hw1_sol Haoyuan Du

Q1.

a.

Q1. a. $f(x|\theta) = \frac{x^n}{\theta^2} \exp\left(-\frac{x^2}{2\theta^2}\right)$

$$\Rightarrow \prod_{i=1}^{n} f(x_i|\theta)$$

$$\hat{\theta}^{MLE} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{n} f(x_i|\theta)$$

$$\log: \Rightarrow \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \ln \frac{x_i^2}{\theta^2} + \frac{-x_i^2}{2\theta^2}$$

$$\frac{\partial f(x|\theta)}{\partial \theta} : \sum_{i=1}^{n} \frac{\theta^2}{x_i^2} \cdot (-2x_i^2 \cdot \theta^{-3}) + x_i^2 \theta^{-3} = 0$$

$$\Rightarrow \sum_{i=1}^{n} -2\theta^{-1} + x_i^2 \theta^{-3} = 0$$

$$2\theta^{-1} = \theta^{-3} \cdot \sum_{i=1}^{n} x_i^2$$

$$2\theta^2 = \sum_{i=1}^{n} x_i^2$$

$$\theta = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{2}}$$

b.

Q1. b. $f(x|\alpha,\theta) = \alpha\theta^{-\alpha}x^{\alpha-1}\exp\left(-\left(\frac{x}{\theta}\right)^{\alpha}\right)$

$$\Rightarrow \prod_{i=1}^{n} \alpha\theta^{-\alpha}x_i^{\alpha-1}\exp\left(-\left(\frac{x_i}{\theta}\right)^{\alpha}\right)$$

log: $$\Rightarrow \sum_{i=1}^{n} \ln\alpha\theta^{-\alpha}x_i^{\alpha-1} - \left(\frac{x_i}{\theta}\right)^{\alpha}$$

$$\Rightarrow \sum_{i=1}^{n} \ln\alpha - \alpha\ln\theta + (\alpha-1)\ln x_i - x_i^{\alpha}\cdot\theta^{-\alpha}$$

$$\Rightarrow n\ln\alpha - \alpha n\ln\theta + (\alpha-1)\sum_{i=1}^{n}\ln x_i - \theta^{-\alpha}\cdot\sum_{i=1}^{n}x_i^{\alpha}$$

$$\frac{\partial \log f(x)}{\partial\theta} = -\alpha n\cdot\frac{1}{\theta} + \alpha\cdot\theta^{-\alpha-1}\cdot\sum_{i=1}^{n}x_i^{\alpha} = 0$$

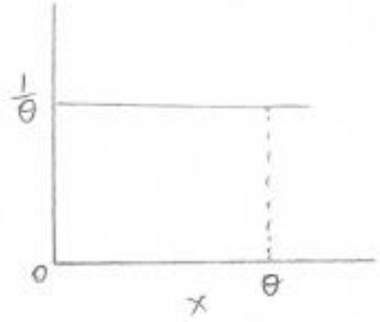
$$n\cdot\frac{1}{\theta} = \theta^{-\alpha-1}\cdot\sum_{i=1}^{n}x_i^{\alpha}$$

$$\frac{1}{\theta^{-\alpha}} = \frac{\sum_{i=1}^{n}x_i^{\alpha}}{n}$$

$$\theta = \sqrt[\alpha]{\frac{\sum_{i=1}^{n}x_i^{\alpha}}{n}}$$

c.

Q1. C



$f(x_i|\theta) = \frac{1}{\theta}$

$0 \le x \le \theta$

max this

$\therefore L(\theta|x_1,\dots,x_n) = \left[\frac{1}{\theta}\right]^n$

$Y_1 = \max(x_1,\cdots,x_n)$

$\therefore \hat{\theta} = \max(x_1,\dots,x_n).$

Q2.

a.

$P(C1|x=4) = P(x=4|C1)P(C1) = 0.5*P(x=4|C1) = 0.5*1/8 = 1/16$

$P(C2|x=4) = P(x=4|C2)P(C2) = 0.5*P(x=4|C2) = 0.5*1/9*(4-2) = 1/9$

$1/16 < 1/9$, thus, when x=4, the answer is C2.

b.

$P(C1|x=6) = P(x=6|C1)P(C1) = 0.7*P(x=6|C1) = 0.7*1/8 = 7/80$

$P(C2|x=6) = P(x=6|C2)P(C2) = 0.3*P(x=6|C2) = 0.3*1/9*(8-6) = 1/15$

$1/15 < 7/80$, thus, when x=6, the answer is C1.

c.

The optimal decision boundary is at the intercept of P(x|C1) and P(x|C2). So we get:

$1/9*(x-2)=1/8 \Rightarrow x = 25/8$

$1/9*(8-x)=1/8 \Rightarrow x = 55/8$

According to $\varphi(x) = (|x - 5|) - \alpha$, assume $x > 5$, then we have $x-5- \alpha=0$ ($x=55/8$). Thus,

$\alpha = 15/8$

As above, $\varphi(x) = (|x - 5|) - 15/8$. When $x > 55/8$ or $x < 25/8$, it will be classified as C1, otherwise C2. The results are as same as what we got from Q(1). So the probability of misclassification is 0.

Q3.
Assume S1 and S2 are learned from the data from each class.
discriminant function:

gi(x) = $-\frac{1}{2}log2\pi - logs_1 - \frac{(x-m1)^2}{2s1^2} + logP(C1)$

gi(x) = $-\frac{1}{2}log2\pi - logs_2 - \frac{(x-m2)^2}{2s2^2} + logP(C2)$

S1 and S2 are generated from class1 and class2 so that we must compare $-logs_i$.

Assume S1 = S2 (learned from the data from both classes).

gi(x) = $-\frac{(x-m1)^2}{2} + logP(C1)$

gi(x) = $-\frac{(x-m2)^2}{2} + logP(C2)$

Because S1=S2, so first two terms are canceled, but P(Ci) are still different (0.7 and 0.3). If C1=C2, then formula becomes gi(x) = $-\frac{(x-mi)^2}{2}$.

Confusion matrices are:
With class-dependent covariance:

	Actual	
Predict	TP: 23	FP: 7
	FN: 7	TN: 63

Type 1 error: 7, Type 2 error: 7. Explanation: We predict 7 records are positive, but it's false. We predict 7 records are negative, but it's false.
Accuracy = (23+63)/100 = 0.86
Precision = TP/(TP+FP) = 0.7667
Recall = TP/(TP+FN) = 0.7667

With class-independent covariance:

	Actual	
Predict	TP: 25	FP: 5
	FN: 5	TN: 65

Type 1 error: 5, Type 2 error: 5. Explanation: We predict 5 records are positive, but it's false. We predict 5 records are negative, but it's false.
Accuracy = (25+65)/100 = 0.90
Precision = TP/(TP+FP) = 0.8333
Recall = TP/(TP+FN) = 0.8333

Diagonal S:

	Actual	
Predict	TP: 26	FP: 14
	FN: 4	TN: 56

Type 1 error: 14, Type 2 error: 4. Explanation: We predict 14 records are positive, but it's false. We predict 4 records are negative, but it's false.
Accuracy = (26+56)/100 = 0.82
Precision = TP/(TP+FP) = 0.65
Recall = TP/(TP+FN) = 0.8667