What Can ML Do For Algorithms?

Sergei Vassilvitskii

Google

Theme

Machine Learning is everywhere…

- Self driving cars
- Speech to speech translation
- Search ranking
- …

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Machine Learning is everywhere…

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…but it's not helping us get better theorems

Motivating Example

Given a sorted array of integers A[1…n], and a query q check if q is in the array.

2	4	7	11	16	22	37	38	44	88	89	93	94	95	96	97	98

7

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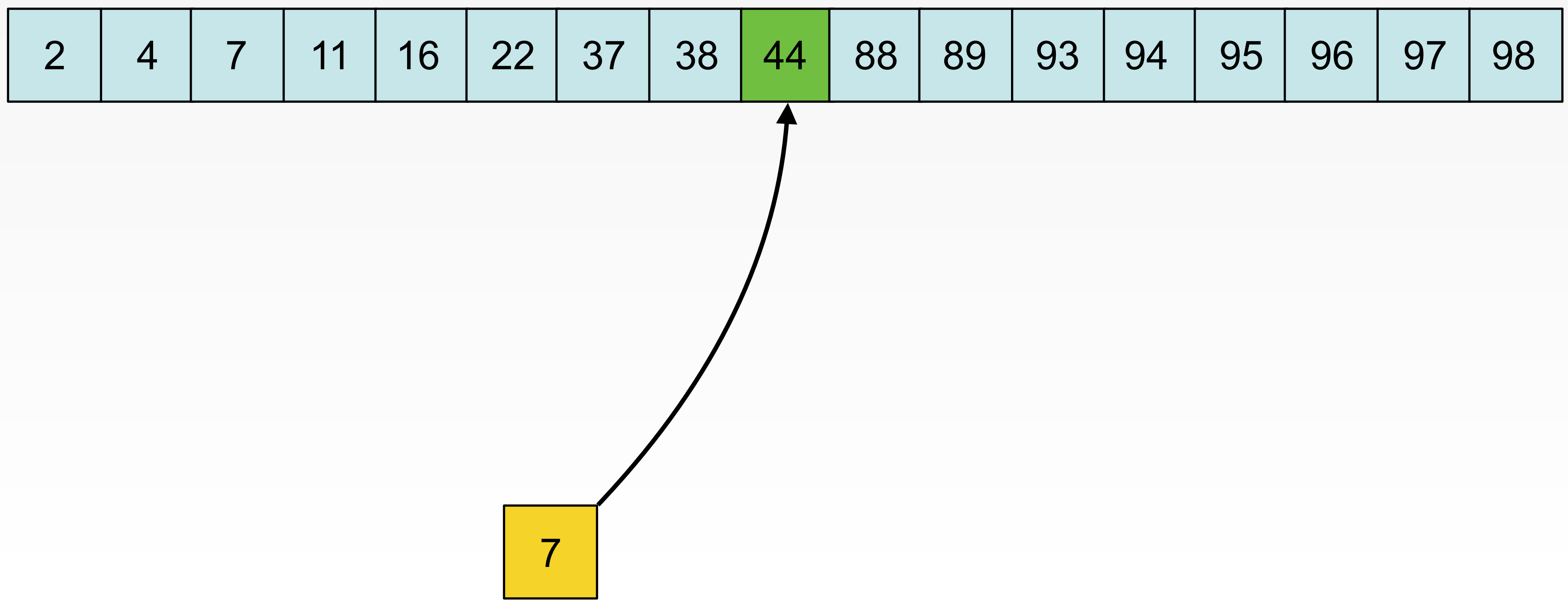

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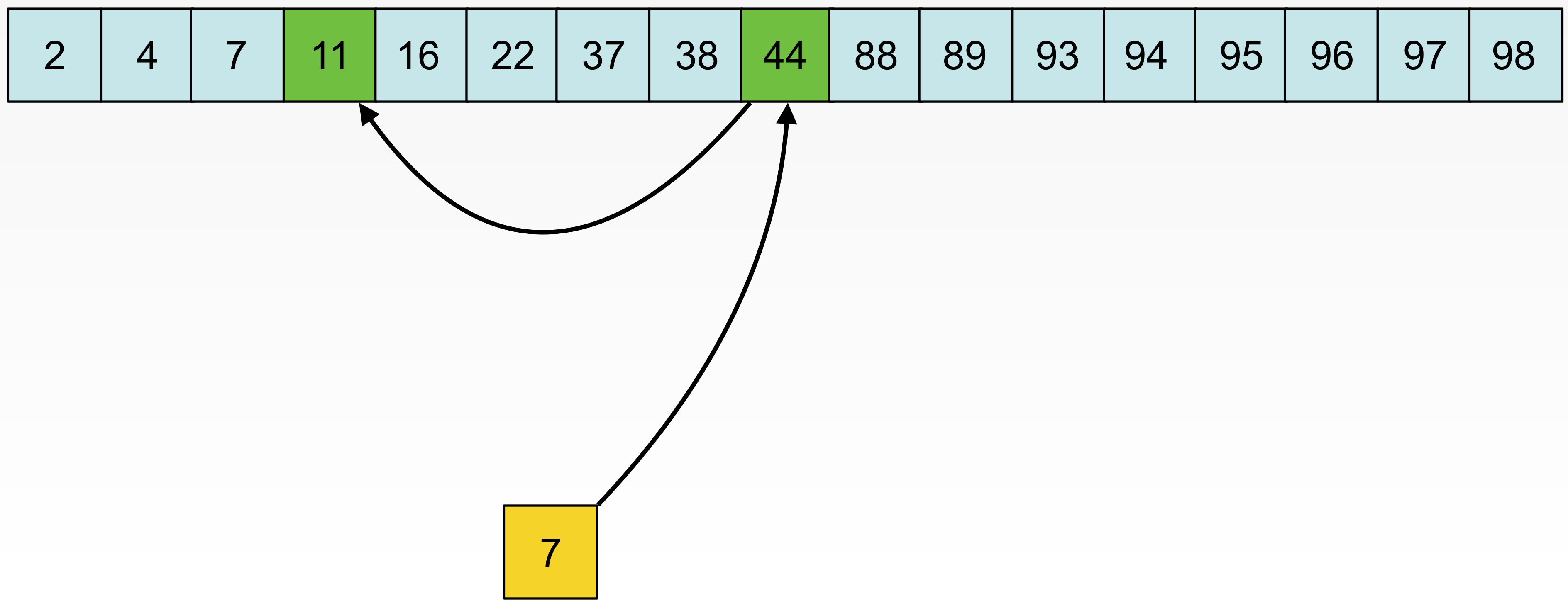

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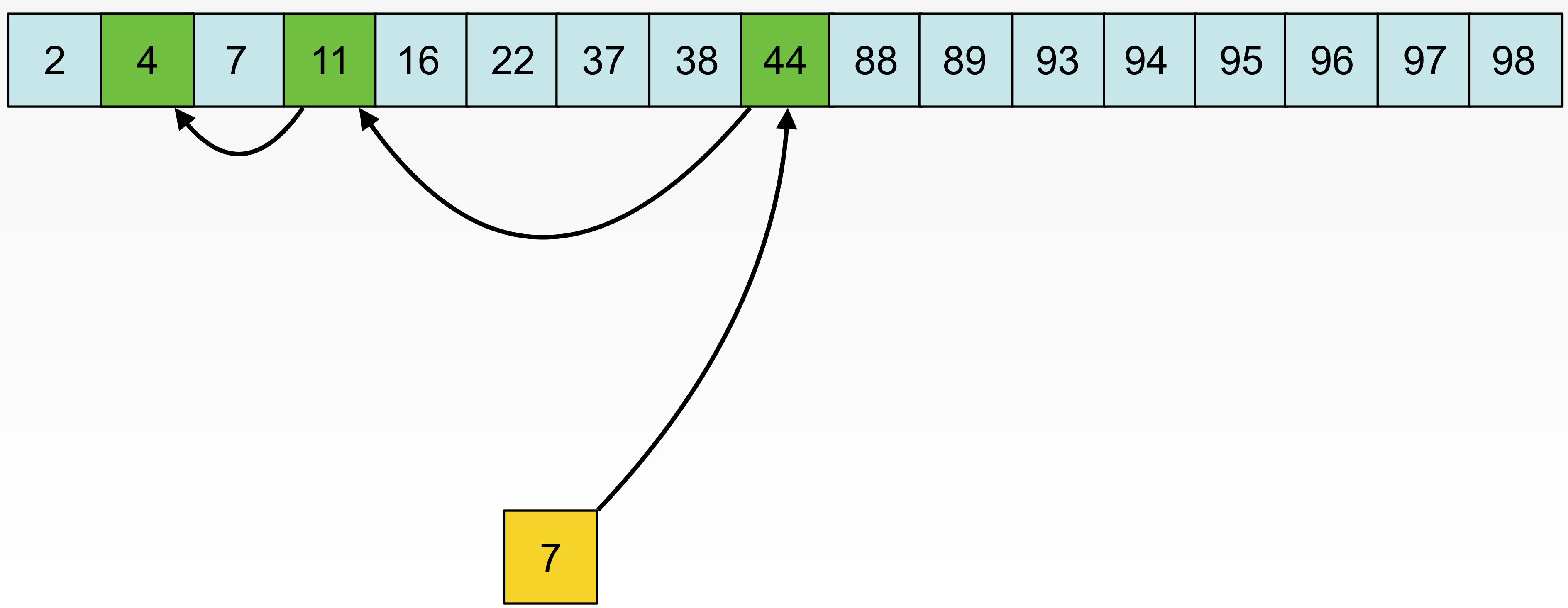

