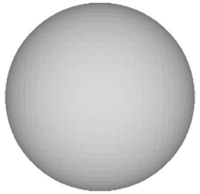
3D Scanning & Motion Capture

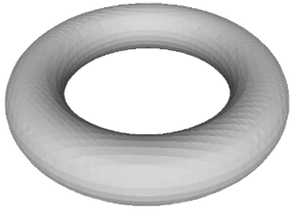
Exercise - 3

Justus Thies

Implicit Functions – Sphere / Torus

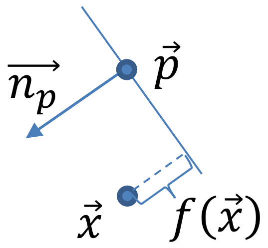


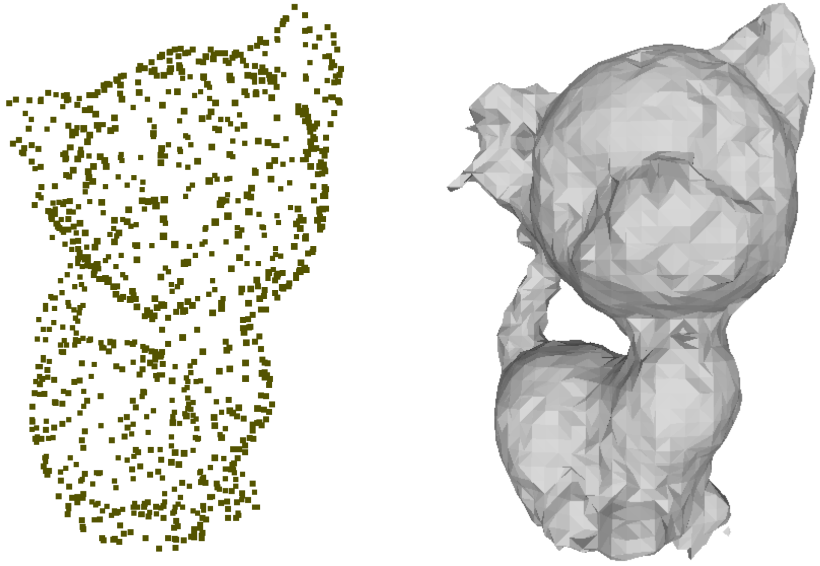
$$f(x, y, z) = x^2 + y^2 + z^2 - R^2$$



$$f(x, y, z) = (x^2 + y^2 + z^2 + R^2 - a^2)^2 - 4R^2(x^2 + y^2)$$

Implicit Functions – Hoppe



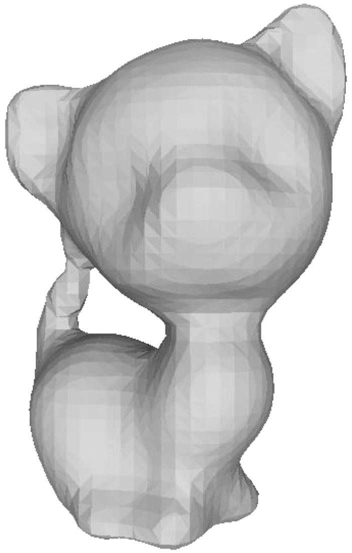


Implicit Functions – RBF

$$f(\vec{x}) = \sum_i \boldsymbol{\alpha_i} \cdot \|\vec{p_i} - \vec{x}\|^3 + \vec{\mathbf{b}} \cdot \vec{x} + \boldsymbol{d}$$

$$\begin{matrix} \text{on surface points} \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \\ \text{off surface points} \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \end{matrix} \underbrace{\begin{bmatrix} \varphi_{1,1} & \cdots & \varphi_{1,n} & p_{1,x} & p_{1,y} & p_{1,z} & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{n,1} & \cdots & \varphi_{n,n} & p_{n,x} & p_{n,y} & p_{n,z} & 1 \\ \varphi_{n+1,1} & \cdots & \varphi_{n+1,n} & p_{n+1,x} & p_{n+1,y} & p_{n+1,z} & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{2\cdot n,1} & \cdots & \varphi_{2\cdot n,n} & p_{2\cdot n,x} & p_{2\cdot n,y} & p_{2\cdot n,z} & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ b_1 \\ b_2 \\ b_3 \\ d \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} h_1 \\ \vdots \\ h_{2\cdot n} \end{bmatrix}}_{\vec{b}}$$

