1}-(x_i+t_{ij})^{a-1}]/2$.

Step 4. Calculate the object function value $Z$.

Step 5. If

$$\frac{Z^a-Z^{a-1}}{Z^{a-1}}\le\varepsilon$$

($\varepsilon$ is a given small positive number), or the iterative time reach the predetermined time. Let $\boldsymbol{X}^*$ :=$\boldsymbol{X}$, $Z^*$ :=$Z^a$; go to Step 2, for $a$=$a$+1.

Step 6. Generate the departure interval $d$.

Step 7. Put out the vehicle schedule.

## IV. Case Study

In this section, we present the applications to a case in Guangzhou. We select 5 transit routes in Guangzhou to make a small public transportation network for this case, shown in Figure 1. The 5 transit routes get through each zone in Guangzhou. We present the scheduling constraints in table Ⅱ.We show the initial data of the transportation network in table Ⅲ. The first column of the table presents the station which are the key station in the network, whereas the first row contains the routes No., mean, variance, and so on. The remaining cells of the table Ⅲ include the time for vehicles from the origin station to a given station.

Figure 2 shows that as the iterative time increases, the value of (3) reduces (157 vs. 79), where the optimization value appears at the 28th iterative within the 30 predetermined time. The optimization results of this application case are showed in table Ⅳ. These results show that the application is more efficient.

For the VSP, it has been necessary to divide the day into several optimization periods. As the time in table Ⅲ are the mean time of the peak hour (8:00 to 9:00 a.m), thus we let optimization period $T$:=8:00 to 9:00 a.m. In this period the departure interval $d$ is 9min. Table Ⅴ shows the vehicle scheduling of this network in 8:00 to 9:00 a.m. In this table, the data are convergence to be the same.

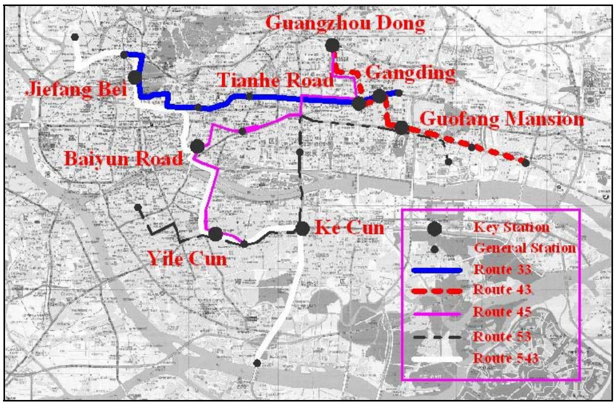


Figure 1. A small public transportation network

TABLE II. SCHEDULING CONSTRAINTS

| Variables | Route No. | | | | |
|---|---|---|---|---|---|
| | 33 | 43 | 45 | 53 | 543 |
| $T_j$ (Min) | 48 | 38 | 55 | 46 | 57 |
| $B_j$ (US$/bus-h) | 29 | 26 | 27 | 27 | 26 |
| $b_j$ (US$/h) | 357 | 343 | 343 | 350 | 350 |
| $C_j$ (pass/bus) | 75 | 75 | 75 | 65 | 65 |
| $Q_j$ (pass/h) | 400 | 380 | 375 | 330 | 340 |

TABLE III. INITIAL DATA OF THE CASE

| Station | Route No. | | | | | Mean | Variance | Total |
|---|---|---|---|---|---|---|---|---|
| | 33 | 43 | 45 | 53 | 543 | | | |
| Guangzhou Dong | +∞ | 0 | 0 | +∞ | +∞ | 0 | 0 | 0 |
| Tianhe Road | +∞ | 10.25 | 10.25 | +∞ | +∞ | 10.25 | 0 | 0 |
| Gangding | 3.19 | 15.37 | +∞ | +∞ | +∞ | 9.28 | 74.18 | 12.18 |
| Guofang Mansion | +∞ | 22.66 | +∞ | 4.77 | +∞ | 13.72 | 160.03 | 17.89 |
| Baiyun Road | +∞ | +∞ | 40.28 | +∞ | 20.00 | 30.14 | 205.64 | 20.28 |
| Jiefang Bei | 40.56 | +∞ | +∞ | +∞ | 3.57 | 22.07 | 684.13 | 36.99 |
| Yile Cun | +∞ | +∞ | 54.67 | 36.78 | 31.80 | 41.08 | 144.65 | 45.74 |
| Ke Cun | +∞ | +∞ | +∞ | 22.59 | 46.49 | 34.54 | 285.605 | 23.90 |
| Sum | | | | | | | | 157 |