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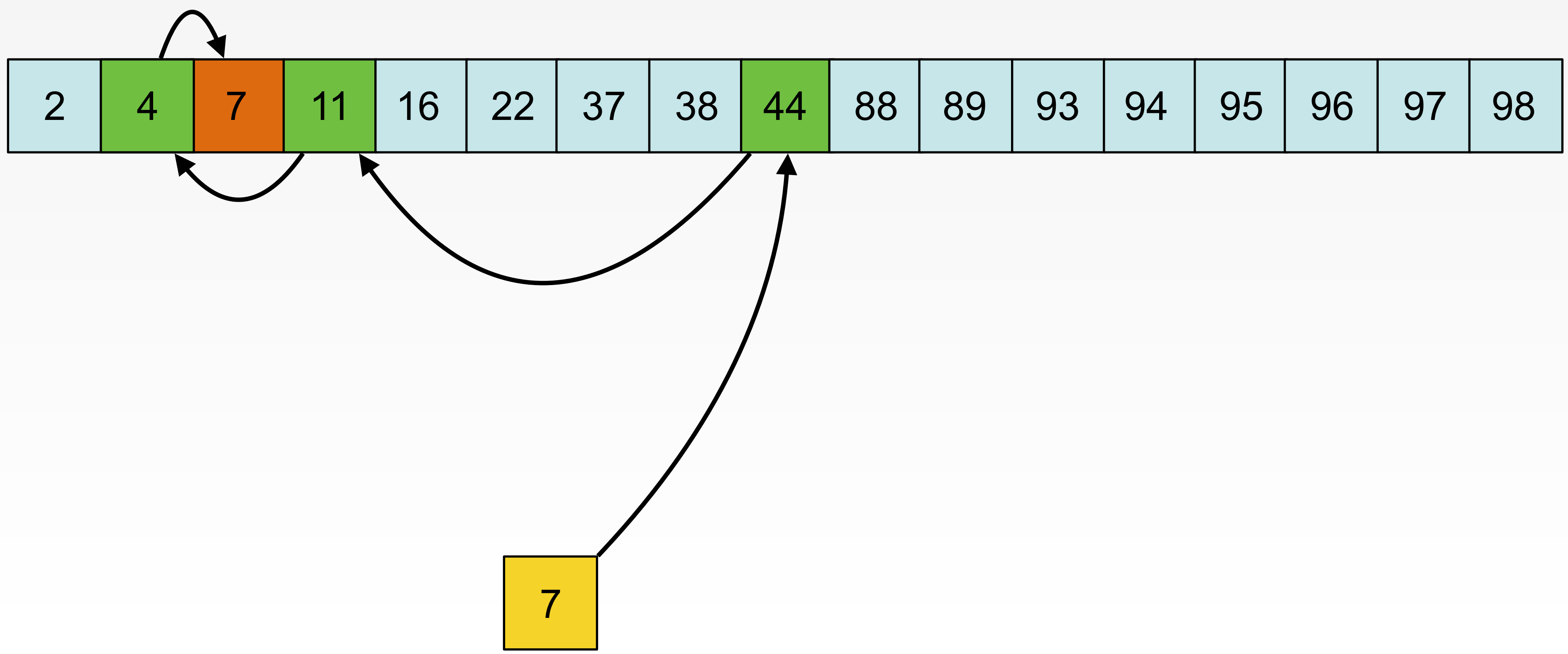

– Look up time: $O(\log n)$

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– Then proceed via doubling binary search

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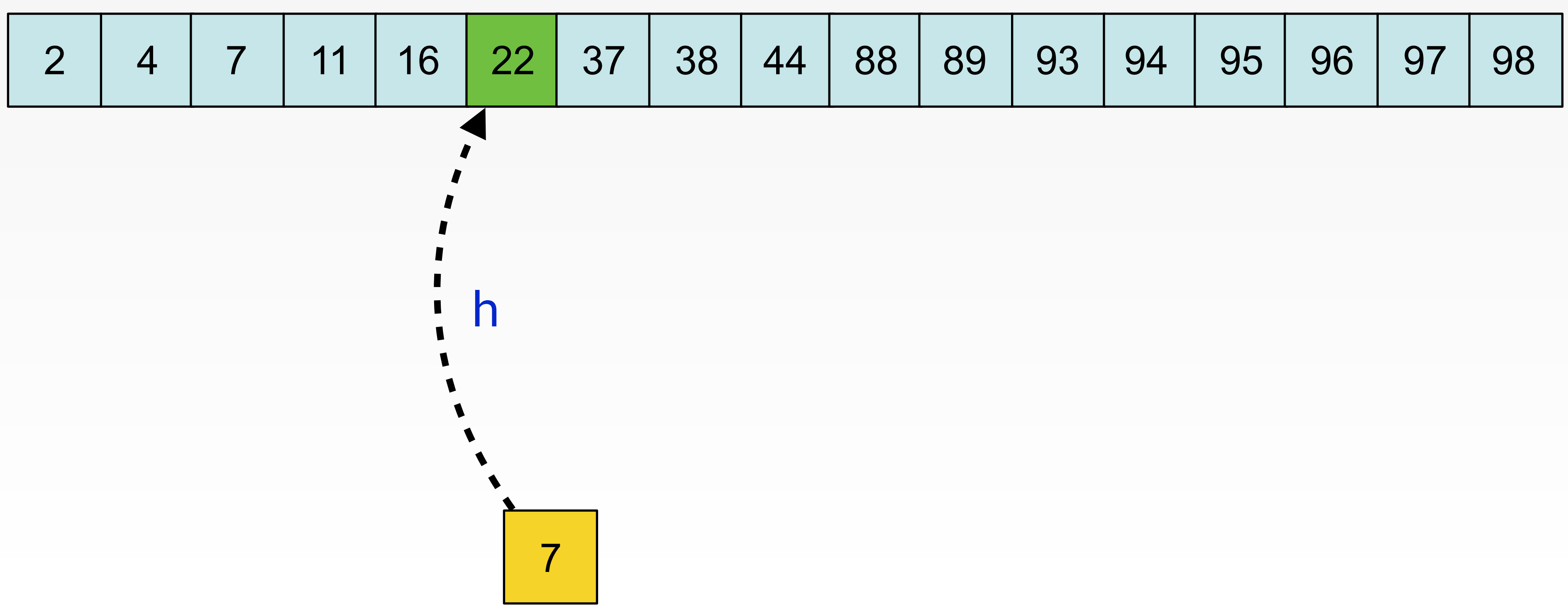

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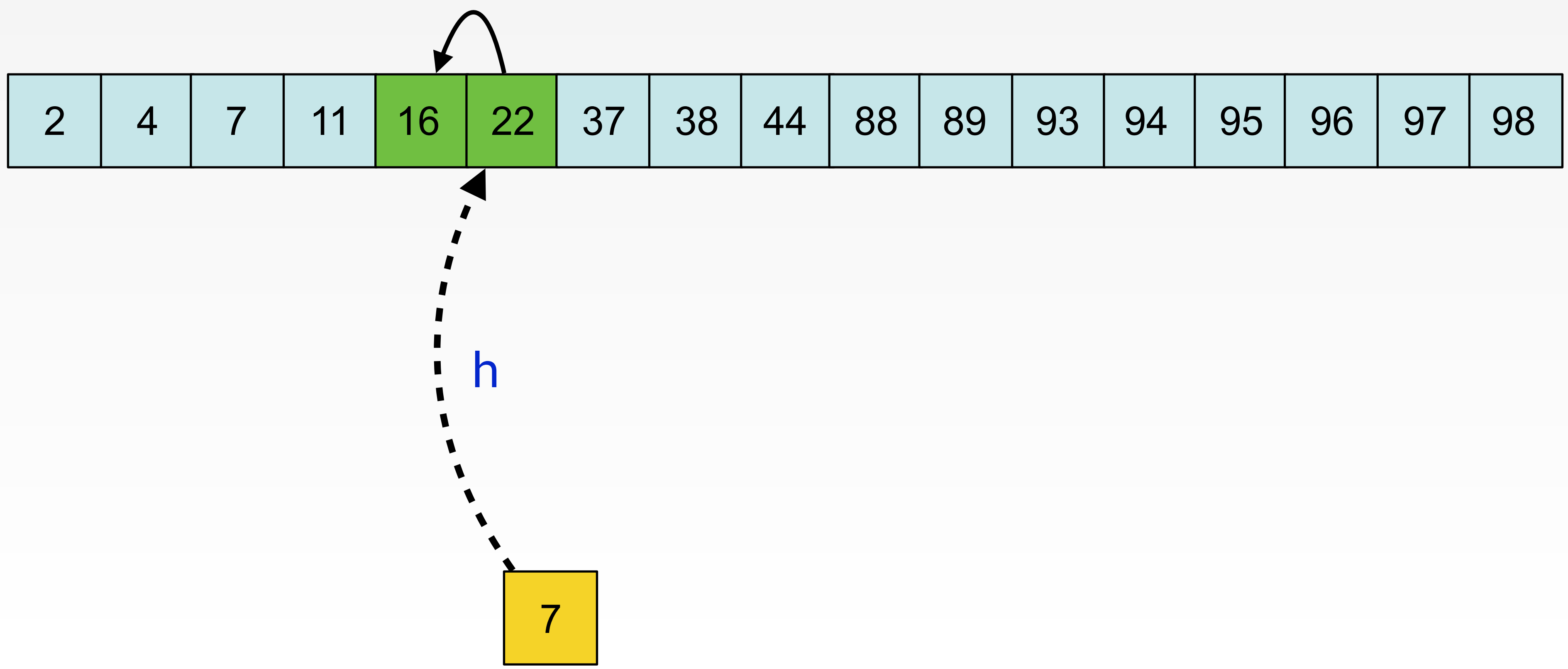