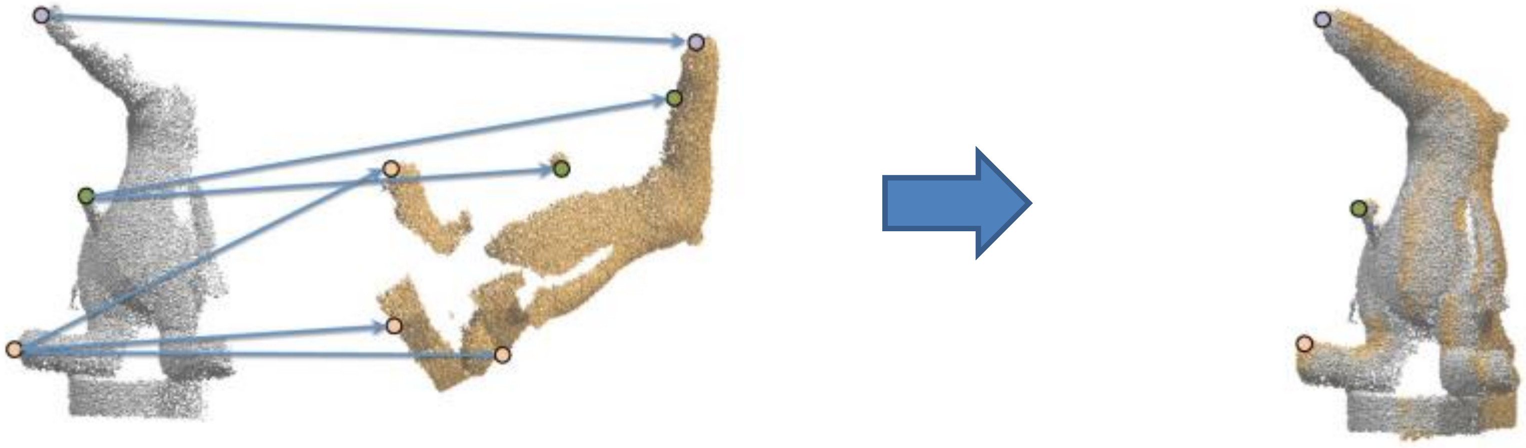
$$A \cdot \vec{x} = \vec{b}$$



Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

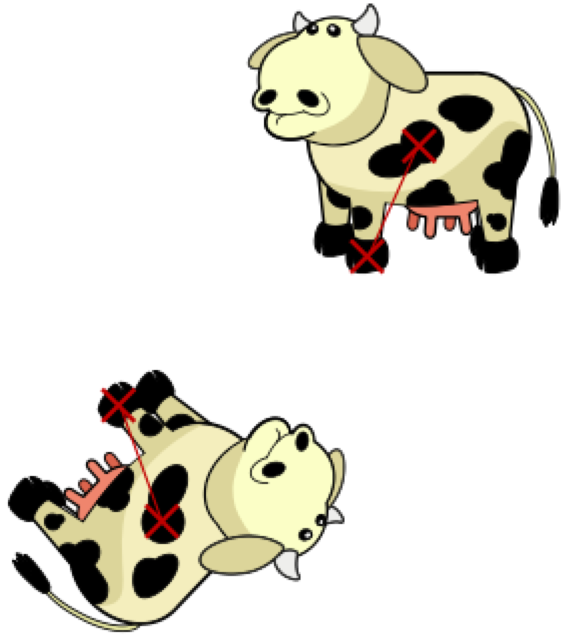


Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - Compute center of gravity of both objects
  - Scale one object to match the avg. distance from all vertices to the center of gravity



Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

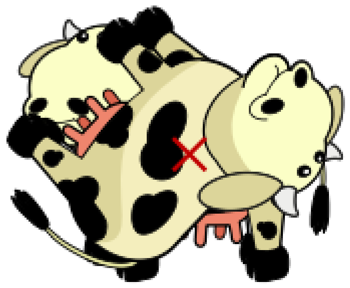
  - Translation is given by the vector between the center of gravity of both objects

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - Assume objects that are zero centered
    - Target object: $\{x_0, .. x_{n-1}\}$
    - Moving object: $\{\hat{x}_0\ , .. \hat{x}_{n-1}\}$



$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \ \rightarrow min$$

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

Procrustes

- Problem: Align two objects using known correspondences

  → scaling, translation, rotation

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

$\|A\|_F^2 = trace(A^T A)$

Cyclic invariance of trace:

$trace(ABC) = trace(CAB)$

$$\left\|X - \hat{X}R^T\right\|_F^2 = trace(X^T X - X^T \hat{X} R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T (\hat{X}R^T)) \rightarrow min$$

$$trace(-X^T \hat{X} R^T - (\hat{X}R^T)^T X + (\hat{X}R^T)^T (\hat{X}R^T)) \rightarrow min$$

