Arithmetic convolutions

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I will survey certain properties of the following arithmetic convolutions. Arithmetic and asymptotic properties of related arithmetical functions will also be considered.

- Dirichlet convolution:

$$(f * g)(n) = \sum_{d|n} f(d)g(n/d), \tag{1}$$

where the sum is over the positive divisors d of n.

- Binomial convolution:

$$(f \circ g)(n) = \sum_{d|n} \left(\prod_p \binom{\nu_p(n)}{\nu_p(d)} \right) f(d)g(n/d), \tag{2}$$

where $n = \prod_p p^{\nu_p(n)}$, $d = \prod_p p^{\nu_p(d)}$ and $\binom{a}{b}$ is the binomial coefficient.

- Unitary convolution:

$$(f \times g)(n) = \sum_{d||n} f(d)g(n/d), \tag{3}$$

where the sum is over the unitary divisors d of n, i. e., positive divisors d of n such that $(d, n/d) = 1$.

- A-convolution:

$$(f *_A g)(n) = \sum_{d \in A(n)} f(d)g(n/d), \tag{4}$$

where A is a mapping from the set $\mathbb{N} := \{1, 2, \ldots\}$ to the set of subsets of $\mathbb{N}$ such that $A(n) \subseteq D(n)$ for each n, $D(n)$ denoting the set of all (positive) divisors of n.

- Exponential convolution:

$$(f \odot g)(n) = \sum_{b_1 c_1 = a_1} \cdots \sum_{b_r c_r = a_r} f(p_1^{b_1} \cdots p_r^{b_r}) g(p_1^{c_1} \cdots p_r^{c_r}), \tag{5}$$

where $n = p_1^{a_1} \cdots p_r^{a_r}$.