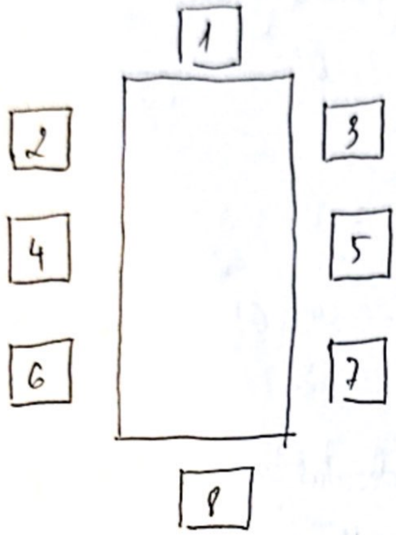
Written Assignment 2

1)



The seating plan :

a) Because Alice is the guest of honour and should sit at the head of the table

$\Rightarrow$ Alice sit at 1 or 8 $\Rightarrow$ Alice have 2 seats

And 7 another guest can sit anywhere

$\Rightarrow$ There are $7! = 5040$ different seating arrangements possible

$\Rightarrow$ There are $5040 \times 2 = 10080$ different arrangements now

b) I will ~~cal cacl~~ caculate how many seating arrangements that Bob and Chalie sit next to or accross the shorter dimension of the table.

+) When Bob sit in 1: Charlie can sit in 2 or 3

+) When Bob sit in 2 : Charlie sit in 3, 1, 4

+) When Bob sit in 3 : 2, 1, 5 (Charlie)

+) When Bob sit in 4 : 2, 5, 6 (Charlie)

+) When Bob sit in 5 : 4, 3, 7 (Charlie)

+) When Bob sit in 6 : 7, 4, 8 (Charlie)

+) When Bob sit in 7 : 6, 5, 8 (Charlie)

+) When Bob sit in 8: 6, 7 (Charlie)

=> We have 22 seats of Charlie

And 6 another guests can sit anywhere

So => There are $22 . 6!$ seating arrangements when Bob and Charlie sit next to or ...

=> There are $8! - 22 . 6! = 34 . 6! = 24480$ seating arrangements sastify this restriction

c) Because Bob and Charlie decided not to turn up.

=> There are 6 guests left.

We need to choose 6 seats from 8 seats

And there are C_8^6 ways to do it

And 6 people can sit anywhere in that 6 seats

=> There are $C_8^6 . 6! = 20160$ ways.

2)

a) +) There are 4 suits in total, so multiply 4 of the result

+) Each suit has 13 cards, so the number of ways to choose 7 cards from one suit is C_{13}^7

+) The total of ways to choose 7 cards from 52 cards is C_{52}^7

=> The probability is $\dfrac{4 \times C_{13}^7}{C_{\Omega}^7} = \dfrac{4 \times \frac{13!}{7! \times 6!}}{\frac{52!}{7! \times 45!}}$

$= \dfrac{4 \times 13! \times 7! \times 45!}{7! \times 6! \times 52!} = \dfrac{4 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13}{46 \times 47 \times 48 \times 49 \times 50 \times 51 \times 52} = 5,1 \times 10^{-5}$

b) There are 13 possible ranks

$\Rightarrow$ Only 13 ways to choose 4 cards from a rank.

+) Choosing the rank for the 3 of a kind

After choosing the rank, we are left with 12 possible rank.

$\Rightarrow$ There are $C_{12}^1 \cdot C_4^3 = 12.4 = 48$ (ways)

$\Rightarrow$ There ~~624~~. $13 \times 48 = 624$ (ways) to choose a "super - full house"

$$P = \frac{624}{C_{52}^7} = \frac{624}{\frac{52!}{7!.45!}} = 4,17.10^{-6}$$

c) A 3-pair consists of 2 cards of one rank, 2 cards of seccond rank, 2 cards of third ranks, and one remaining card of a different rank.

+) We need to ~~ce~~ choose 3 ranks from 13 ranks

The number of ways is: C_{13}^3

+) For each rank, there are 4 cards and we need choose 2 of them

The number is $C_4^2 = 6$

but there 3 ~~three~~ pairs

$\Rightarrow$ So the total ways to do this is: $6^3 = 216$.

+) Choosing the remaining card: There are 10 ranks avaiable, and we need to choose 1 of them $\Rightarrow C_{10}^1$ ways.

we have 4 cards in this rank as well

$\Rightarrow C_{10}^1 \cdot 4 = 40$ ways

So the total ways to choose 3 pair : $40 . 216 . C_{13}^3$

$$\Rightarrow P = \frac{40 . 216 . C_{13}^3}{C_{52}^7} = 1,8.10^{-2}$$

3)

a) We have these information

+) Probability of the star player being fit for the match: 0,3 => $\boxed{P(\text{not fit}) = 1-0,3 = 0,7}$

+) If the star player play:

-) Win : 0,8

-) Draw : 0,1 (1 - 0,8 - 0,1 = 0,1)

-) Loss : 0,1

+) If the star player does not play ;

-) Win : 0,5

-) Draw : 0,3 (1 - 0,5 - 0,2 = 0,3

-) Loss : 0,2

=> Point if star players is fit :

$$0,3 \times [3 \times 0,8 + 1 \times 0,1 + 0 \times 0,1] = 0,75$$

Point if star players isn't fit :

$$0,7 \times [3 \times 0,5 + 1 \times 0,3 + 0 \times 0,2] = 1,26$$

=> Total point = 1,26 + 0,75 = 2,01 (points)

b) Let's analyze some information:

Strategy 1: +) Star players play in 1st match :

point = $3 \times 0,8 + 1 \times 0,1 = 2,5$

+) 2nd match: we have some

-) If Star players can play :

P (win) = 0,6

P (loss) = 0,1 => Points = $3 \times 0,6 + 1 \times 0,3 = 2,1$

P (draw) = 0,3

-) If star players can't play :

P(win) = 0,3

P (loss) = 0,4 => Points = $3 \times 0,3 + 1 \times 0,3 = 1,2$

P (draw) = 0,3

=> Total point for strategy 1: = 0,5 . (2,1 + 1,2) + 2,5 = 4, Strategy 2: The star player did not play the 1st match

+> Similarity to calculate:

Point for 1st match is : $3 \times 0{,}5 + 1 \times 0{,}3$

$= 1{,}8$ (points)

+) 2nd match:

+> Star player can play : $0{,}9 \times 2{,}1 = 1{,}89$

+> Star can not play : $0{,}1 \times 1{,}2 = 0{,}12$

=> Total point for 2nd match

$1{,}89 + 0{,}12 = 2{,}01$ (points)

=> Total point of 2 matches :

$2{,}01 + 1{,}8 = 3{,}81$ (points)

=> The best strategy is strategy 1 with 4,15 points