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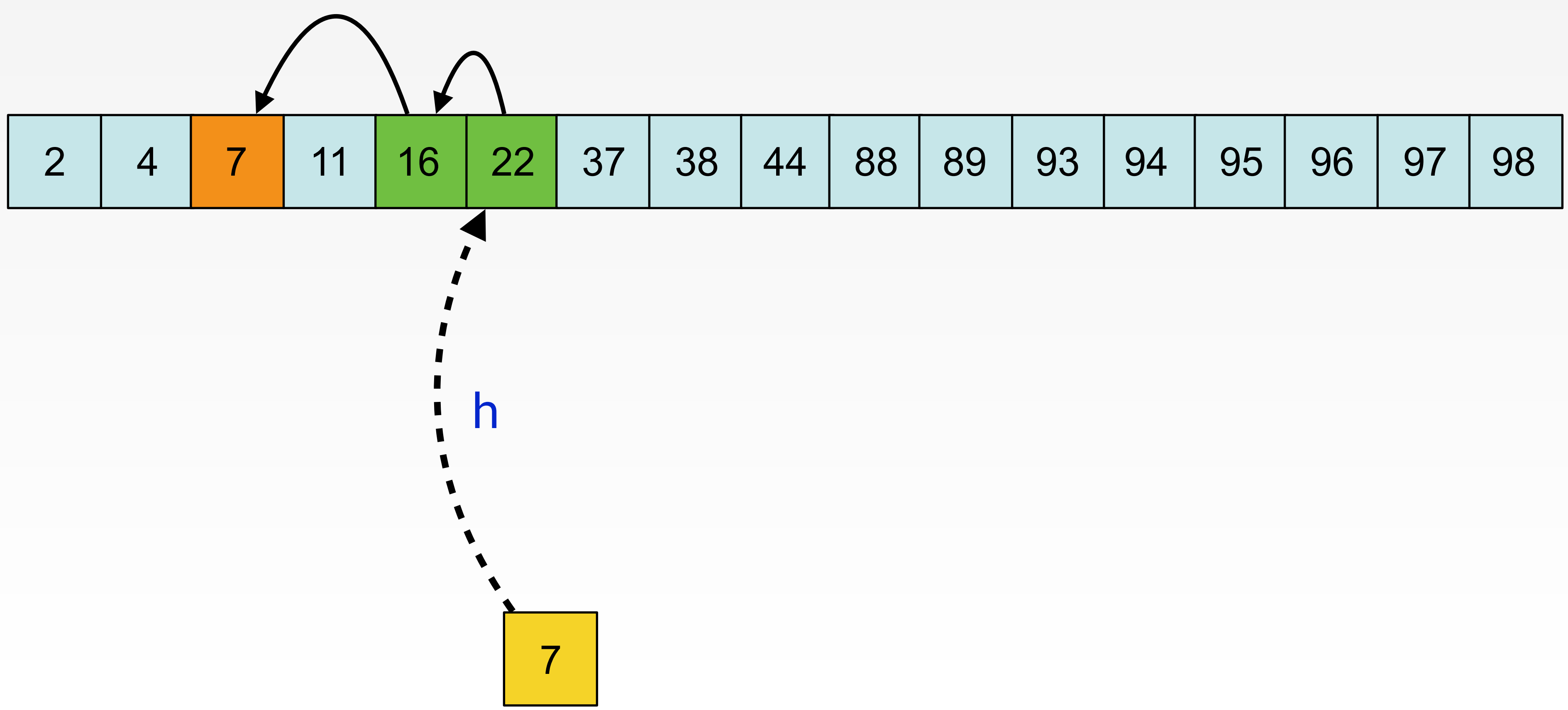

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– Then proceed via doubling binary search

Empirical Slide [Kraska et al. 2018]

Type	Config	Map Data		
		Size (MB)	Lookup (ns)	Model (ns)
Btree	page size: 32	52.45 (4.00x)	274 (0.97x)	198 (72.3%)
	page size: 64	26.23 (2.00x)	277 (0.96x)	172 (62.0%)
	page size: 128	13.11 (1.00x)	265 (1.00x)	134 (50.8%)
	page size: 256	6.56 (0.50x)	267 (0.99x)	114 (42.7%)
	page size: 512	3.28 (0.25x)	286 (0.93x)	101 (35.3%)
Learned Index	2nd stage models: 10k	0.15 (0.01x)	98 (2.70x)	31 (31.6%)
	2nd stage models: 50k	0.76 (0.06x)	85 (3.11x)	39 (45.9%)
	2nd stage models: 100k	1.53 (0.12x)	82 (3.21x)	41 (50.2%)
	2nd stage models: 200k	3.05 (0.23x)	86 (3.08x)	50 (58.1%)

– Smaller Index

– Faster lookups when error is low, including ML cost

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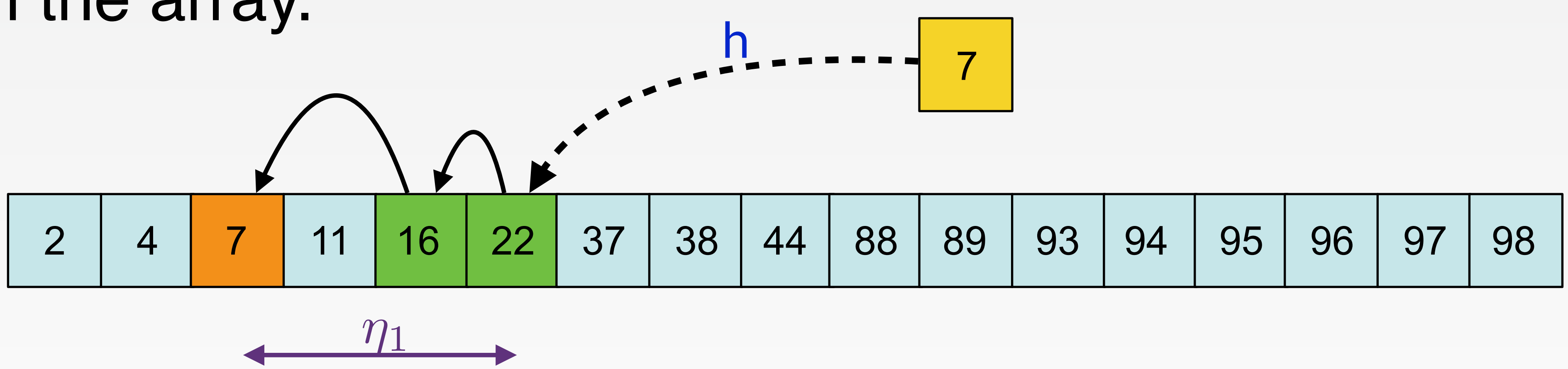

Analysis:

– Let $\eta_1 = |h(q) - \mathrm{OPT}(q)|$ be the error of the predicted position

– Running time: $O(\log \eta_1)$

- Can be made practical (must worry about speed & accuracy of predictions)

More on the analysis

Comparing

- Classical: $O(\log n)$
- Learning augmented: $O(\log \eta_1)$

Results:

- Consistent: perfect predictions recover optimal (constant) lookup times.
- Robust: even if predictions are bad, not (much) worse than classical

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Punchline:

- Use Machine Learning together with Classical Algorithms to get better results.

Outline

Introduction

Motivating Example

Learning Augmented Algorithms

- Overview
- Online Algorithms
- Streaming Algorithms
- Data Structures

Conclusion

Learning Augmented Algorithms

Nascent Area with a number of recent results:

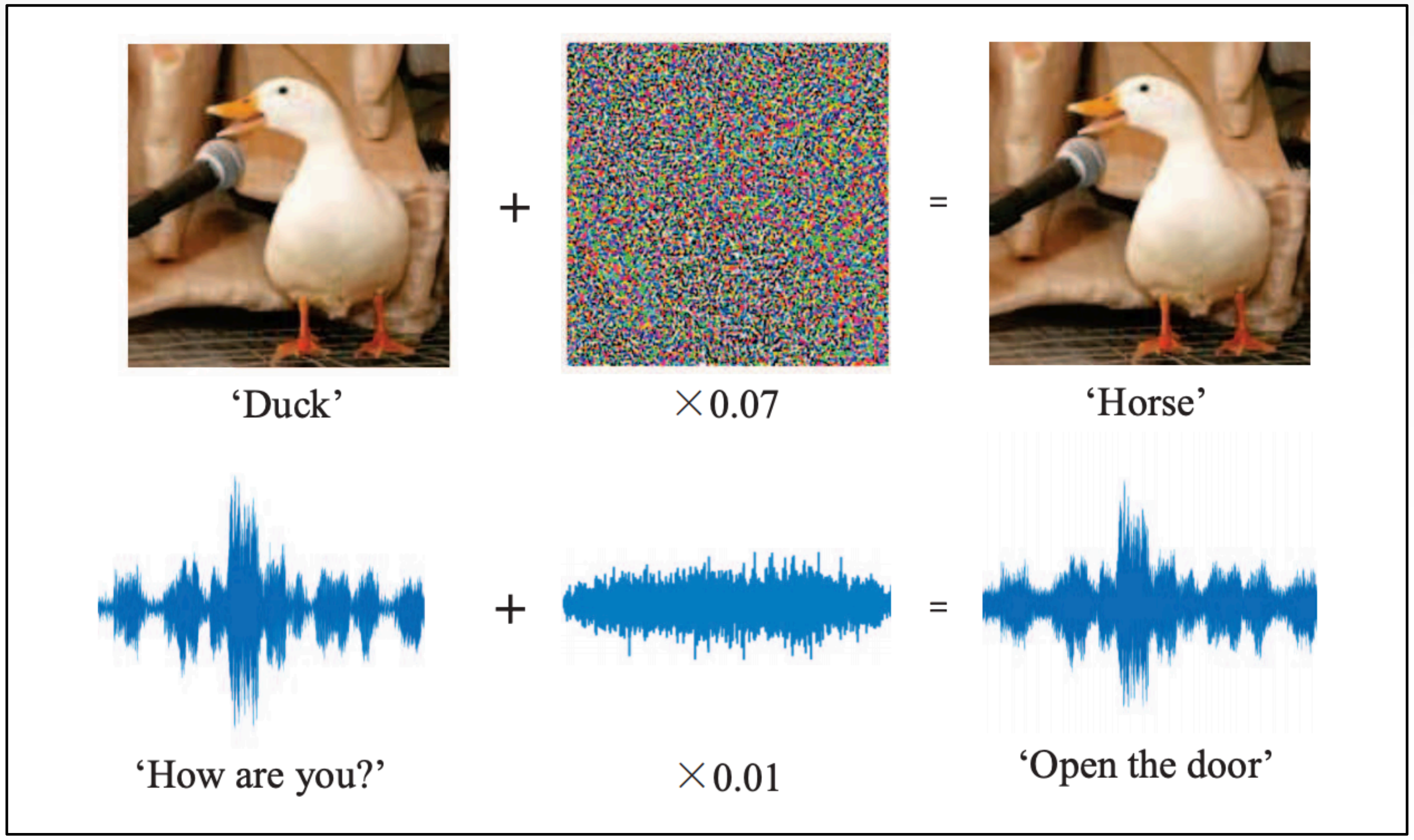
- Build better data structures
  - Indexing: Kraska et al. 2018
  - Bloom Filters: Mitzenmacher 2018
- Improve Competitive and Approximation Ratios
  - Pricing : MedinaV 2017,
  - Caching: LykourisV 2018
  - Scheduling: Kumar et al. 2018, Lattanzi et al. 2019, Mitzenmacher 2019
- Reduce running times
  - Branch and Bound: Balcan et al. 2018
- Reduce space complexity
  - Streaming Heavy Hitters: Hsu et al. 2019

Limitations of Machine Learning

Limitations of Machine Learning

Limit 1. Machine learning is imperfect.

- Algorithms must be robust to errors



Limitations of Machine Learning

Limit 1. Machine learning is imperfect.

- Algorithms must be robust to errors

Limit 2. ML is best at learning a few things

- Generalization is hard, especially with little data
- e.g. predicting the whole instance is unreasonable

Limitations of Machine Learning

Limit 1. Machine learning is imperfect.

