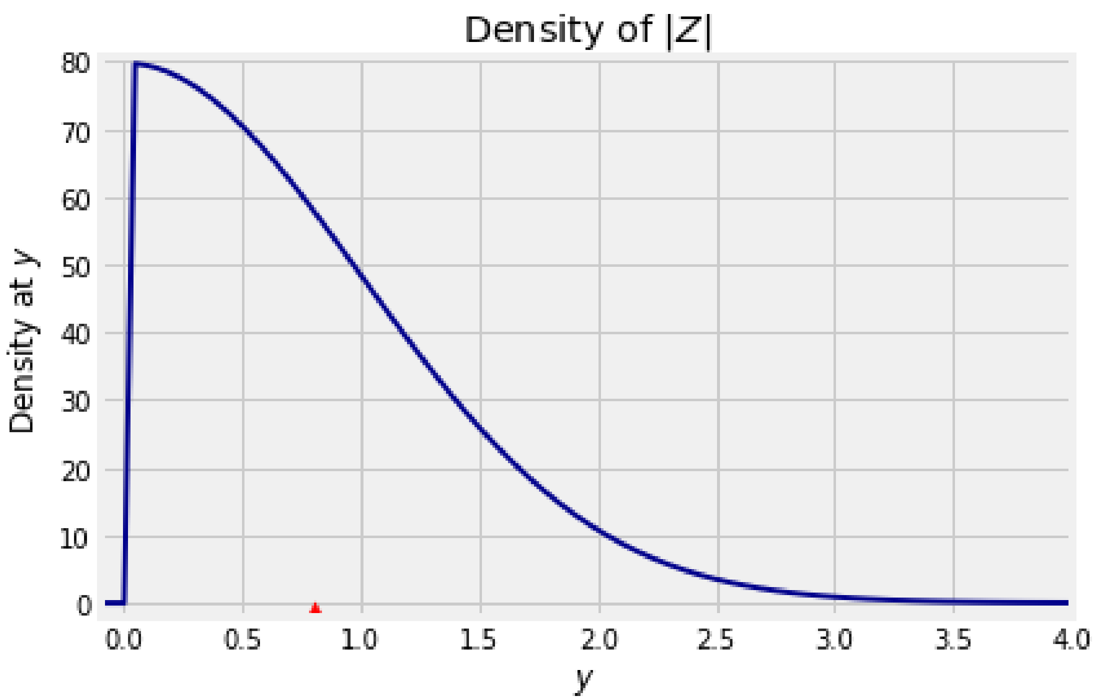
Notebook

April 1, 2019

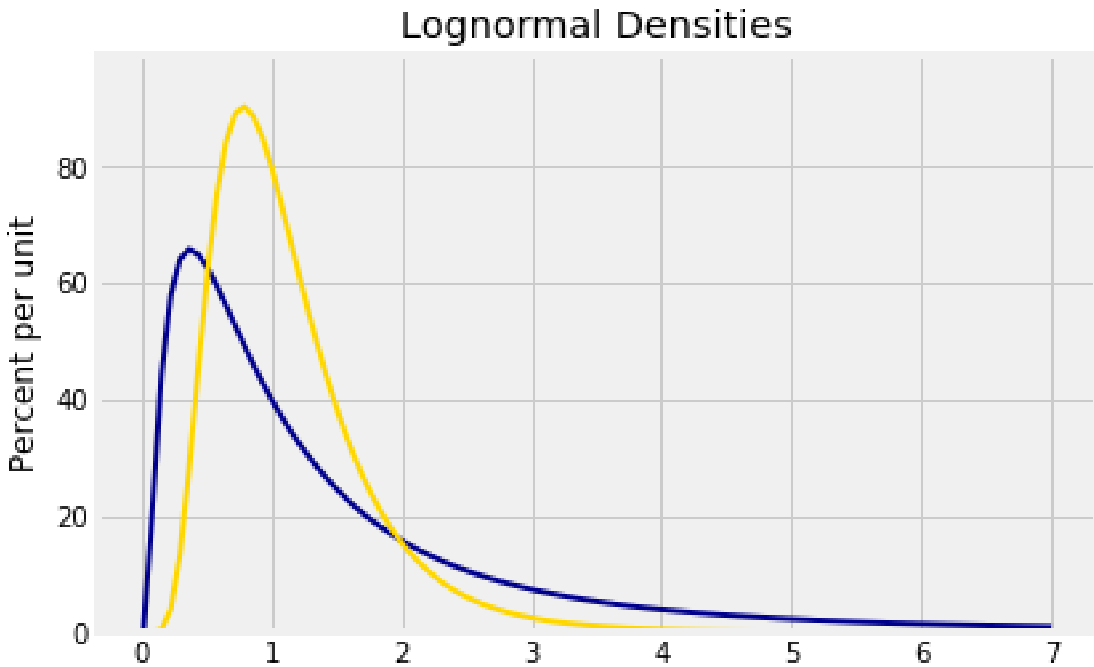
Local date & time is : 04/01/2019 11:44:06 PDT

