Solar Grazing Modelling

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1 Introduction

We want to create a model that will maximize the profitability of a solar grazing operation, measured by the Net Present Value of yearly cash flows over a time horizon of T years. The farmer starts with an initial sheep flock size. To reach the maximum flock size, the farmer will deploy his flock, can buy sheep upfront by borrowing, and will grow the remainder.

Parameters

λ_{acres}: Number of acres
$\lambda_{revperacre}$: Annual Revenue per Acre
$\lambda_{sheepperacre}$: Sheep per Acre
$\lambda_{mowNoSheep}$: Mowing Cost per Acre (0 sheep)
λ_{mowMax}: Mowing Cost per Acre (max sheep)

$\lambda_{sheepOnHand}$: Number of Sheep On Hand
$\lambda_{maxflock}$: Maximum Flock Size ($\lambda_{acres} \times \lambda_{sheepperacre}$)
λ_{repro}: Reproduction Rate
λ_{cull}: Annual Cull Rate
λ_{death}: Annual Death Rate
$\lambda_{eweCost}$: Cost per Ewe
$\lambda_{revRamLamb}$: Revenue per Ram Lamb
$\lambda_{revEweLamb}$: Revenue per Ewe Lamb
$\lambda_{revCull}$: Revenue per Ewe Culled

$\lambda_{intRate}$: Interest Rate
$\lambda_{loanYears}$: Loan Duration (Years)
$\lambda_{discRate}$: Discount Rate

$C_{catchment}$: Catchment Systems Cost
C_{misc}: Miscellaneous Costs
$C_{upfront}$: Upfront Cost ($C_{catchment} \times C_{misc}$)

$C_{mowingFixed}$: Mowing Fixed Cost per Year
$C_{fixedYearAcre}$: Fixed Cost per Year per Acre ($C_{mowingFixed}$)

$C_{mowingHead}$: Mowing Cost per Head per Year $(-(\lambda_{mowNoSheep} - \lambda_{mowMax}) \times \lambda_{acres}/\lambda_{maxflock})$
$C_{transportation}$: Transportation Cost per Head per Year
$C_{vetCare}$: Vet Care Cost per Head per Year
$C_{minerals}$: Minerals Cost per Head per Year
$C_{overwintering}$: Overwintering Cost per Head per Year
$C_{staffingHead}$: Staffing Cost per Head per Year
$C_{costHeadYear}$: Cost per Head per Year $(C_{mowingHead} + C_{transportation} + C_{vetCare} + C_{minerals} + C_{overwintering} + C_{staffingHead})$

$C_{insurance}$: Insurance Cost per Year
$C_{waterHaul}$: Water Hauling Cost per Year
$C_{staffing}$: Fixed Staffing Cost per Year
$C_{guardDogs}$: Guard Dogs Cost per Year
$C_{fixedYear}$: Fixed Cost per Year $(C_{insurance} + C_{waterHaul} + C_{staffing} + C_{guardDogs})$

$C_{rentHeadCost}$: Rental Cost per Head per Year
$C_{rentMineralCost}$: Mineral Cost per Rented Head
$C_{transportationCost}$: Cost per Mile per Truck with Capacity for 300 Sheep

2 Base Model - Deploy Beginning Flock, Purchase Some with Loan, Grow Remainder

The farmer can decide to deploy his/her starting flock, purchase some sheep with a loan, and then grow the remainder until reaching the maximum flock size.

Variables

$x =$ flock purchase size

$R_t =$ total revenue in year t

$C_t =$ total cost in year t

$P =$ yearly loan payment

$m_t =$ number of ewes to sell at end of year t

$n_t =$ number of ewes at end of year t

Mixed Integer Linear Program

$$\max \quad Z_{IP1} = -(C_{upfront} + \lambda_{sheepOnHand}\lambda_{eweCost}) + \sum_{t=1}^{T} \frac{(R_t - C_t)}{(1+\lambda_{discRate})^t} \tag{1}$$

$$\text{s.t.} \quad \lambda_{acres}\lambda_{revperacre} + n_{t-1}(\frac{\lambda_{repro}}{2}\lambda_{revRamLamb} + \lambda_{revCull}\lambda_{cull}) + \lambda_{revEweLamb}m_t = R_t, \quad \forall t = 1, ..., T \tag{2}$$

$$P = \frac{x(\lambda_{eweCost})(\lambda_{intRate})}{1 - (1+\lambda_{intRate})^{-\lambda_{loanYears}}} \tag{3}$$

$$P + \lambda_{acres}C_{fixedYearAcre} + \frac{(n_{t-1} + n_t)}{2}C_{costHeadYear} + C_{fixedYear} = C_t, \quad \forall t = 1, ..., \lambda_{loanYears} \tag{4}$$

$$\lambda_{acres}C_{fixedYearAcre} + \frac{(n_{t-1} + n_t)}{2}C_{costHeadYear} + C_{fixedYear} = C_t, \quad \forall t = \lambda_{loanYears} + 1, ..., T \tag{5}$$

$$\lambda_{sheepOnHand} + x = n_0 \tag{6}$$

$$n_{t-1}\underbrace{(1 + \frac{\lambda_{repro}}{2} - \lambda_{death} - \lambda_{cull})}_{\text{Growth Rate}} - m_t = n_t, \quad \forall t = 1, ..., T \tag{7}$$