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Limit 2. ML is best at learning a few things

- Generalization is hard, especially with little data
- e.g. predicting the whole instance is unreasonable

Limit 3. Most ML minimizes a few different functions

- Squared loss is most popular
- Esoteric loss functions are hard to optimize (e.g. pricing)

But.. the power of ML

Machine learning reduces uncertainty

- Image recognition : uncertainty of what is in the image
- Click prediction: uncertainty about which ad will be clicked
- …

Online Algorithms with ML Advice

Augment online algorithms with some information about the future.

Goals:

- If the ML prediction is good : algorithm should perform well
  - Ideally: perfect predictions lead to competitive ratio of 1
- If the ML prediction is bad : revert back to the non augmented optimum
  - Then trusting the prediction is "free"

- Isolate the role of the prediction as a plug and play mechanism.
  - Allow to plug in richer ML models.
  - Ensure that better predictions lead to better algorithm performance.

Online Algorithms with ML Advice

Augment online algorithms with some information about the future.

Not a new idea:

- Advice Model : minimize the number of bits of perfect advice to recover OPT
- Noisy Advice: minimize the number of bits of imperfect advice to recover OPT

What is new:

- Look at quality of natural prediction tasks rather than measuring # of bits.

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- Overview
- *Online Algorithms: Paging*
- Streaming Algorithms: Heavy Hitters
- Data Structures: Bloom Filters

Conclusion

Caching (aka Paging)

Caching problem:

Have a cache of size k.

Elements arrive one a time.

- If arriving element is in the cache: cache hit, cost 0.
- If arriving element is not in the cache. Cache miss. Pay cost of 1.
  - Evict one element from the cache, and place the arriving element in its slot

State of the Art (in theory)

Bad News:

- Any deterministic algorithm is k-competitive
- There exist randomized algorithms that are $\log k$ competitive
- But no better competitive ratio is possible

A bit unsatisfying:

- Would like a constant competitive algorithm
- Would like to use theory to guide us in selection of a good algorithm

ML Advice

What kind of ML predictions would be helpful?

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Generally:

- The richer the prediction space, the harder it is to learn
- Lots of learning theory results quantifying this exactly

- Intuition: need enough examples for every possible outcome.

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What to predict for caching?

Offline Optimum

What is the offline optimum solution?

Offline Optimum

What is the offline optimum solution?

Simple greedy scheme (Belady's rule)

- Evict element that reappears <u>furthest</u> in the future

- Intuition: greedy stays ahead (makes fewest evictions) as compared to any other strategy.

What to Predict?

What do we need to implement Belady's rule?

Predict: the next appearance time of each element upon arrival.

Notes:

- One prediction at every time step
- No need to worry about consistency of predictions from one time step to the next

Measuring Error

Tempting:

- Use the performance of the predictor, h, in the caching algorithm

Better:

- Use a standard error function
- For example squared loss, absolute loss, etc.

Why Better?

- Most ML methods are used to optimize squared loss
- Want the training to be independent of how the predictor is used
- Decomposes the problem into (i) find a good prediction and (ii) use this prediction effectively

A bit more formal

Optimum Algorithm:

– Always evict element that appears furthest in the future.

Prediction:

– Every time an element arrives, predict when it will appear next

– Today consider absolute loss:

$$\eta = \sum_i |h(i) - t(i)|$$

Actual Arrival Time (integral)

Predicted Arrival Time

Using the predictions

Now have a prediction. What’s next?

Blindly Following the Oracle

Algorithm:

– Evict element that is predicted to appear furthest in the future

Blindly Following the Oracle

Elements

- x in position 2r
- y in position 2r+1
- c at position 1,T

Predictions of next arrival

- For x : always correct
- For y : always correct
- For c : 1

c x y x y x y x y x y x y x y x y … c

Evict Element Predicted Furthest in the Future

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↑

Algorithm :

- [t = 2] Initial Cache: `[c,x]`

Evict Element Predicted Furthest in the Future

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Algorithm :

- [t = 2] Initial Cache: `[c,x]`
- [t = 3] Evict x, place y: `[c,y]`

Evict Element Predicted Furthest in the Future

Blindly Following the Oracle

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- x in position 2r
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Algorithm :

- [t = 2] Initial Cache: [c,x]
- [t = 3] Evict x, place y: [c,y]
- [t = 4] Evict y, place x: [c,y]
- …

Error :

- Constant on average

Evict Element Predicted Furthest in the Future

Using the Prediction

Blindly following the oracle:

- Not a good idea
- Constant average error can lead to super-constant competitive ratio

Algorithms to the rescue!

Using the Prediction

Marker Algorithm:

- In beginning of a phase all elements unmarked
- When an element arrives, mark it.
- When need to evict, pick a random unmarked element
- When all elements are marked, start a new phase, and unmark all elements
- Theorem: $2\log k$ - competitive [Fiat+'91].

Predictive Marker [LykourisV'18]

Marker Algorithm:

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Notes:

- If predictions are perfect, almost follows Belady's rule. Recover a 2-competitive algorithm.
- When predictions are terrible, algorithm is k-competitive, small tweaks can ensure log k competitive in the worst case.

Proof Intuition

What causes cache misses?

- Elements appearing that have not been seen for a long time
  - OPT has to pay for these as well
- Recent elements being evicted
  - Tried to minimize this (subject to predictions)
  - Charge these to error of the predictor
  - Phases defined by marker cap the maximum impact of errors

Analysis

Main claim:

- Suppose the absolute error of predictor during the phase is η. Then number of misses due to mispredictions is at most $O(\sqrt{\eta})$.
- Intuition: loss on two length t sequences: a,b,c,…,t and t,…,c,b,a is $\Omega(t^2)$.

Altogether:

- Given a predictor with total error η, predictive marker has competitive ratio of $O(1 + \sqrt{1 + 4\eta/OPT})$
- Can tune to recover worst case bounds: $\min(O(\frac{\sqrt{\eta/OPT}}{\epsilon}), (2+\epsilon)\log k)$

Empirical Slide

Algorithm	Britekite Competitive ratio	Citi Bike Competitive ratio
BlindOracle	2.049	2.023
LRU	1.280	1.859
Marker	1.310	1.869
Predictive Marker	1.266	1.810

Discussion:

– Blind Oracle is too sensitive to errors in the data

– LRU tends to outperform Marker (latter is too pessimistic)

– Predictive marker consistently outperforms LRU.

Online Algorithms

Other algorithms analyzed in this setting:

- Ski Rental
- Non clairvoyant job scheduling
- Online scheduling with restricted assignment
- Online matching
- Online pricing

Many open problems:

- Clustering
- Submodular Maximization
- k-server
- …

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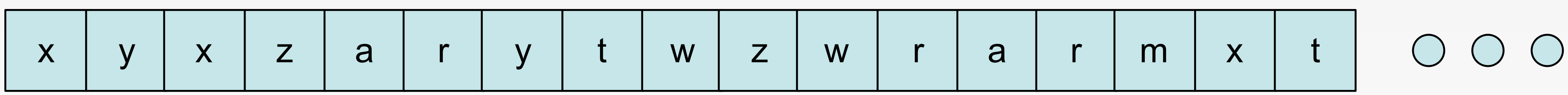
Streaming Algorithms

See a never ending stream of elements, only allowed to use small (typically logarithmic) amount of memory.



Canonical question:

- Frequency estimation: compute the frequency of every element in the stream
- If elements are drawn from U trivial to do in O(|U|) space
- How to use less space?

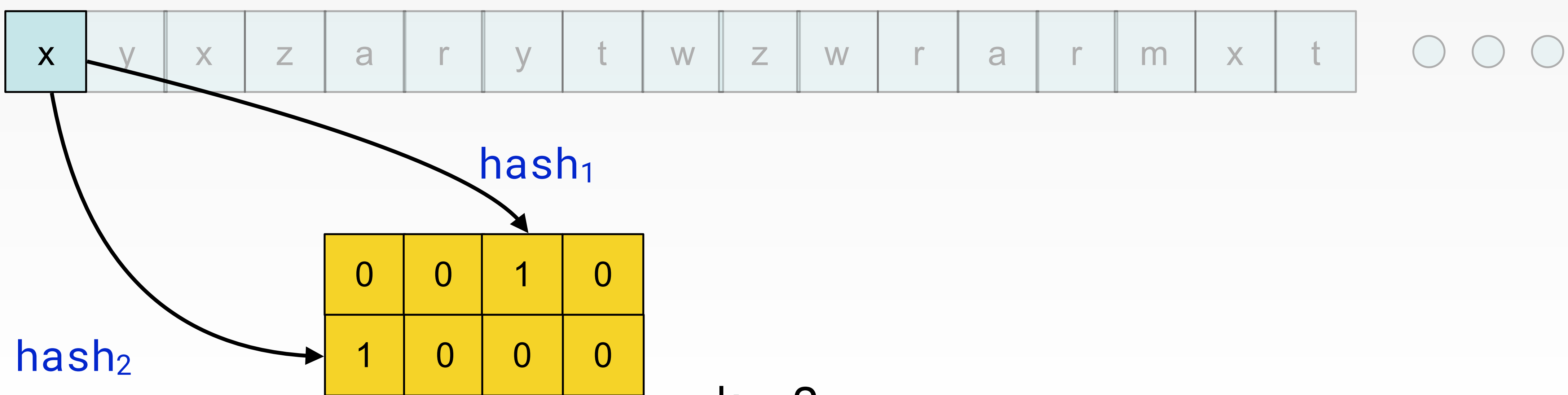
Frequency Estimation: Count Min Sketch

CountMin:

– Prepare k hash functions to use B/k buckets each.

