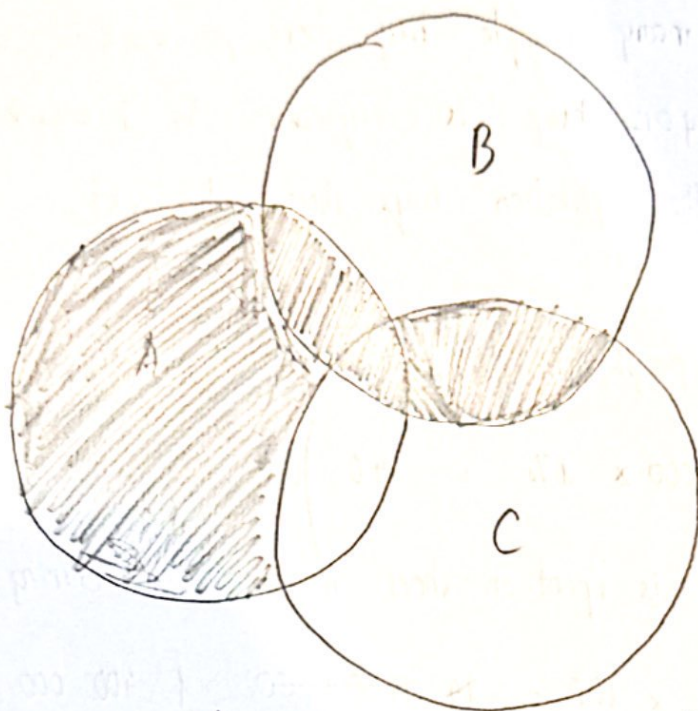


## Assignment 1

1)

i) a Venn diagram show  $(A \cap B) \cup (A \setminus C) \cup (B \cap C)$



- The parts that are covered by gray color is the answer.

ii) +)  $A = \{-1; \frac{1}{2}; 2; 5\}$

+ )  $B = [0, 2) \in \mathbb{R}$

$$\Rightarrow B = \{x \in \mathbb{R} \mid 0 \leq x < 2\}$$

+ )  $C = \{n \in \mathbb{Z} \mid \frac{n+1}{2} \in \mathbb{Z}\}$

$\Rightarrow (n+1)$  is even mean  $n$  is odd

we have :  $(A \cap B) = \{\frac{1}{2}\}$

$$(A \setminus C) = \{-1; \frac{1}{2}; 2\}$$

$$(B \cap C) = \{1\}$$

$$\Rightarrow (A \cap B) \cup (A \setminus C) \cup (B \cap C) = \{-1; \frac{1}{2}; 1; 2\}$$

2)

→ There is nearly 1.4 million people in Adelaide in 2024

→ The cost of a pair of shoes can vary, but I assume that an average price is \$100 per pair

→ Estimate how many people buy shoes per week:

- Not everyone buys shoes every week. So I assume that around 1% of the population buys shoes each week.

⇒ The number of people who buy shoes weekly is:

$$1\,400\,000 \times 1\% = 14\,000 \text{ (people)}$$

⇒ Money in total is spent on shoes in Adelaide during a week is:

$$14\,000 \times 100 = 1\,400\,000 \text{ (dollars)}$$

3) function  $f$  with domain  $[0, 6]$

$$\Rightarrow 0 \leq x \leq 6$$

$$g(x) = \begin{cases} 2 & \text{if } 0 \leq x < 1 \\ \left[\frac{x}{4} + 1\right] & \text{if } x \geq 1 \end{cases}$$

i)  $g(f(4))$

We have  $f(4) = 3$  so  $g(f(4)) = g(3) = \left[\frac{3}{4} + 1\right] = 1$

ii)  $f(x) = g(x)$

→ It can be seen that,  $0 \leq x < 1$ ,  $g(x) = 2$

but in if  $0 \leq x < 1$ ,  $f(x) > 3$

→ with  $1 \leq x < 4$ ,  $g(x) = 1$

and  $f(x) = 1$  when  $x = 2$

so when  $x = 2$ ,  $f(x) = g(x)$



+) with  $4 \leq x \leq 6$

$$g(x) = 2$$

and ~~when  $4 \leq x \leq 6$ ,  $f(x) = 2$~~

when  $x = 5$ ,  $f(x) = 2$

$$\rightarrow x = 2, f(x) = g(x)$$

So  $f(x) = g(x)$  when  $x \in \{2, 5\}$

$$\text{iii) Assume } \sum_{i=1}^4 \sum_{j=2}^3 f(i)g(j) = A$$

We have  $g(2) = 1$

$$g(3) = 1$$

$$f(1) = 3$$

$$f(2) = 1$$

$$f(3) = 3$$

$$f(4) = 3$$

$$A = f(1) \left( \sum_{j=2}^3 g(j) \right) + f(2) \cdot \sum_{j=2}^3 g(j) + f(3) \cdot \sum_{j=2}^3 g(j) + f(4) \cdot \sum_{j=2}^3 g(j)$$

$$A = 3 \times (1+1) + 1 \times (1+1) + 3 \times (1+1) + 3 \times 2$$

$$= 6 + 6 + 6 + 2 = 14 + 6 = 20$$