

# Notebook

April 12, 2019

Local date & time is : 04/12/2019 15:14:17 PDT



## 1 newpage

In [25]: *#each interval is poisson distribution with  $\mu = 40$ . find  $P(X \geq 30)$  for each interval and mul*

```
(1-stats.poisson.cdf(29, 40))**2
```

Out[25]: 0.9154113546571047



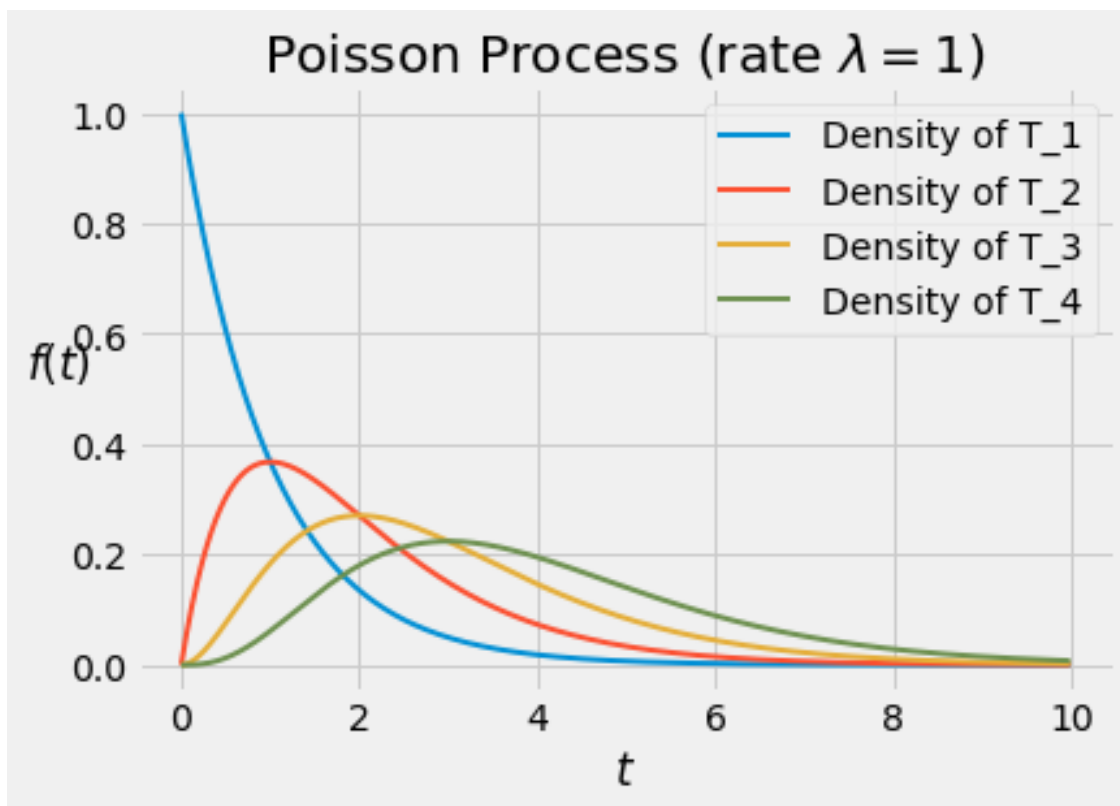
## 2 newpage

Distribution of  $G_2$  is same as distribution of  $T_1$  which is exponential distribution with parameter  $\lambda$ .

It is because each  $T_i$  is independent from the other. So the waiting time for  $T_2$  from  $T_1$  is same distribution as  $T_1$ .

It is an exponential distribution with parameter  $n \cdot \lambda$

```
In [30]: t = np.arange(0, 10, 0.01)
         for r in np.arange(1, 5):
             f = stats.gamma.pdf(t, r, scale = 1)
             plt.plot(t, f, lw=2, label='Density of T_'+str(r)) # plot the density of T_r
         plt.xlabel('$t$')
         plt.ylabel('$f(t)$', rotation=0)
         plt.legend()
         plt.title('Poisson Process (rate $\lambda = 1$)');
```



```
In [32]: # lambda = 1, t = 15/2
         # P(T10 > t)
         stats.poisson.cdf(9, 7.5)
```

```
Out[32]: 0.7764076130197146
```

```
In [33]: #consider second call as origin
         #r = 3, lambda = 1, t = 7/2
         #P(Tr < 7/2)
         1- stats.poisson.cdf(2, 3.5)
```

```
Out[33]: 0.6791528011378658
```



### 3 newpage

For each  $t > 0$ , the total number of points  $N_{(0,t)}$  has the Poisson  $\lambda t$  distribution, and the number of blue points  $B_{(0,t)}$  has the Poisson  $p\lambda t$  distribution.

We know from above that  $N(0,t)$  has poisson  $\lambda t$  distribution. And considering blue points as the number of successes in Bernoulli trials, number of blue points will have Poisson  $p\lambda t$  distribution.

Yes. Following from our argument part 1, that disjoint time intervals are independent for the total number of arrivals, since blue points are also part of the total arrivals, blue points will also be independent if intervals are disjoint

The blue process is a Poisson process with rate  $p\lambda$  per unit time.

$R_{(0,t)}$  has the poisson  $(1-p)\lambda t$  distribution.