– Keep a histogram on frequency of each hash function

– Return the minimum hashed value for any element



k = 2

B = 8

Frequency Estimation: Count Min Sketch

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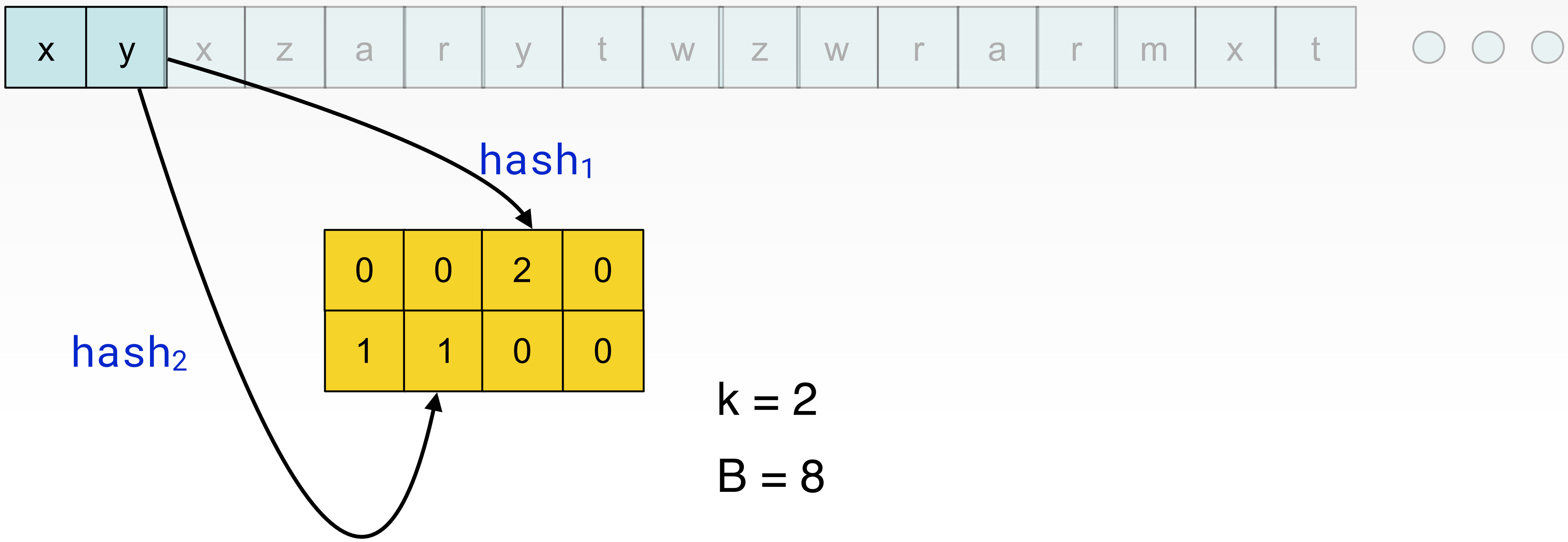

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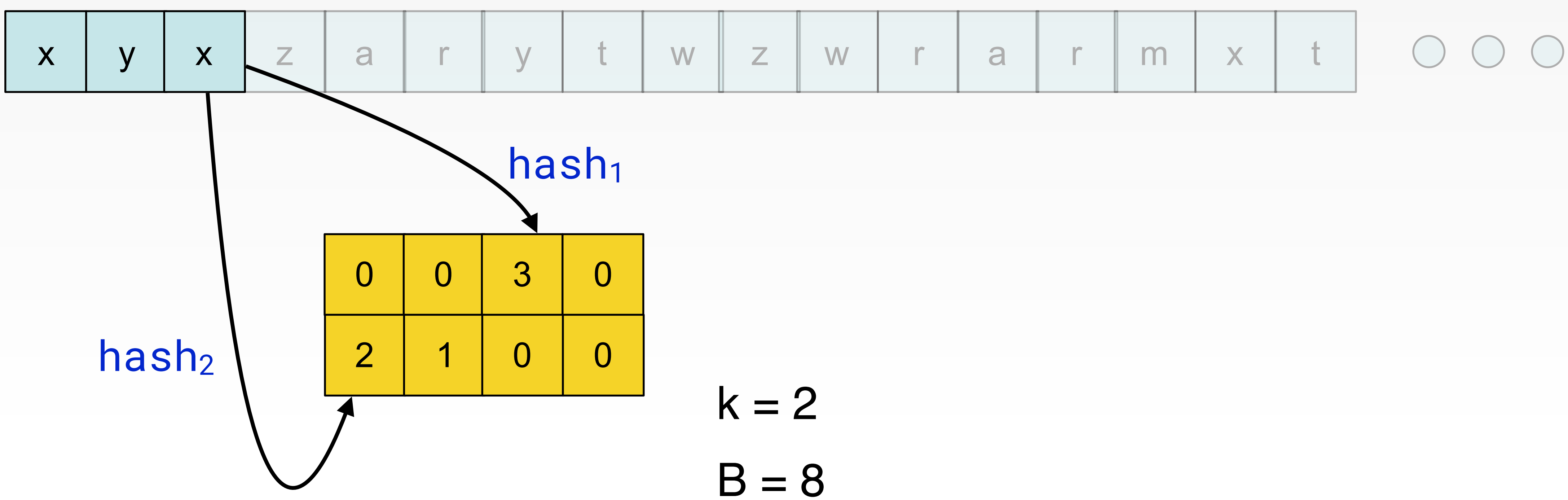

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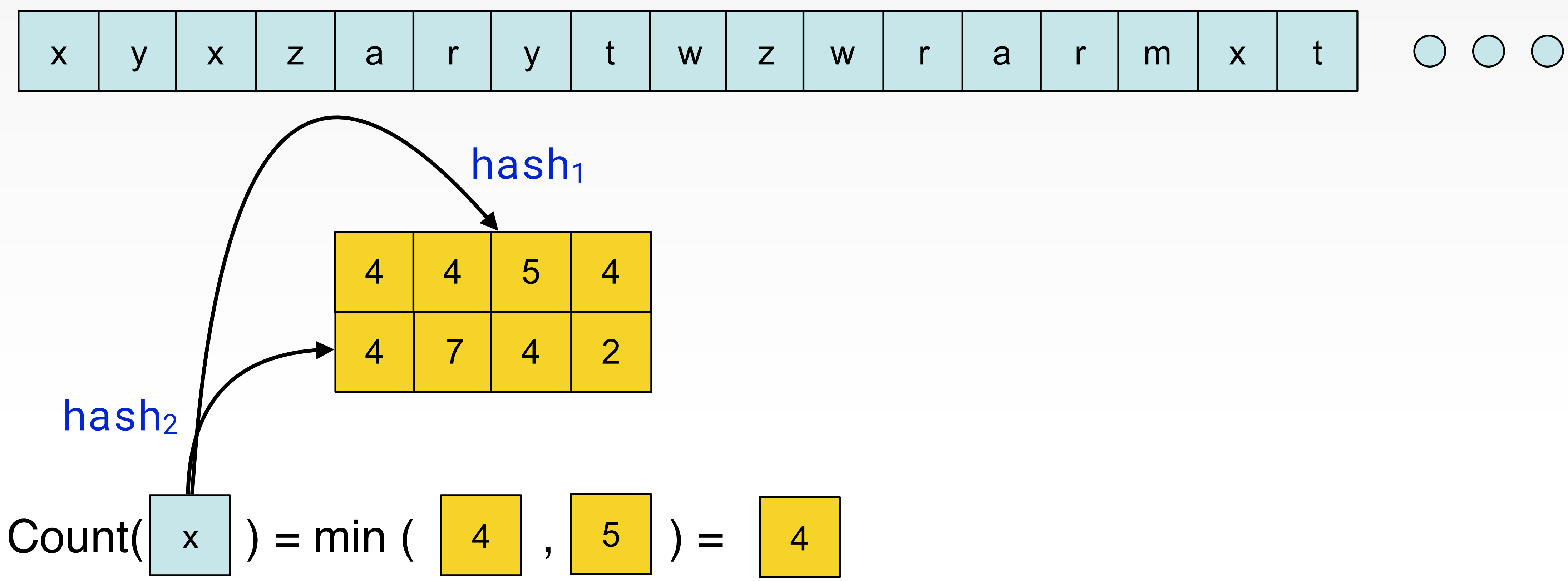


Learned CountMin [Hsu+'19]

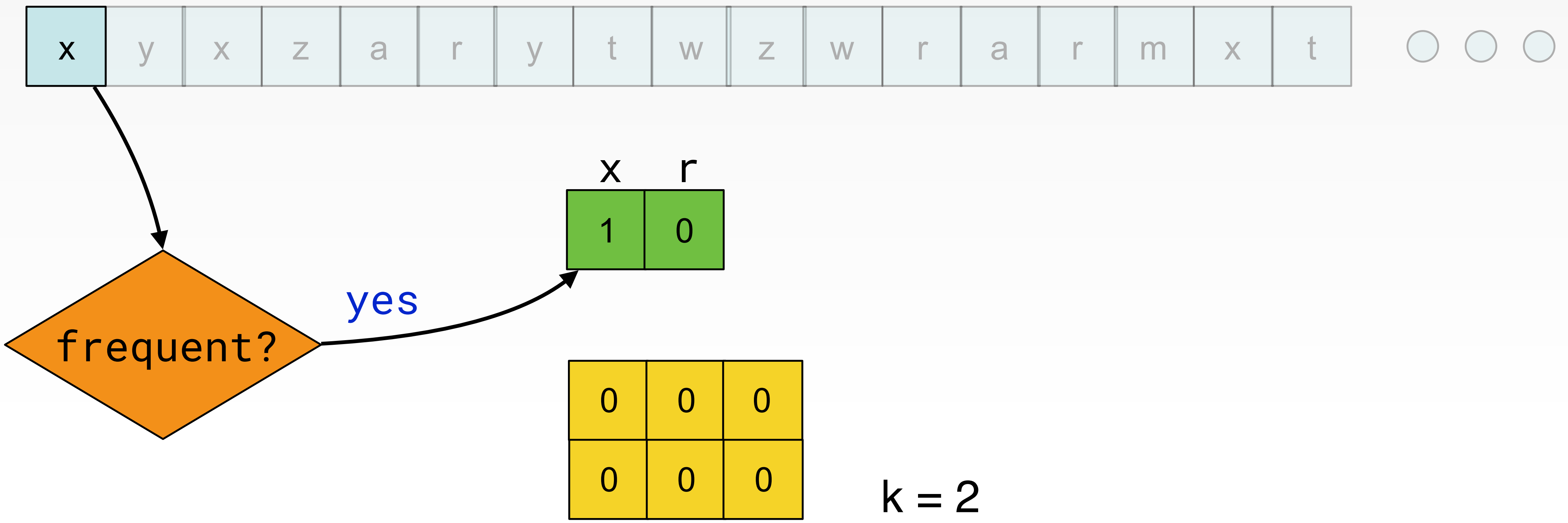
Idea:

- Train a classifier to predict whether an item is a heavy hitter
- For those predicted to be frequent elements, keep their counts exactly
- For the rest, use a CountMin sketch

Frequency Estimation: Count Min Sketch

Learned CountMin:

- Predict whether an element is frequent
- If so, keep its count exactly
- Otherwise, use CountMin