Note: The +∞ is not taken into account

TABLE IV. RESULTS OF THE CASE

| Station | Route No. | | | | | Mean | Variance | Total |
|---|---|---|---|---|---|---|---|---|
| | 33 | 43 | 45 | 53 | 543 | | | |
| Guangzhou Dong | +∞ | -4.2 | -3.3 | +∞ | +∞ | -3.7 | 0.4 | 0.9 |
| Tianhe Road | +∞ | 6.1 | 7.0 | +∞ | +∞ | 6.5 | 0.4 | 0.9 |
| Gangding | -2.8 | 11.2 | +∞ | +∞ | +∞ | 4.2 | 97.9 | 14.0 |
| Guofang Mansion | +∞ | 18.5 | +∞ | 28.1 | +∞ | 23.3 | 45.9 | 9.6 |
| Baiyun Road | +∞ | +∞ | 37.0 | +∞ | 37.8 | 37.4 | 0.3 | 0.8 |
| Jiefang Bei | 34.6 | +∞ | +∞ | +∞ | 21.4 | 28.0 | 86.8 | 13.2 |
| Yile Cun | +∞ | +∞ | 51.4 | 60.1 | 49.6 | 53.7 | 31.3 | 20.9 |
| Ke Cun | +∞ | +∞ | +∞ | 45.9 | 64.3 | 55.1 | 169.7 | 18.4 |
| Sum | | | | | | | | 79 |

Note: The +∞ is not taken into account

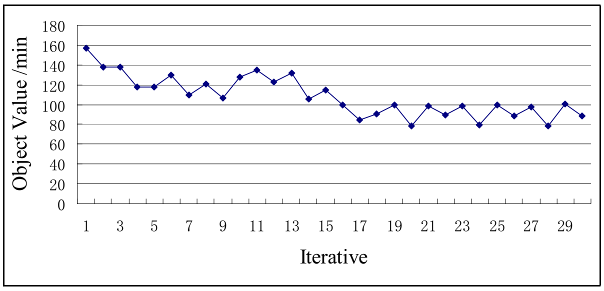


Figure 2. Curve of object value

TABLE V. VEHICLE SCHEDULING (8:00-9:00)

| Vehicle | Route No. | | | | |
|---|---|---|---|---|---|
| | **33** | **43** | **45** | **53** | **543** |
| 1 | 8:00 | 8:02 | 8:03 | 8:02 | 8:06 |
| 2 | 8:09 | 8:11 | 8:12 | 8:11 | 8:15 |
| 3 | 8:18 | 8:20 | 8:21 | 8:20 | 8:24 |
| 4 | 8:27 | 8:29 | 8:30 | 8:29 | 8:33 |
| 5 | 8:36 | 8:38 | 8:39 | 8:38 | 8:42 |
| 6 | 8:45 | 8:47 | 8:48 | 8:47 | 8:51 |
| 7 | 8:54 | 8:56 | 8:57 | 8:56 | 9:00 |

## V. CONCLUSION

New optimization models for the vehicle scheduling problem have been presented in this paper, where the models are the departure time model and departure interval model.. Also, it presented the heuristic algorithms to above models. The computational results of a study case presented in this paper show that the solutions are more efficient. Finally, the proposed solution methodology is applied to a case study, and the computational results are presented.

To the real vehicle scheduling problem point of view, the time from origin station to a given station usually is not constant, and we should assume the travel time and its reliability. These suggest us further improving the solution approach based on dynamic vehicle scheduling problem. Furthermore, the computational time would be greater in a real complex network with more stations, routes, and constraints. We suggest using the immune arithmetic to divide the transportation network in several subsets, and using multi-agent to solve the VSP in public transportation system.

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