They are independent. The for poissonization of binomial, the distribution of success and failure are independent

The blue process and the red process are independent Poisson processes. The blue process has rate  $p\lambda$  and the red process has rate  $(1-p)\lambda$ .

```
In [35]: #lambda = 3, t = 5
         #Number of cell phone = poisson distribution with 3*5*0.05
         #Number of other items = poisson distribution with 3*5*0.95
         #P(cell >= 1)*p(other <= 10)
         (1- stats.poisson.pmf(0, 3*5*0.05)) * (stats.poisson.cdf(10, 3*5*0.95))

Out[35]: 0.08426146935195485

In [59]: #P(other = 15)*P(cell = 1)
         0.95**15*0.05

Out[59]: 0.023164561507987652

In [62]: """Among the first 22 items that arrive, there should be less than 3 cell phones."""
         stats.binom.cdf(2, 22, 0.05)

Out[62]: 0.905176954081569
```





## 4 newpage

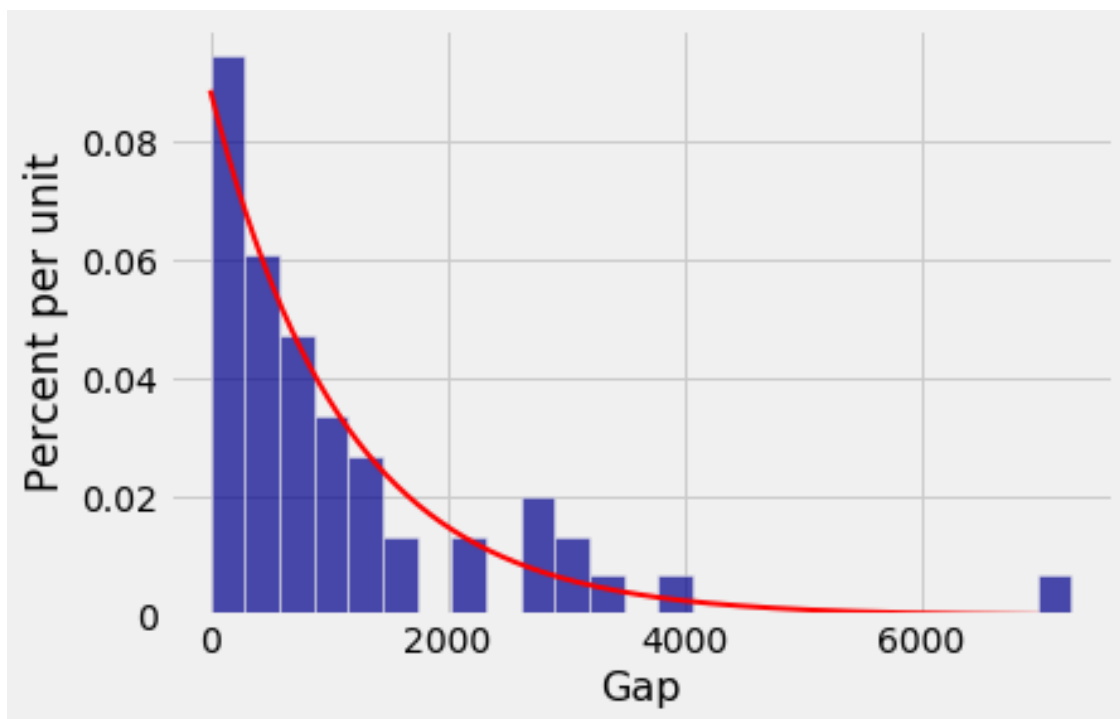
```
In [66]: gaps = quakes.column('Gap')
         mean_gap = np.mean(gaps)
```

```
         lam = 1 / np.mean(gaps)
```

```
         mean_gap, lam
```

```
Out[66]: (1128.8235294117646, 0.0008858780614903596)
```

```
In [67]: quakes.hist('Gap', bins=25)
         t = np.arange(7000)
         f = stats.expon.pdf(t, scale = 1/lam)
         plt.plot(t, f, color='red', lw=2);
```



```
In [68]: # P(at least one quake within a year from now)
         # t = 365, lambda = lam
         1-stats.poisson.pmf(0, 365*lam)
```

```
Out[68]: 0.2762762299248045
```

```
In [69]: # number of quakes with magnitude at least 6.0
         num_big = sum(quakes.column('Magnitude') > 6)

         # proportion of quakes with magnitude at least 6.0
         prop_big = num_big / 51

         num_big, prop_big
```

Out[69]: (35, 0.6862745098039216)

```
In [76]: # P(1 quake of magnitude at least 6.0 and at least one quake of magnitude in [4.9, 6.0))
big = prop_big*365*lam
small = (1-prop_big)*365*lam

stats.poisson.pmf(1, big)*(1-stats.poisson.pmf(0, small))
```

Out[76]: 0.017146209255629666

```
In [81]: # number of years n such that
# P(at least quake of magnitude 6.0 will happen in n years) = 0.99
# want 1 - stats.poisson.pmf(0, 365*n*lam*prop_big) = 0.99
# stats.poisson.pmf(0, 365*n*lam*prop_big) = e^(-365*n*lam*prop_big) = 0.01
n = np.log(0.01) / (-365*lam*prop_big)
n
```

Out[81]: 20.75300568354868