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B = 6

Frequency Estimation: Count Min Sketch

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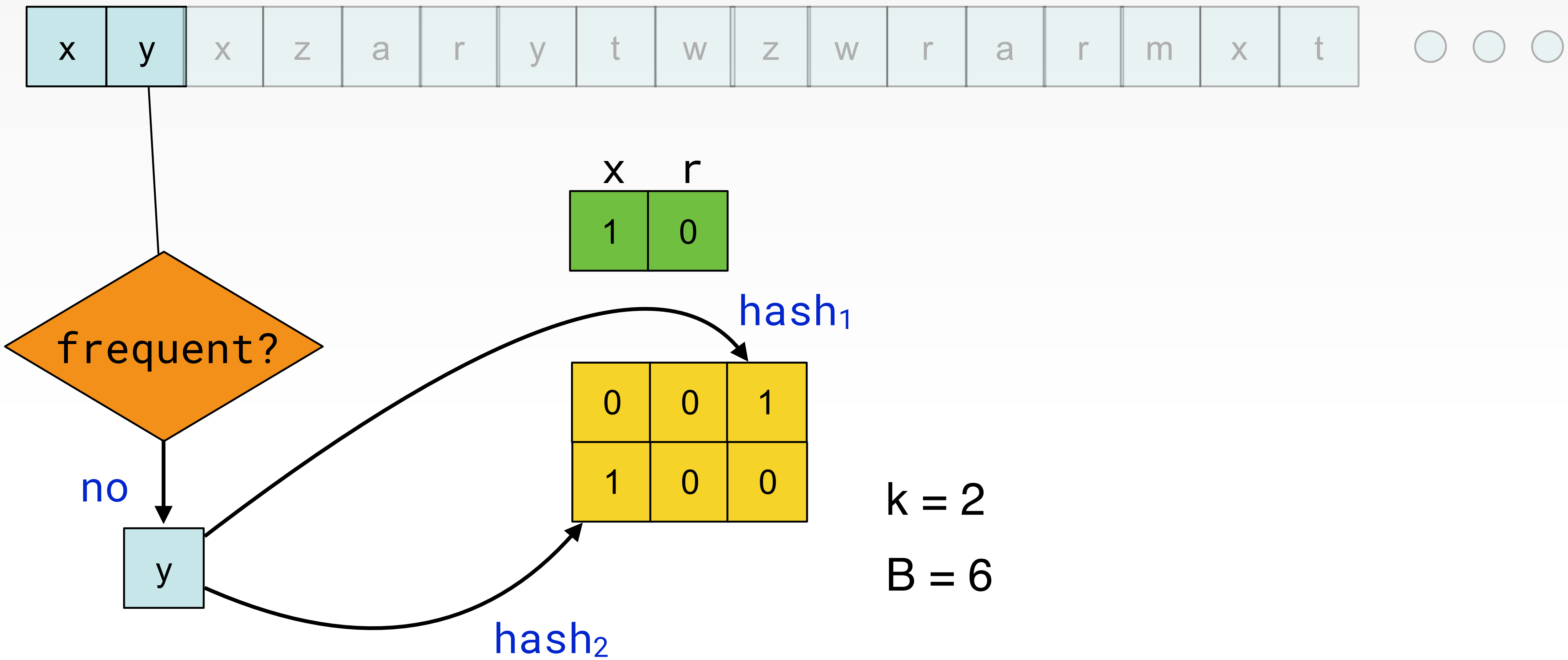

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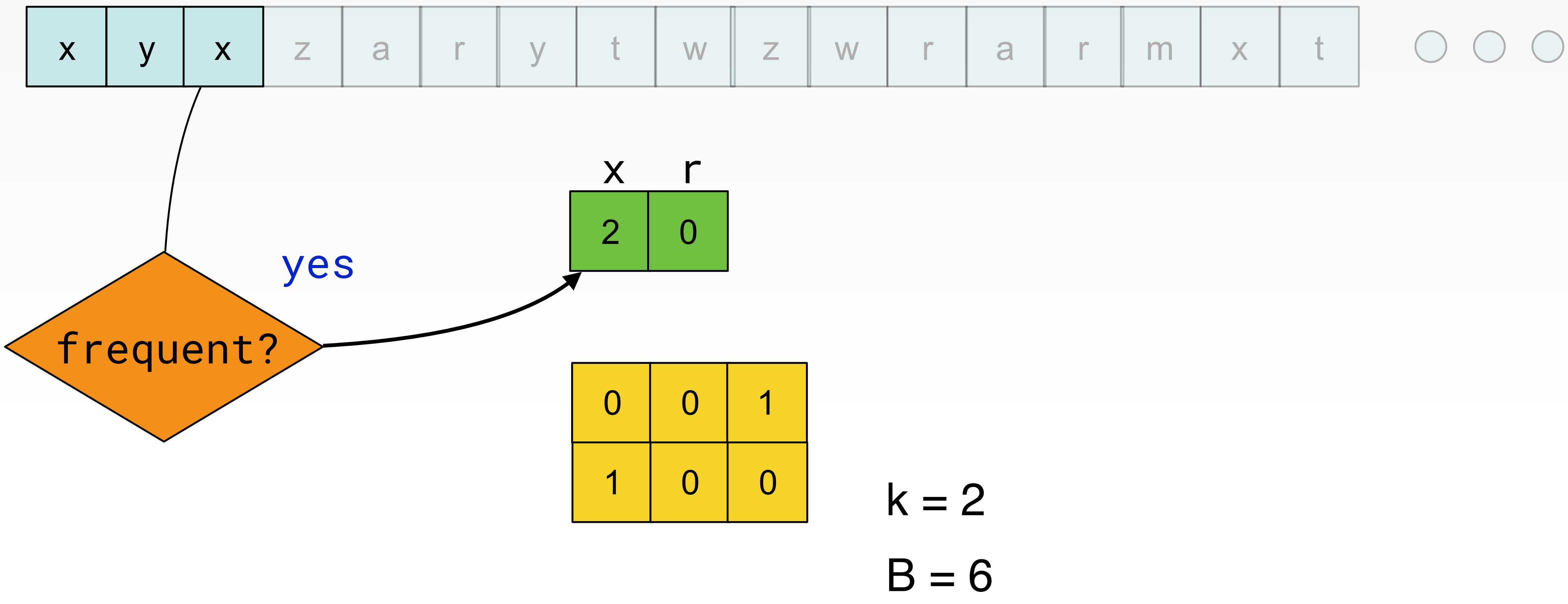


Analysis

Main question:

- Space vs. Accuracy trade-off.
- Fix space of B buckets. Measure accuracy

Error Function:

- "Expected" error
- Given true counts f_i and estimated counts $\hat{f}_i$.

$$\textsc{Err}(f, \hat{f}) = \frac{1}{N} \sum_i |f_i - \hat{f}_i| \cdot f_i$$

Analysis of Learned CountMin

For Zipf Distributions:

– Vanilla Count Min: $$O\left(\frac{k \ln n \ln(\frac{kn}{B})}{B}\right)$$

– Perfect Predictions: $$O\left(\frac{\ln^2 \frac{n}{B}}{B}\right)$$

– Noisy Predictions: $$O\left(\frac{\delta^2 \ln^2 B + \ln^2 \frac{n}{B}}{B}\right)$$

Analysis of Learned CountMin

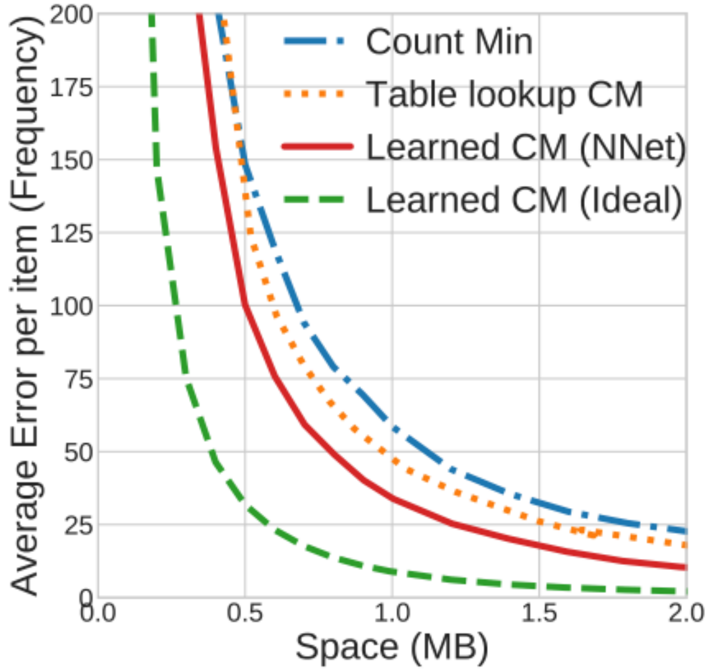
For Zipf Distributions: When $B = \Theta(n)$

– Vanilla Count Min: $O\left(\frac{k \ln n \ln(\frac{kn}{B})}{B}\right)$ $O\left(\frac{\ln n}{n}\right)$

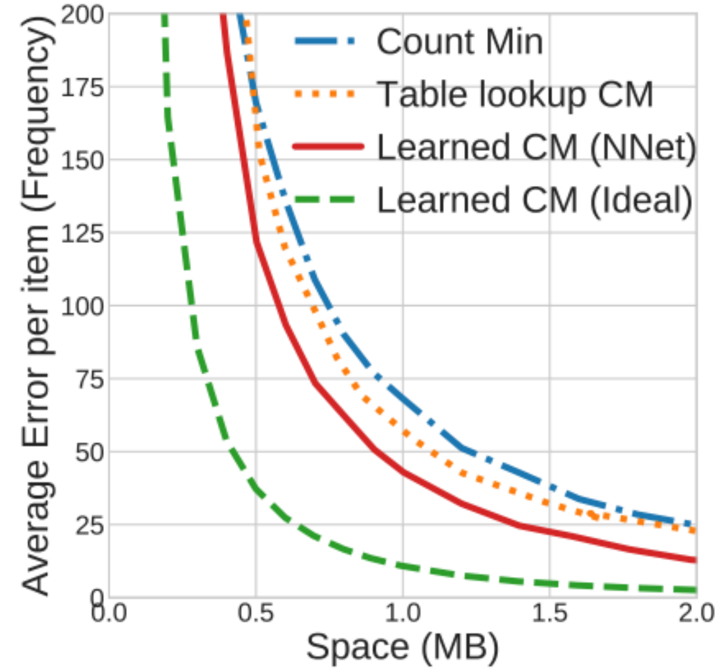
– Perfect Predictions: $O\left(\frac{\ln^2 \frac{n}{B}}{B}\right)$ $O\left(\frac{1}{n}\right)$

