Q1. a. $f(x|\theta) = \frac{x^n}{\theta^2} \exp\left(-\frac{x^2}{2\theta^2}\right)$

$$\Rightarrow \prod_{i=1}^{n} f(x_i|\theta)$$

$$\hat{\theta}^{MLE} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{n} f(x_i|\theta)$$

$$\log: \Rightarrow \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \ln \frac{x_i^2}{\theta^2} + \frac{-x_i^2}{2\theta^2}$$

$$\frac{\partial f(x|\theta)}{\partial \theta} : \sum_{i=1}^{n} \frac{\theta^2}{x_i^2} \cdot (-2x_i^2 \cdot \theta^{-3}) + x_i^2 \theta^{-3} = 0$$

$$\Rightarrow \sum_{i=1}^{n} -2\theta^{-1} + x_i^2 \theta^{-3} = 0$$

$$2\theta^{-1} = \theta^{-3} \cdot \sum_{i=1}^{n} x_i^2$$

$$2\theta^2 = \sum_{i=1}^{n} x_i^2$$

$$\theta = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{2}}$$

Q1. b. $f(x|\alpha,\theta) = \alpha\theta^{-\alpha}x^{\alpha-1}\exp\left(-\left(\frac{x}{\theta}\right)^{\alpha}\right)$

$$\Rightarrow \prod_{i=1}^{n} \alpha\theta^{-\alpha}x_i^{\alpha-1}\exp\left(-\left(\frac{x_i}{\theta}\right)^{\alpha}\right)$$

log: $$\Rightarrow \sum_{i=1}^{n} \ln \alpha\theta^{-\alpha} x_i^{\alpha-1} - \left(\frac{x_i}{\theta}\right)^{\alpha}$$

$$\Rightarrow \sum_{i=1}^{n} \ln\alpha - \alpha\ln\theta + (\alpha-1)\ln x_i - x_i^{\alpha}\cdot\theta^{-\alpha}$$

$$\Rightarrow n\ln\alpha - \alpha n\ln\theta + (\alpha-1)\sum_{i=1}^{n}\ln x_i - \theta^{-\alpha}\cdot\sum_{i=1}^{n} x_i^{\alpha}$$

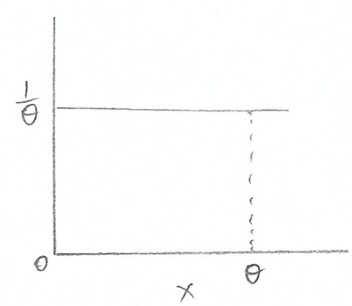
$$\frac{\partial \log f(x)}{\partial\theta} = -\alpha n\cdot\frac{1}{\theta} + \alpha\cdot\theta^{-\alpha-1}\cdot\sum_{i=1}^{n} x_i^{\alpha} = 0$$

$$n\cdot\frac{1}{\theta} = \theta^{-\alpha-1}\cdot\sum_{i=1}^{n} x_i^{\alpha}$$

$$\frac{1}{\theta^{-\alpha}} = \frac{\sum_{i=1}^{n} x_i^{\alpha}}{n}$$

$$\theta = \sqrt[\alpha]{\frac{\sum_{i=1}^{n} x_i^{\alpha}}{n}}$$

Q1. C



$f(x_i|\theta) = \frac{1}{\theta}$

$0 \le x \le \theta$

max this

$\therefore L(\theta|x_1,\ldots,x_n) = \left[\frac{1}{\theta}\right]^n$

$Y_1 = \max(x_1,\cdots,x_n)$

$\therefore \hat{\theta} = \max(x_1,\ldots,x_n).$