Lecture Notes

Non Life Insurance

First Draft

Prof. Dr. Ricardo Gatto

Switzerland-Spain-Ecuador

Índice

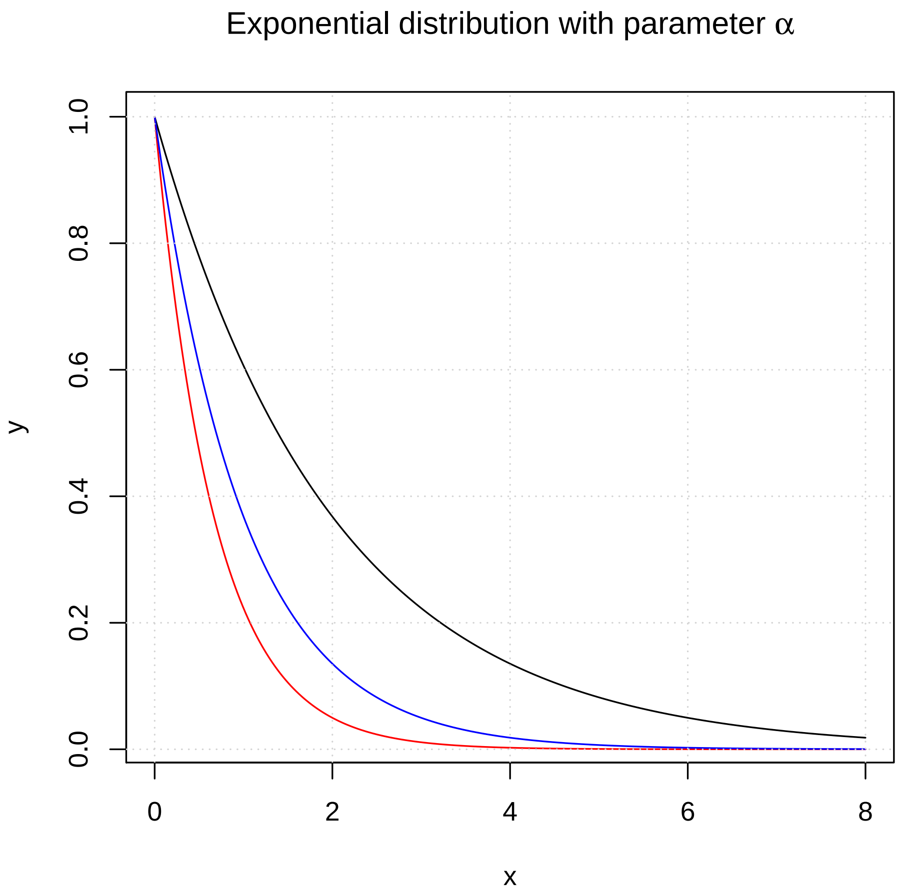
1. Individual Risk and Distributions

A non negative random variable is called a **loss** and it its distribution a **loss distribution**.
$X \sim Exponential(\alpha)$ means that X has density $f_X(x) = \alpha e^{-\alpha x}$ and distribution function (d.f) $F_X(x) = 1 - e^{-\alpha x}$ $\forall x > 0$ and $\alpha > 0$.

Let $Y = e^x$,

$$\begin{aligned} F_Y(Y) &= F_X(logY) \\ &= 1 - e^{\alpha log(y)} \\ &= 1 - y^{-\alpha} \end{aligned}$$

Is called the **Pareto Distribution**. If Y follows a Pareto distribution, denoted $Y \sim Pareto(\alpha)$



$X \sim Exponential(\lambda)$ and $Y \sim X^{\frac{1}{\tau}}$ $\forall \tau > 0$

$$\begin{aligned} F_Y(Y) &= F_X(Y^\tau) \\ &= 1 - e^{-\lambda y^\tau} \qquad \forall y > 0 \end{aligned}$$

Y follows the **Weibull distribution**, τ is called the Weibull index.
It is denoted by $Y \sim Weibull(\tau, \lambda)$

2. Thursday 09/03/17

2.1. Distribution of the largest claim amount

The distribution of the largest loss is very important in **risk management**.
We will derive asymptotic aproximation of standardized maxima.
Let $X_1, ..., X_n$ be independent losses with distribution function (d.f) F and define

$$M_n = max\{X_1, ..., X_n\}$$

$$\begin{aligned} P[M_n \leq n] &= P[X_1, .., X_n \leq x] \\ &= F^n(x) \quad \forall x > 0 \end{aligned}$$

Let $\bar{x} = sup\{x > 0 | F(x) < 1\}$.
Assume $E[M_n] < \infty$, then $E[M_n] = \int_0^{\bar{x}} \{1 - F^n(x)\}dx \xrightarrow{n\to\infty} \bar{x}$.
Assume $E[M_n^2] < \infty$, then $E[M_n^2] = \int_0^{\bar{x}} x\{1 - F^n(x)\}dx \xrightarrow{n\to\infty} \bar{x}^2$
$Var(M_n) = E[M_n^2] - E^2[M_n] \xrightarrow{n\to\infty} \bar{x}^2 - \bar{x}^2 = 0$, assuming $\bar{x} = 0$.
Thus the asymptotic distribution of M_n is degenerate (the total mass is over $\bar{x}$). SO if we want to compute this asymptotic distribution, we must consider the standardization $\frac{M_n - b_n}{a_n}$.
Before studying these asymptotic approximation we give some examples with finite sample.

2.2. Examples

The distribution of the monthly largest loss is Gumbel $F(x) = G(\frac{x-\mu}{\sigma})$ where $G(x) = exp\{-e^{-x}\}$ $x \in \mathbb{R}$, what is the distribution of the annual maximum?

$$\begin{aligned} F^12 &= exp\{-12e^{-\frac{x-\mu}{\sigma}}\} \\ &= exp\{-e^{-\frac{x-\mu}{\sigma} + log12}\} \\ &= exp\{-e^{-\frac{x-(\mu+\sigma log12)}{\sigma}}\} \end{aligned}$$

It is thus agian GUmbel, with another location parameter with Frechet monthly largest loss, viz. with $G(x) = exp\{-x^{-\alpha}\}$ $x > 0$, we have $F^{12}(x) = exp\{-12\frac{x-\mu}{\sigma}^{-\alpha}\} = exp\{-(\frac{x-\mu}{12^{\frac{1}{\alpha}}\sigma})^{-\alpha}\}$
It is again Fréchet with another scale parameter. Because of this algebraic closure property, the Gumbel and the Frechet distributions are called max-stable. We consider the slight generalization where the sample size is the random variable N.
Let $M_N = max\{X_1, ..., X_N\}$. Assume N independent of $X_1, X_2, ...$

$$\begin{aligned} P[M_N \leq x] &= \sum_{n=0}^{\infty} P[M_N \leq x | N = n]P[N = n] \\ &= \sum_{n=0}^{\infty} F^n(x)P[N = n] \\ &= G_N(F(x)) \quad \forall x \geq 0 \end{aligned}$$

Where $M_0 = 0$ and $G_N(v) = \sum_{n=0}^{\infty} v^n P[N = n]$ is the generating function of N.
Thus $P[M_N \leq 0]$ if $F(0) = 0$