– Noisy Predictions: $O\left(\frac{\delta^2 \ln^2 B + \ln^2 \frac{n}{B}}{B}\right)$ $O\left(\frac{\delta^2 \ln^2 n}{n}\right)$

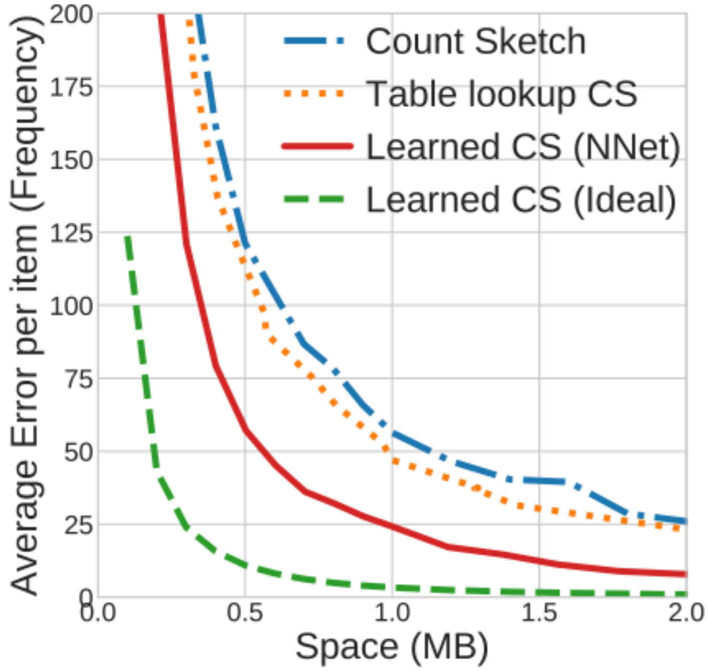
Empirical Slide



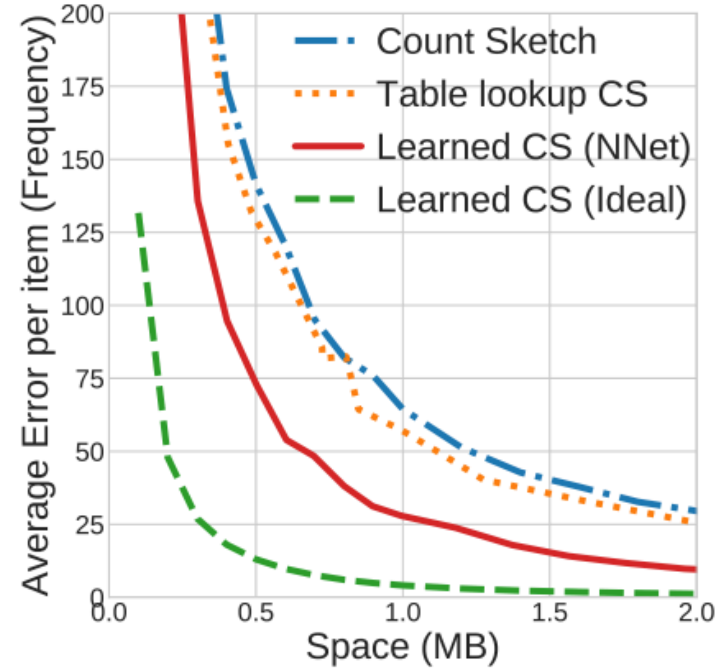
(a) Learned Count-Min - 20th test minute



(b) Learned Count-Min - 50th test minute



(c) Learned Count-Sketch - 20th test minute



(d) Learned Count-Sketch - 50th test minute

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Streaming Algorithms

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Already saw “learned indexes” [Kraska+’18, LykourisV’18]

- Predict offset rather than doing binary search

New idea:

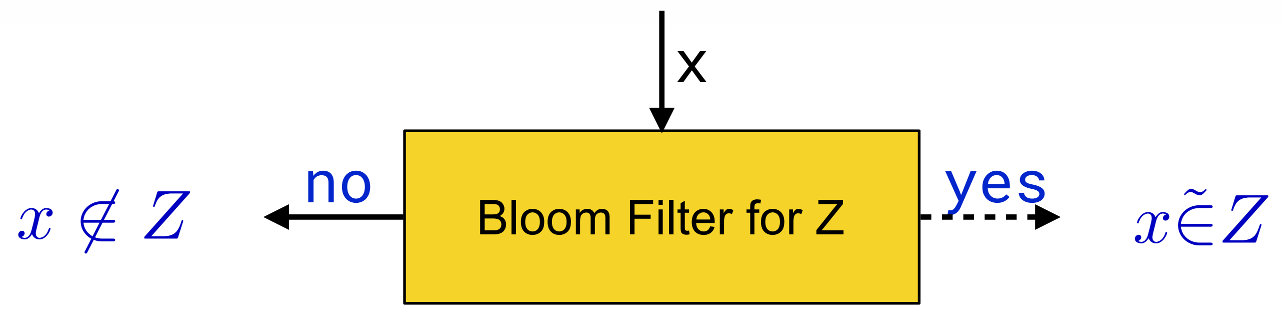
- Learned Bloom Filters.

Bloom Filters Review

Bloom Filter

- Data Structure to test set membership
- Never returns a false negative (elements in the set always returned as in the set)
- Sometimes returns a false positive (elements not in the set are claimed to be in the set)

Trade-off between space & false positive probability.



Learned Bloom Filters [Mitzenmacher ’18]

Train a predictor on whether an element is in the set.

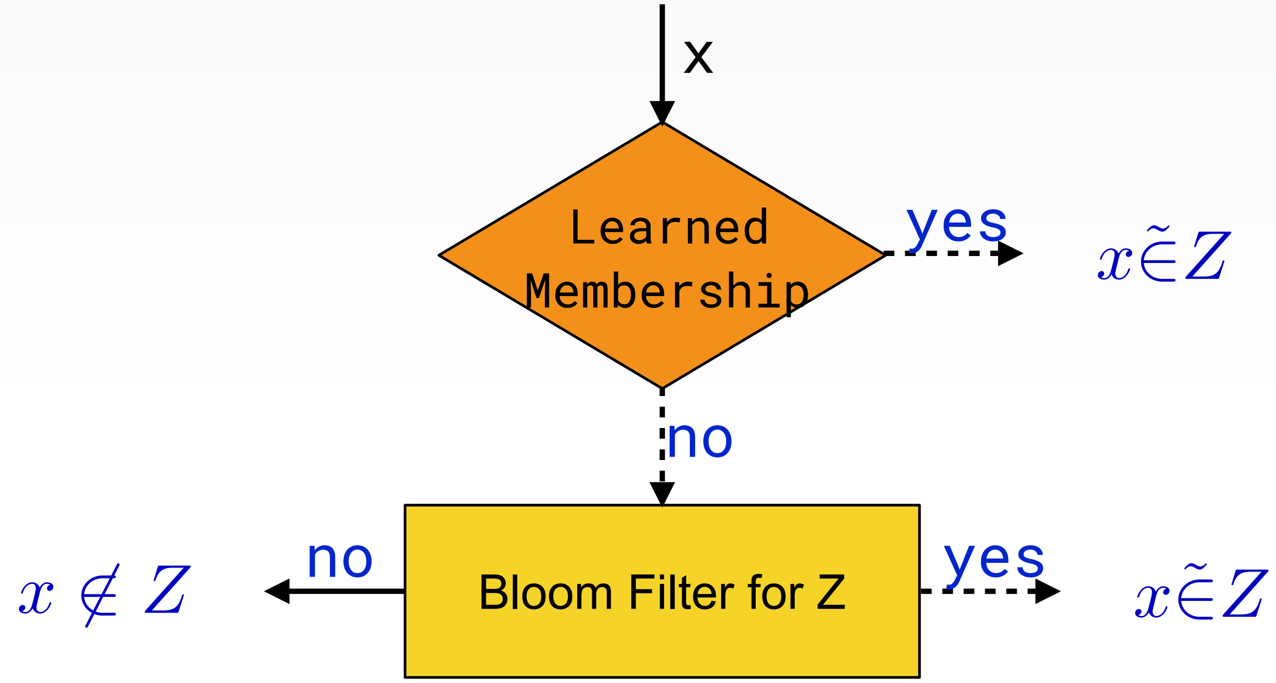
– Prediction has both false positive & false negative rates

Learned Bloom Filters

Train a predictor on whether an element is in the set.

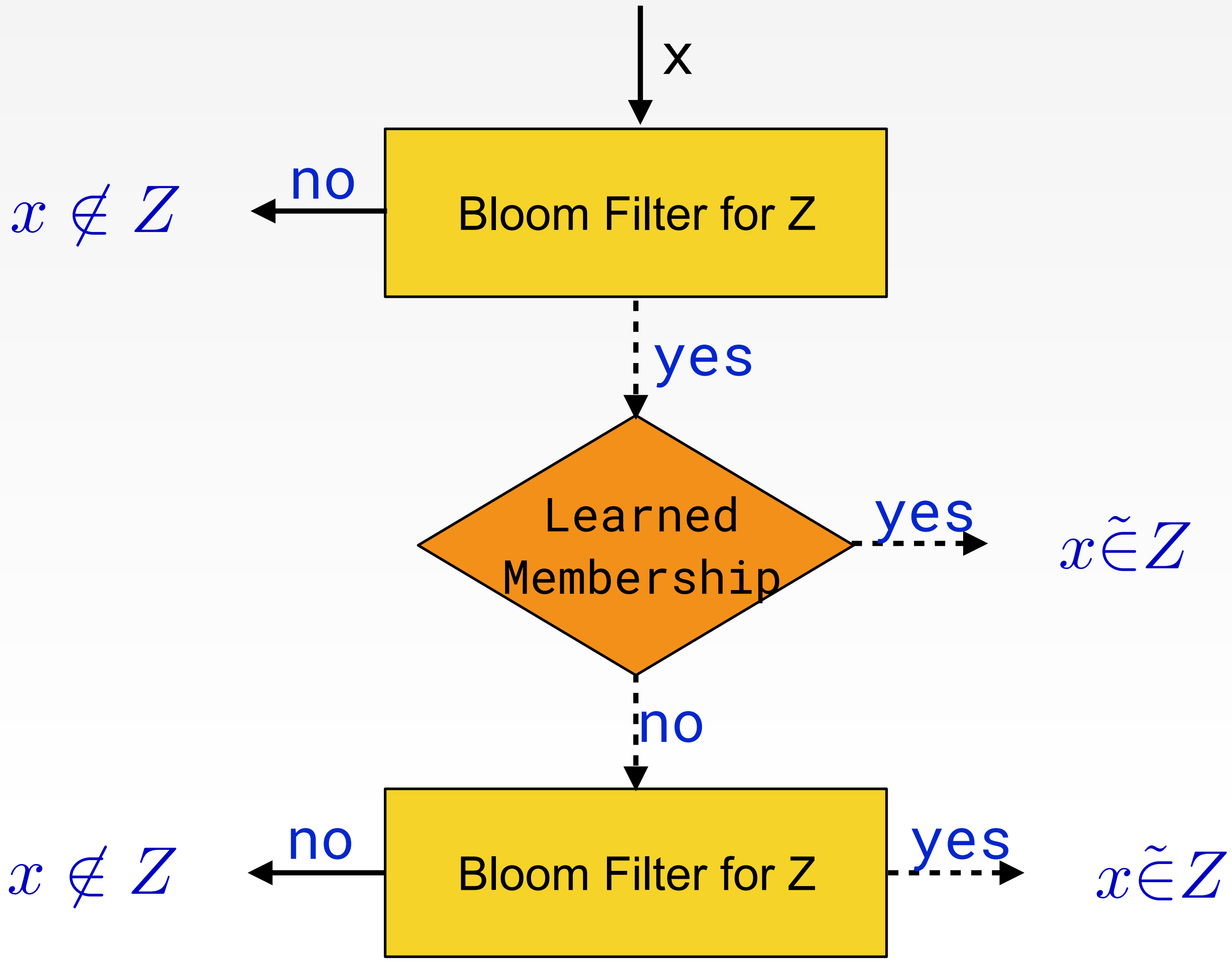
- Prediction has both false positive & false negative rates