$$R_t \geq 0, \forall t = 1, ..., T \tag{8}$$

$$C_t \geq 0, \forall t = 1, ..., T \tag{9}$$

$$\lambda_{maxflock} \geq n_t \geq 0, \ \text{integer}, \forall t = 0, ..., T \tag{10}$$

$$m_t \geq 0, \ \text{integer}, \forall t = 1, ..., T \tag{11}$$

$$\lambda_{maxFlock} - \lambda_{sheepOnHand} \geq x \geq 0, \ \text{integer} \tag{12}$$

$$\tag{13}$$

3 Model 2 - Base Model with Ability to Cull from Younger Age Groups

The farmer is under a similar situation as before with respect to making a decision on flock purchase size. However, in addition to culling sheep from the oldest age group, the farmer now has full discretion in culling sheep from the younger age groups, whereas previously he/she was restricted to culling the number of sheep in excess of the maximum value. We assume that the age distributions of the initial flock purchase and the sheep on hand are uniform. We assume that sheep sold throughout the year are priced the same across both genders and differently across ages and years, introducing a new parameter $p_t(i)$ to indicate the selling price of a sheep aged i years during year t. Similarly, we let $C_{costHeadYear}$ be indexed by time with the variable $c(t)$.

Variables

$x_t(i)$ = number of sheep aged i years at end of year t, $\quad t = 0, 1, ..., T, \quad i = 1, 2, 3, 4, 5$

$y_t(i)$ = number of sheep sold aged i years sold at end of year t, just after $x_t(i)$ is measured, $\quad t = 0, 1, ..., T, \quad i = 1, 2, 3, 4, 5$

$n(i)$ = number of ewes aged i years purchased with loan

R_t = total revenue in year t

C_t = total cost in year t

P = yearly loan payment

New Parameters

$p_t(i)$ = cull price of sheep aged i years during year t (previously $\lambda_{rev\_}$)

c_t = total cost per head during year t (mainly wintering and mowing; previously $C_{totalHeadYear}$)

Mixed Integer Linear Program

$$
\begin{aligned}
\max \quad & Z_{IP2} = -(C_{upfront} + \lambda_{sheepOnHand}\lambda_{eweCost}) + \sum_{t=1}^{T} \frac{(R_t - C_t)}{(1+\lambda_{discRate})^t} && (14)\\
\text{s.t.} \quad & \lambda_{acres}\lambda_{revperacre} + \sum_{i=1}^{5} p_t(i) y_t(i) = R_t, \quad \forall t = 1, ..., T && (15)\\
& \frac{(\lambda_{intRate})\lambda_{eweCost} \sum_{i=1}^{5} n(i)}{1 - (1+\lambda_{intRate})^{-\lambda_{loanYears}}} = P && (16)\\
& P + C_{fixedYear} + \lambda_{acres} C_{fixedYearAcre} + c_t \sum_{i=1}^{5} [x_t(i) - y_t(i)] = C_t, \quad \forall t = 1, ..., \lambda_{loanYears} && (17)\\
& C_{fixedYear} + \lambda_{acres} C_{fixedYearAcre} + c_t \sum_{i=1}^{5} [x_t(i) - y_t(i)] = C_t, \quad \forall t = \lambda_{loanYears} + 1, ..., T && (18)\\
& n(i) + \frac{\lambda_{sheepOnHand}}{5} = x_0(i), \quad \forall i = 1, ..., 5 && (19)\\
& \lambda_{maxFlock} \geq \sum_{i=1}^{5} [x_t(i) - y_t(i)], \quad \forall t = 0, 1, 2, ..., T && (20)\\
& (1 - \lambda_{death}) \lambda_{repro} \sum_{i=2}^{5} x_t(i) = x_{t+1}(1), \quad \forall t = 0, 1, ..., T-1 && (21)\\
& (1 - \lambda_{death}) [x_t(i) - y_t(i)] = x_{t+1}(i+1), \quad \forall i = 1, ..., 4, \;\; \forall t = 0, 1, ..., T-1 && (22)\\
& \underbrace{(1 - \lambda_{death}) \frac{\lambda_{repro}}{2} \sum_{i=2}^{5} x_t(i)}_{\text{male lambs at time } t+1} \leq y_{t+1}(1), \quad \forall t = 0, 1, ..., T-1 && (23)\\
& x_t(5) = y_t(5), \quad \forall t = 1, 2, ..., T && (24)\\
& y_0(i) = 0, \quad \forall i = 1, ..., 5 && (25)\\
& R_t \geq 0, \quad \forall t = 1, ..., T && (26)\\
& C_t \geq 0, \quad \forall t = 1, ..., T && (27)\\
& x_t(i) \geq 0, \quad \forall i = 1, ..., 5, \;\; \forall t = 0, ..., T && (28)\\
& \lambda_{maxFlock} \geq n(i) \geq 0, \quad \forall i = 1, ..., 5 && (29)\\
& x_t(i) \geq y_t(i) \geq 0, \quad \forall i = 1, ..., 5, \;\; \forall t = 0, ..., T && (30)\\
& P \geq 0 && (31)\\
& && (32)
\end{aligned}
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