In [ ]:

In [38]:
```
# answer to 1c

def abs_norm_density(y):
    if y > 0:
        return 2*1/(2*np.pi)**0.5*np.exp(-0.5*y**2)
    else:
        return 0

Plot_continuous([-0.5, 4], abs_norm_density)

ev = 2 / (2*np.pi)**0.5
plt.scatter(ev, -0.01, marker = '^', color='red', s=50)

# Labels and axes; do not edit below this line
plt.xlabel('$y$')
plt.ylabel('Density at $y$')
plt.title('Density of $|Z|$')
plt.xlim(-0.1, 4)
plt.ylim(-0.02, 0.81);
```

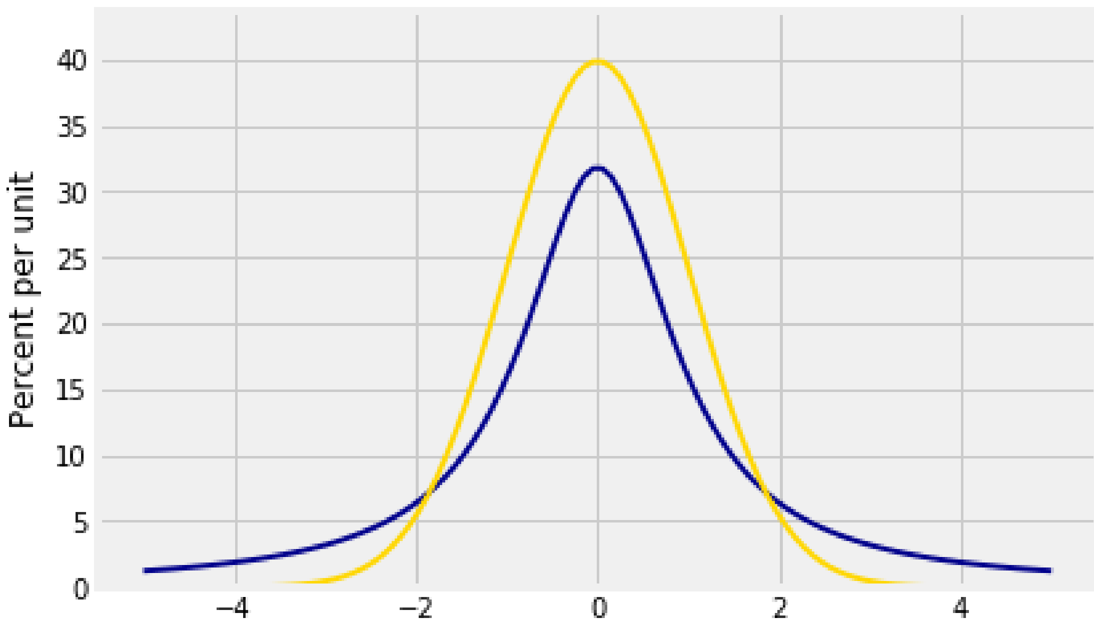


In [39]:
```
# answer to 1e

# Density of e^Z where Z is standard normal
def lognorm_density_1(y):
    return 1/y * stats.norm.pdf(np.log(y), 0, 1)


# Density of e^X where X is normal with mean 0 and SD 0.5
def lognorm_density_05(y):
    return 1/y * stats.norm.pdf(np.log(y), 0, 0.5)

Plot_continuous([0.01, 7], lognorm_density_1)
Plot_continuous([0.01, 7], lognorm_density_05, color='gold')
plt.title('Lognormal Densities');
```