- Combine the two:



Learned Bloom Filters

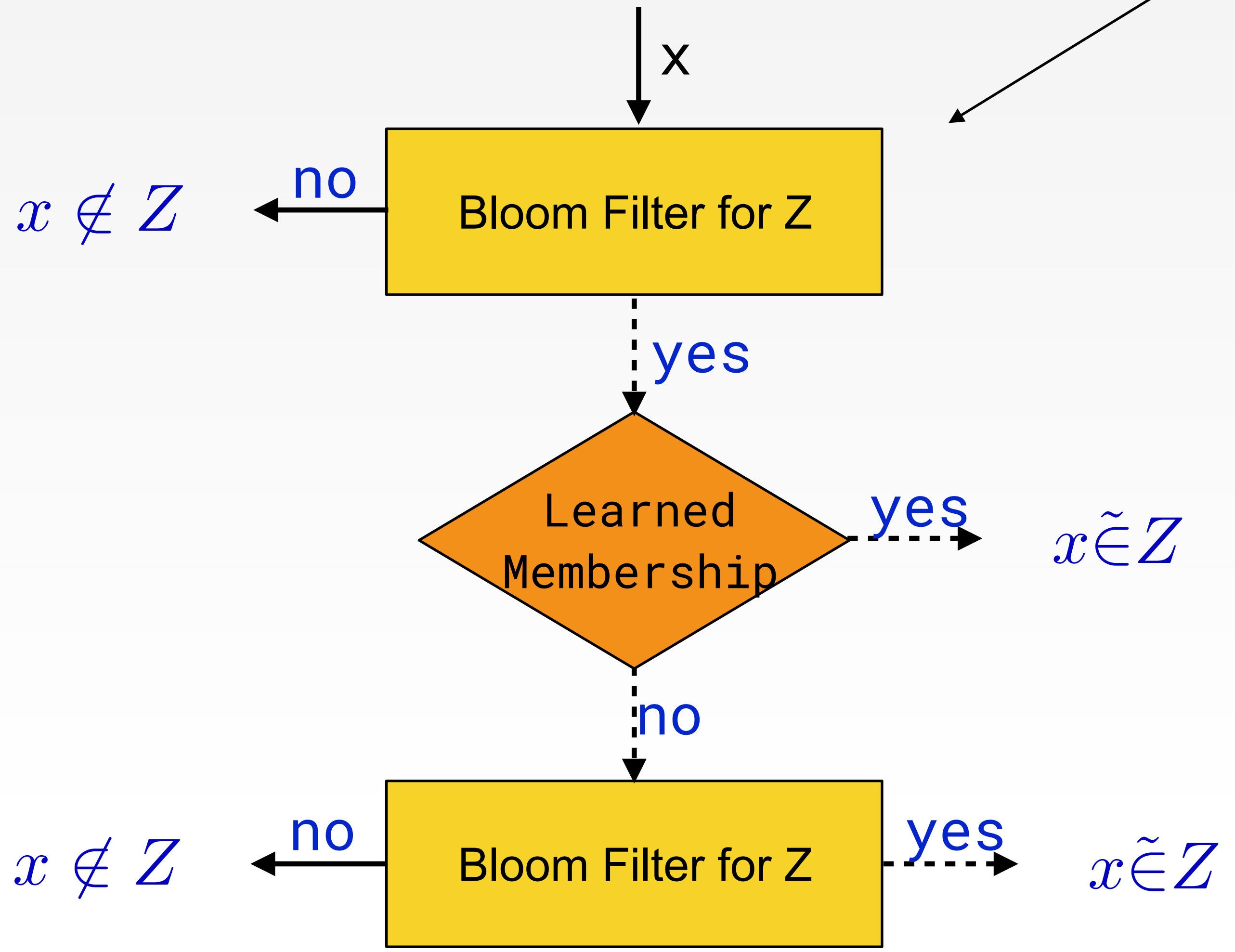
Do a step better:



Learned Bloom Filters

Do a step better:

Filter out easy negatives to make learning easier



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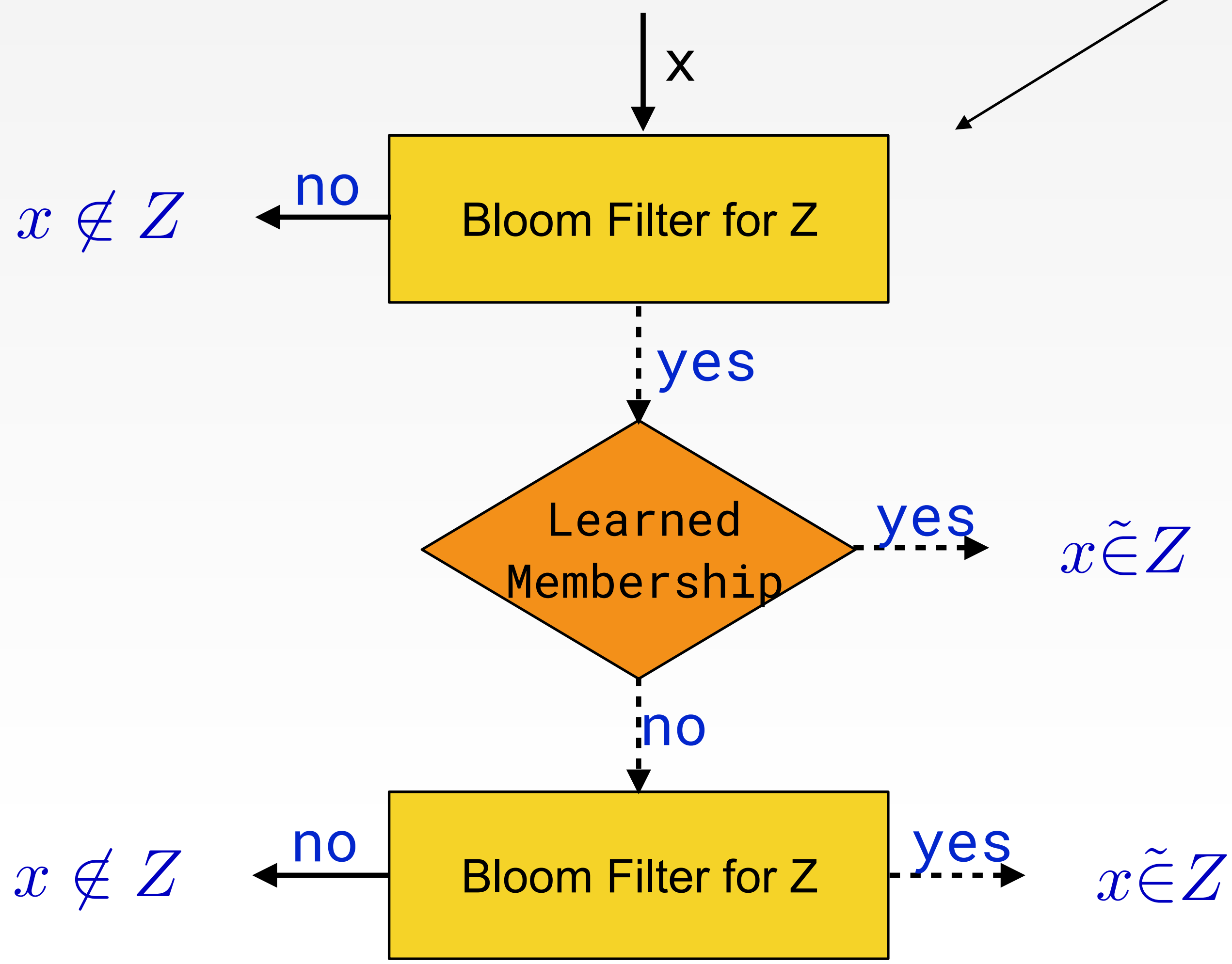

Back up filter to deal with prediction errors

Learned Bloom Filter Analysis

Trade-off between error rates and false positive / negative rates.

Main takeaways:

- The forward bloom filter makes the learning robust (if, for instance, examples are from a different distribution)
- The backup bloom filter does not grow with input size (it depends more on the quality of the learner)

Conclusion

Overall Question

How to incorporate (noisy, non-uniform) ML predictions to improve performance (time, space, approximation/competitive ratios) of classical algorithms.

Two Subproblems

Decide on what to predict.

- Predictions should be concise & compact
- Should use traditional loss functions

Incorporate predictions into algorithms.

- Full power of algorithm design and analysis
- Typically need a “trust but verify” approach

Final Thought

Another way to go beyond worst case analysis.

- Parametrize difficulty of the problem by the quality of the prediction
- Formally cast heuristics (e.g. LRU) as learning problems and evaluate their quality

Thank You