$$-2 \cdot trace(X^T \hat{X} R^T) \rightarrow min$$

$$trace(X^T \hat{X} R^T) \rightarrow max$$

$SVD\colon X^T \hat{X} = USV^T$

$$trace(USV^T R^T) \rightarrow max$$

$$trace(SV^T R^T U) \rightarrow max$$

Singular values
→positive

Product of orthogonal matrices
→max if equal to Identity

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

  - Compute SVD of the Cross-Covariance Matrix

$$X^T\hat{X} = USV^T$$

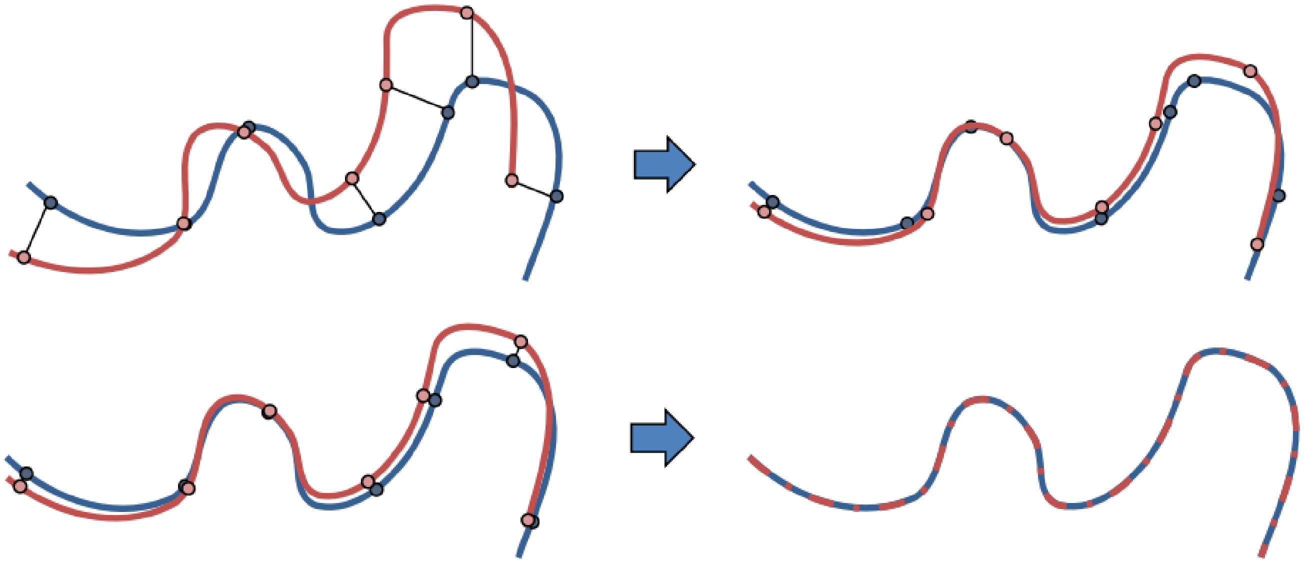
  - Compute the rotation

$$R = UV^T$$

ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
        - » Procrustes

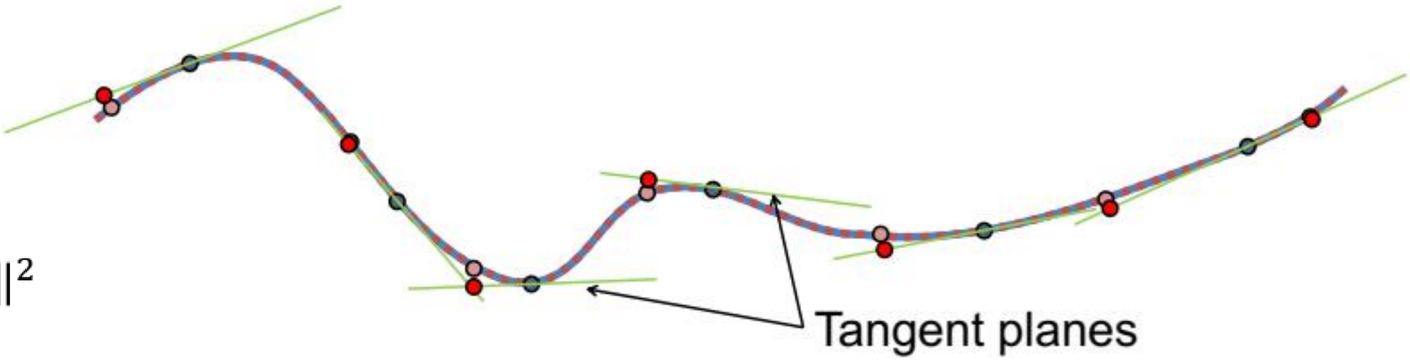
$$\min_{R,t} \sum_i \|p_i - (Rq_i + t)\|^2$$



ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
      - Point-to-plane distances
        - » Faster convergence
        - » Non-linear!

$$\min_{R,t} \sum_i \|(p_i - (Rq_i + t)) \cdot n_i\|^2$$



ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
      - Point-to-plane distances
    - Use weighting of correspondences and pruning
      - Good correspondences are close, have similar normal, …
      - Prune correspondences to border

$$\min_{R,t} \sum_i w_i \|p_i - (Rq_i + t)\|^2$$