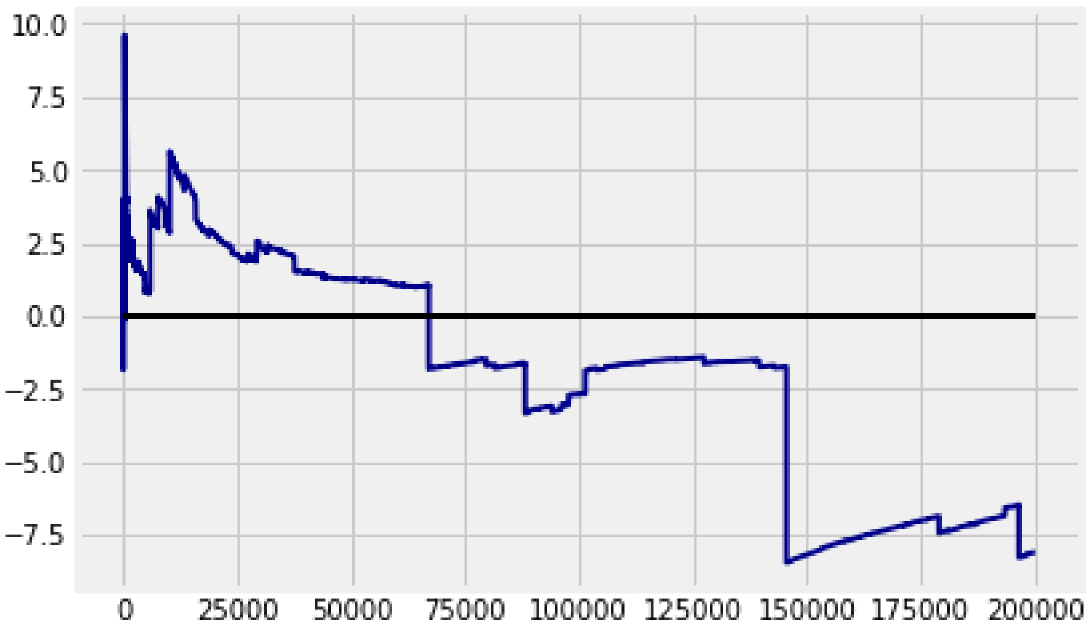

Lognormal Densities
Percent per unit
0
20
40
60
80
0
1
2
3
4
5
6
7

1 newpage

In [40]:
```
# answer to 2b

# Plotting interval on the horizontal axis: -5 to 5
# The first plot should be the Cauchy.
# The gold plot should be the standard normal curve.

Plot_continuous([-5, 5], stats.cauchy.pdf)
Plot_continuous([-5, 5], stats.norm.pdf, color='gold')
```



In [41]:
```
#2d: run this
N = 200000
n = np.arange(1, N+1)
x = stats.cauchy.rvs(size = N)
y = np.cumsum(x)/n
plt.plot(n, y, color='darkblue', lw=2)
plt.plot([0, N], [0, 0], color='k', lw=2);
```


10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0
25000
50000
75000
100000
125000
150000
175000
200000

2 newpage

3 newpage

4 newpage

5 newpage

6 newpage

In [46]:
```
# answer to 7b

h = (y**2 - x**2)*exp(-y)
c = 1 / integrate(h, (x, -y, y), (y, 0, oo))
c
```

Out[46]:

$$\frac{1}{8}$$

In [48]:
```
# answer to 7c

f = c*h
Integral(f, (x, -y, y), (y, 0, oo)).doit()
```

Out[48]:

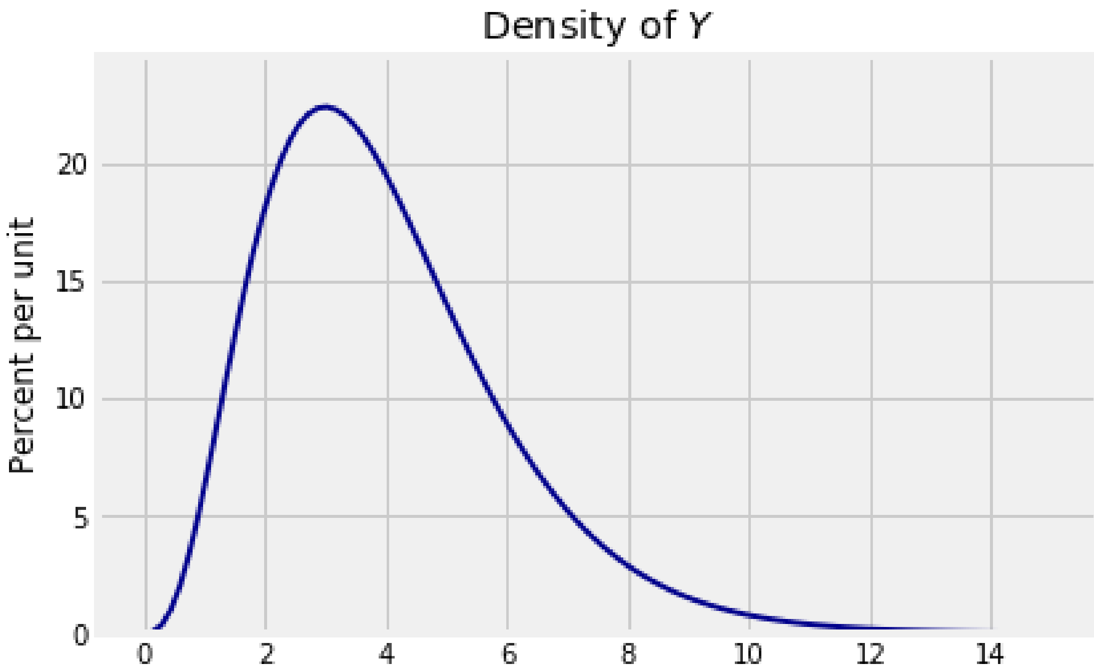
$$1$$

In [51]:
```
# answer to 7e

f_Y = Integral(f, (x, -y, y)).doit()
f_Y
```

Out[51]:

$$\frac{y^3}{6}e^{-y}$$

In [53]:
```
# 7f

"""Y has the gamma (4, 1) distribution.
By an earlier problem in this Homework, E(Y) = 4"""

Plot_continuous([0, 15], f_Y)
plt.xticks(np.arange(0, 15, 2))
plt.title('Density of $Y$');
```


Density of Y
Percent per unit
0
5
10
15
20
0
2
4
6
8
10
12
14

7 newpage

In [54]:
```
# answer to 8a

# Conditional density of X given Y=y for fixed y > 0:

"""Given Y=y, the possible values of X are -y to y"""

f_X_given_Y_is_y = f / f_Y

f_X_given_Y_is_y
```

Out[54]:

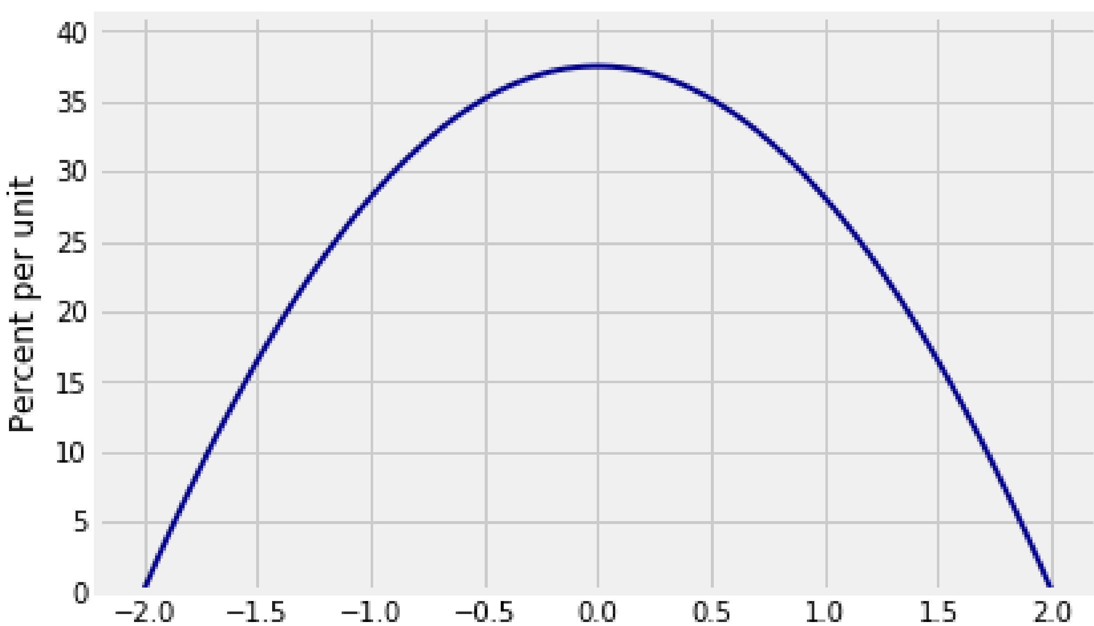
$$\frac{3}{4y^3}\left(-x^2+y^2\right)$$

In [56]:
```
# answer to 8b

Integral(f_X_given_Y_is_y, (x, -y, y)).doit()
```

Out[56]:

$$1$$

In [57]:
```
# answer to 8c

f_X_given_Y_is_2 = f_X_given_Y_is_y.subs(y, 2)
Plot_continuous([-2, 2], f_X_given_Y_is_2)
```



In [58]:
```python
# answer to 8g

# E(|X| | Y=y)

expectation_absX_given_Y_is_y = 2 * Integral(x*f_X_given_Y_is_y, (x, 0, y)).doit()
expectation_absX_given_Y_is_y
```

Out[58]:

$$\frac{3y}{8}$$